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Weakly symmetric and weakly ricci–symmetric conditions on $(LCS)_n-$manifolds

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In this paper, we have researched the necessary and sufficient conditions for a $(LCS)_n-$manifold to be weakly symmetric and weakly Ricci–symmetric and examined the conditions over 1–forms involved in the definitions of weakly symmetric and weakly Ricci–symmetric conditions. We have evaluated some certain results of situations in which a weakly symmetric $(LCS)_n-$manifold is with $\eta-$parallel or cyclic Ricci tensor. Finally, an example is used to demonstrate that the method presented in this paper is effective.

Key words: (LCS)manifold, Weakly Symmetric, Weakly Ricci Symmetric.

INTRODUCTION

In 1989, Tamassy and Binh (1989, 1993) introduced the notion of weakly symmetric and weakly Ricci–symmetric Riemannian manifolds; studied such structures on Sasakian manifolds and proved that such a structure does not always exist. Weakly symmetric and Ricci–symmetric structures were also studied by Shaikh and Jana (2006, 2007a, b).

Recently, Shaikh (2003) introduced the notion of Lorentzian noncircular structure manifolds (briefly $(LCS)_n-$manifolds) with an example, which generalizes the notion of *LP*–Sasakian manifolds introduced by Matsumoto (1989).

A non–flat Riemannian manifold $(M^n,g)\,(n>2)$ is called a weakly symmetric manifold if its curvature tensor R of type $(0,4)$ satisfies the condition:

$$(\nabla_X R)(Y,Z,U,V)=\tau(X)R(Y,Z,U,V)+\beta(Y)R(X,Z,U,V)+\gamma(Z)R(Y,X,U,V)$$
$$+\delta(U)R(Y,Z,X,V)+\sigma(V)R(Y,Z,U,X) \tag{1}$$

for all vectors fields $X,Y,Z,U,V\in\chi(M)$, where τ,β,γ,δ and σ are 1–forms (non-zero simultaneously) and ∇ is the operator of covariant differentiation with respect to the Riemannian metric g.

A weakly symmetric manifold is said to be proper if $\tau=\beta=\gamma=\delta=\sigma=0$ is not the case. A Riemannian manifold is called weakly Ricci–symmetric if there is exist 1–forms λ,μ,ν such that

$$(\nabla_X S)(Y,Z)=\lambda(X)S(Y,Z)+\mu(Y)S(X,Z)+\nu(Z)S(X,Y) \tag{2}$$

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for vector fields $X,Y,Z \in (M^n, g)$, where S is the Ricci tensor of type $(0,2)$ of manifold (M^n, g). A weakly Ricci–symmetric manifold is said to be proper if $\lambda = \mu = \nu = 0$ is not the case.

Recently, De and Bandyopadhyay (1999) proved that in a weakly symmetric manifolds the associated 1–forms $\beta = \gamma$ and $\delta = \sigma$. Hence Equation (1) reduces to the following form:

$$(\nabla_X R)(Y,Z,U,V) = \tau(X)R(Y,Z,U,V) + \beta(Y)R(X,Z,U,V) + \beta(Z)R(Y,X,U,V) + \delta(U)R(Y,Z,X,V) + \delta(V)R(Y,Z,U,X). \tag{3}$$

Weakly symmetric manifolds and concircular structures have been studied by various authors.

For example Shaikh and Baishya, (2005, 2006); Shaikh et al. (2007, 2008); Shaikh and Jana (2007) and Shaikh, 2009).

PRELIMINARIES

An $n-$dimensional Lorentzian manifold M is a smooth connected para–compact hausdorff manifold with a Lorentzian metric g, that is, M admits a smooth symmetric tensor field g of type $(0,2)$ such that for each point $p \in M$, the tensor $g_p : T_pM \times T_pM \to R$ is a non–degenerate inner product of signature $(-,+,...,+)$, where T_pM denotes the tangent vector space of M at p. A non–zero vector $\upsilon \in T_pM$ is said to be time - like (resp., non–space - like, null, space - like) if it satisfies $g_p(\upsilon,\upsilon) < 0$ (resp., $\leq 0, = 0, > 0$) (O'Neill, 1983).

Let M^n be a Lorentzian manifold admitting a unit time–like concircular vector field ξ, called the characteristic vector field of the manifold. Then we have:

$$g(\xi,\xi) = -1 \tag{4}$$

And

$$(\nabla_X \eta)(Y) = \alpha\left[g(X,Y) + \eta(X)\eta(Y)\right], \ (\alpha \neq 0) \tag{5}$$

(Shaikh, 2003) for all vector fields X,Y where ∇ denotes the operator of covariant differentiation with respect to the Lorentzian metric g, η is 1–form associated to

$$\xi \ g(X,\xi) = \eta(X) \tag{6}$$

and α is non–zero scalar function satisfies

$$X\alpha = \rho\eta(X) \tag{7}$$

where ρ being a certain scalar function given by

$$\rho = -(\xi\alpha). \tag{8}$$

If we put

$$\phi X = \frac{1}{\alpha}\nabla_X \xi, \tag{9}$$

then from Equations (6) and (9) we have

$$\phi X = X + \eta(X)\xi. \tag{10}$$

Applying ϕ to Equation (10), we conclude that

$$\phi^2 X = X + \eta(X)\xi \tag{11}$$

from which that ϕ is a symmetric (1,1) tensor. Thus Lorentzian manifold M^n together with the unit time–like concircular vector field ξ, its associated 1–form η and (1,1) tensor field ϕ is said to be a Lorentzian concircular structure manifold (briefly $(LCS)_n -$manifold) (Shaikh, 2003). Especially, if we take $\alpha = 1$, we can obtain the $LP-$Sasakian structure of Matsumoto (1989).

In a $(LCS)_n -$manifold, the following relations hold (Shaikh, 2003):

$$\eta(\xi) = -1, \quad \phi\xi = 0, \quad \eta(\phi X) = 0 \tag{12}$$

$$g(\phi X, \phi Y) = g(X,Y) + \eta(X)\eta(Y) \tag{13}$$

$$\eta(R(X,Y)Z) = (\alpha^2 - \rho)\left[g(Y,Z)\eta(X) - g(X,Z)\eta(X)\right] \tag{14}$$

$$S(X,\xi) = (n-1)(\alpha^2 - \rho)\eta(X) \tag{15}$$

$$R(X,Y)\xi = (\alpha^2 - \rho)\left[\eta(Y)X - \eta(X)Y\right] \tag{16}$$

$$(\nabla_X \phi)(Y) = \alpha\left\{g(X,Y)\xi + 2\eta(X)\eta(Y)\xi + \eta(Y)X\right\} \tag{17}$$

for any vector fields $X,Y,Z,$ where R,S denote respectively the curvature tensor and Ricci tensor of the manifold.

Lemma 1

In a $(LCS)_n-$ manifold $M,$ the following relation holds:

$$X(\rho)=-\xi(\rho)\eta(X) \tag{18}$$

for any vector field $X\in\chi(M)$ (Shaikh and Binh, 2009).

Definition 1

The Ricci tensor of weakly Ricci–symmetric $(LCS)_n-$ manifolds is called $\eta-$parallel if it satisfies

$$(\nabla_X S)(\phi Y,\phi Z)=0$$

for all vector fields $X,Y,Z\in\chi(M)$.

Weakly symmetric and weakly ricci–symmetric $(LCS)_n-$ manifolds with $(\nabla_\xi S)(Z,U)=0$

Theorem 1

If an $(LCS)_n-$ manifold M is weakly symmetric or weakly Ricci–symmetric under the condition of $\alpha^2-\rho\neq0$, then the sum of $\tau+\beta+\delta$ of 1- forms on ξ vector field is equal to the sum $\lambda+\mu+\nu$ of 1–forms on the same vector.

Proof

Let $\{e_i\},1\le i\le n,$ be an orthonormal basis of the tangent space at a point of the $(LCS)_n-$ manifold $M.$ Choosing $Y=V=e_i$ in Equation (3), $1\le i\le n,$ then we obtain:

$$\begin{aligned}(\nabla_X R)(e_i,Z,U,e_i)&=\tau(X)R(e_i,Z,U,e_i)+\beta(e_i)R(X,Z,U,e_i)\\&+\beta(Z)R(e_i,X,U,e_i)+\delta(U)R(e_i,Z,X,e_i)\\&+\delta(e_i)R(e_i,Z,U,X),\end{aligned}$$

Where

$$\beta(e_i)R(X,Z,U,e_i)=\beta(R(X,Z)U)$$

And

$$\begin{aligned}\delta(e_i)R(e_i,Z,U,X)&=-\delta(e_i)R(e_i,Z,X,U)=\delta(e_i)R(Z,e_i,X,U)\\&=\delta(e_i)R(X,U,Z,e_i)=\delta(R(X,U)Z).\end{aligned}$$

From the definition from Ricci tensor, we can easily see that

$$\begin{aligned}(\nabla_X S)(Z,U)&=\tau(X)S(Z,U)+\beta(Z)S(X,U)+\delta(U)S(Z,X)\\&+\beta(R(X,Z)U)+\delta(R(X,U)Z).\end{aligned} \tag{19}$$

Let $X=\xi$ be in Equation (19), then

$$\begin{aligned}(\nabla_\xi S)(Z,U)&=\tau(\xi)S(Z,U)+\beta(Z)S(\xi,U)+\delta(U)S(Z,\xi)\\&+\beta(R(\xi,Z)U)+\delta(R(\xi,U)Z).\end{aligned}$$

In view of Equation (15), we have

$$\begin{aligned}(\nabla_\xi S)(Z,U)&=\tau(\xi)S(Z,U)+(n-1)(\alpha^2-\rho)[\beta(Z)\eta(U)+\delta(U)\eta(Z)]\\&+\beta(R(\xi,Z)U)+\delta(R(\xi,U)Z).\end{aligned} \tag{20}$$

Let $X=\xi$ be in Equation (2), then we have:

$$(\nabla_\xi S)(Z,U)=\lambda(\xi)S(Z,U)+\mu(Z)S(\xi,U)+\nu(U)S(\xi,Z).$$

Again, from Equation (15), we obtain:

$$(\nabla_\xi S)(Z,U)=\lambda(\xi)S(Z,U)+(n-1)(\alpha^2-\rho)[\mu(Z)\eta(U)+\nu(U)\eta(Z)] \tag{21}$$

Since $(\nabla_\xi S)(Z,U)=0$ and making use of Equations (20) and (21), we have:

$$\begin{aligned}&\lambda(\xi)S(\xi,\xi)+(n-1)(\alpha^2-\rho)\{\mu(\xi)\eta(\xi)+\nu(\xi)\eta(\xi)\}=\tau(\xi)S(\xi,\xi)\\&+(n-1)(\alpha^2-\rho)\{\beta(\xi)\eta(\xi)+\delta(\xi)\eta(\xi)\}+\delta(R(\xi,\xi)\xi)+\beta(R(\xi,\xi)\xi)\end{aligned}$$

Whence

$$(n-1)(\alpha^2-\rho)\{\tau(\xi)+\beta(\xi)+\delta(\xi)\}=(n-1)(\alpha^2-\rho)\{\lambda(\xi)+\mu(\xi)+\nu(\xi)\},$$

which proves our assertion.

Theorem 2

In weakly Ricci–symmetric $(LCS)_n-$manifold $M,$ the following relation holds:

$$\lambda(\xi)+\mu(\xi)+\nu(\xi)=-\frac{1}{\alpha^2-\rho}\left(\xi(\rho)+2\alpha\rho\right).$$

Proof

Let $X=Y=Z=\xi$ be in Equation (2). Then we have:

$$\left(\nabla_\xi S\right)(\xi,\xi)=\lambda(\xi)S(\xi,\xi)+\mu(\xi)S(\xi,\xi)+\nu(\xi)S(\xi,\xi).$$

Here,

$$\nabla_\xi S(\xi,\xi)-S\left(\nabla_\xi\xi,\xi\right)-S\left(\xi,\nabla_\xi\xi\right)=S(\xi,\xi)\left[\lambda(\xi)+\mu(\xi)+\nu(\xi)\right].$$

From Equations (9), (12) and (15), we obtain:

$$\xi\left[(1-n)\left(\alpha^2-\rho\right)\right]=\left[\lambda(\xi)+\mu(\xi)+\nu(\xi)\right]\left[(1-n)\left(\alpha^2-\rho\right)\right],$$

which leads to

$$(1-n)\left[2\alpha\xi(\alpha)-\xi(\rho)\right]=\left[(1-n)\left(\alpha^2-\rho\right)\right]\left[\lambda(\xi)+\mu(\xi)+\nu(\xi)\right].$$

Also, taking account of Equation (8), we conclude:

$$\lambda(\xi)+\mu(\xi)+\nu(\xi)=-\frac{1}{\alpha^2-\rho}\left(\xi(\rho)+2\alpha\rho\right)$$

which proves our assertion.

$(LCS)_n-$ MANIFOLDS WITH CYCLIC RICCI TENSOR

Theorem 3

If the sum of 1–forms τ,β and δ are identically zero on a $(LCS)_n-$manifold $M,$ then $(LCS)_n-$manifold M is cyclic Ricci–symmetric.

Proof

From Equation (3) we have:

$$\begin{aligned}(\nabla_X S)(Y,Z)&=\tau(X)S(Y,Z)+\beta(Y)S(X,Z)+\delta(Z)S(Y,X)+\beta\left(R(X,Y)Z\right)\\&\quad+\delta\left(R(X,Z)Y\right),\end{aligned}\tag{22}$$

$$\begin{aligned}(\nabla_Y S)(Z,X)&=\tau(Y)S(Z,X)+\beta(Z)S(Y,X)+\delta(X)S(Z,Y)+\beta\left(R(Y,Z)X\right)\\&\quad+\delta\left(R(Y,X)Z\right),\end{aligned}\tag{23}$$

and

$$\begin{aligned}(\nabla_Z S)(X,Y)&=\tau(Z)S(X,Y)+\beta(X)S(Z,Y)+\delta(Y)S(X,Z)+\beta\left(R(Z,X)Y\right)\\&\quad+\delta\left(R(Z,Y)X\right).\end{aligned}\tag{24}$$

Again, taking into account of Equations (22), (23) and (24) we get:

$$\begin{aligned}(\nabla_X S)(Y,Z)+(\nabla_Y S)(Z,X)+(\nabla_Z S)(X,Y)&=S(Y,Z)\left[\tau(X)+\beta(X)+\delta(X)\right]\\&+S(Z,X)\left[\tau(Y)+\beta(Y)+\delta(Y)\right]\\&+S(X,Y)\left[\tau(Z)+\beta(Z)+\delta(Z)\right]\\&+\delta\left(R(X,Z)Y\right)+\delta\left(R(Y,X)Z\right)\\&+\delta\left(R(Z,Y)X\right)+\beta\left(R(X,Y)Z\right)\\&+\beta\left(R(Y,Z)X\right)+\beta\left(R(Z,X)Y\right).\end{aligned}$$

From the firstly Bianchi identity, we get:

$$\begin{aligned}\delta\left(R(X,Z)Y+R(Y,X)Z+R(Z,Y)X\right)&=\beta\left(R(X,Y)Z+R(Y,Z)X+R(Z,X)Y\right)\\&=0,\end{aligned}$$

we can easily to see that

$$\begin{aligned}(\nabla_X S)(Y,Z)+(\nabla_Y S)(Z,X)+(\nabla_Z S)(X,Y)&=S(Y,Z)\left[\tau(X)+\beta(X)+\delta(X)\right]\\&+S(Z,X)\left[\tau(Y)+\beta(Y)+\delta(Y)\right]\\&+S(X,Y)\left[\tau(Z)+\beta(Z)+\delta(Z)\right],\end{aligned}$$

that is,

$$(\nabla_X S)(Y,Z)+(\nabla_Y S)(Z,X)+(\nabla_Z S)(X,Y)=0.$$

the proof is completed.

Theorem 4

$(LCS)_n-$manifold M has the scalar curvature $r=(n-1)\left(\alpha^2-\rho\right)$ if weakly Ricci–symmetric $(LCS)_n-$manifold M is $\eta-$parallel.

Proof

Let us suppose that $(LCS)_n-$manifold M is $\eta-$parallel, then we have:

$$(\nabla_X S)(\phi Y,\phi Z)=0,$$

for $X, Y, Z \in \chi(M)$.

From Equation (2), we obtain:

$$(\nabla_X S)(\phi Z, \phi Y) = \lambda(X) S(\phi Z, \phi Y) + \mu(\phi Z) S(X, \phi Y) + \nu(\phi Y) S(X, \phi Z),$$

which implies

$$\lambda(X) S(\phi Z, \phi Y) + \mu(\phi Z) S(X, \phi Y) + \nu(\phi Y) S(X, \phi Z) = 0 \quad (25)$$

choosing $X = \xi$ in Equation (25), we obtain

$$\lambda(\xi) S(\phi Z, \phi Y) = 0.$$

Also, considering Equation (10), we arrive at

$$\lambda(\xi) S\big(Z + \eta(Z)\xi, Y + \eta(Y)\xi\big) = 0,$$

that is

$$S(Y, Z) + \eta(Z)\eta(Y)\left[-(1-n)\left(\alpha^2 - \rho\right)\right] = 0.$$

Hence for $Y = Z = e_i,\ 1 \le i \le n,\ \ r = (n-1)\left(\alpha^2 - \rho\right),$

which proves our assertion. Now, we present an example to illustrate our result.

Example 1

We consider the $4-$dimensional manifold $M = \left\{(x, y, z, u) \in R^4 : u \neq 0\right\}$ where (x, y, z, u) are standard coordinates in R^4. Let $\{E_1, E_2, E_3, E_4\}$ be linearly independent global frame on M given by

$$E_1 = e^u \left\{ x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z} \right\},\ E_2 = e^u \frac{\partial}{\partial y},$$

$$E_3 = e^u \frac{\partial}{\partial z},\ E_4 = \frac{\partial}{\partial u}.$$

Let g be the Lorentzian metric defined by

$$g(E_1, E_1) = g(E_2, E_2) = g(E_3, E_3) = 1,$$

$$g(E_4, E_4) = -1,\quad g(E_i, E_j) = 0 \text{ for } i \neq j.$$

Let η be the 1–form defined by $\eta(U) = g(U, E_4)$ for any $U \in \chi(M)$.

Let be ϕ be the (1,1) tensor field by $\phi E_1 = E_1,\ \phi E_2 = E_2,\ \phi E_3 = E_3,$ and $\phi E_4 = 0.$

Then using the linearity of ϕ and g, we have:

$$\eta(E_4) = -1,\quad \phi^2 U = U + \eta(U) E_4 \quad g(\phi U, \phi W) = g(U, W) + \eta(U)\eta(W)$$

for any $U, W \in \chi(M)$.

Let ∇ be the Levi–Civita connection with respect to the Lorentzian metric g and R be the Riemannian curvature tensor of $g.$ Then by direct calculations we have:

$$[E_1, E_2] = -e^u E_2,\quad [E_1, E_3] = -e^u E_3,\quad [E_1, E_4] = -E_1,$$

$$[E_2, E_3] = 0,\quad [E_2, E_4] = -E_2,\quad [E_3, E_4] = -E_3.$$

Taking $E_4 = \xi$ and using Koszul formula for the Lorentzian metric $g,$ we can easily calculate:

$$\begin{array}{llll}
\nabla_{E_1} E_1 = -E_4, & \nabla_{E_1} E_2 = 0, & \nabla_{E_1} E_3 = 0, & \nabla_{E_1} E_4 = -E_1, \\
\nabla_{E_2} E_1 = e^u E_2, & \nabla_{E_2} E_2 = -e^u E_1 - E_4, & \nabla_{E_2} E_3 = 0, & \nabla_{E_2} E_4 = -E_2, \\
\nabla_{E_3} E_1 = e^u E_3, & \nabla_{E_3} E_2 = 0, & \nabla_{E_3} E_3 = -e^u E_1 - E_4, & \nabla_{E_3} E_4 = -E_3, \\
\nabla_{E_4} E_1 = 0, & \nabla_{E_4} E_2 = 0, & \nabla_{E_4} E_3 = 0, & \nabla_{E_4} E_4 = 0.
\end{array}$$

From the above, it can be easily seen that $E_4 = \xi$ is a unit time–like concircular vector field and hence (ϕ, ξ, η, g) is $(LCS)_4$ structure on $M.$

Consequently $M^4(\phi, \xi, \eta, g)$ is a $(LCS)_4-$manifold with $\alpha = -1 \neq 0$ and $\rho = 0$. Using the above relations, we can easily calculate the non–vanishing components of the curvature tensor R as follows:

$$\begin{array}{lll}
R(E_1, E_2)E_2 = E_1 - e^{2u} E_1, & R(E_1, E_3)E_3 = E_1 - e^{2u} E_1, & R(E_1, E_4)E_4 = -E_1, \\
R(E_2, E_1)E_1 = E_1 - e^{2u} E_1, & R(E_2, E_3)E_3 = -E_2 - e^{2u} E_2, & R(E_2, E_4)E_4 = -E_2, \\
R(E_3, E_1)E_1 = E_3 - e^{2u} E_3, & R(E_3, E_2)E_2 = E_3 - e^{2u} E_3, & R(E_3, E_4)E_4 = -E_3 \\
R(E_4, E_1)E_1 = E_4, & R(E_4, E_2)E_2 = E_4, & R(E_4, E_3)E_3 = -E_4,
\end{array}$$

and, other components which can be obtained from these by the symmetry properties.

Thus the components of the Ricci Tensor is also:

$$S(E_1, E_1) = S(E_3, E_3) = 1 - 2e^{2u},\ S(E_3, E_3) = -e^{2u},\ S(E_4, E_4) = -3,$$

$$S(E_1, E_2) = S(E_2, E_3) = -1 + 2e^{2u},\ S(E_1, E_4) = S(E_2, E_4) = S(E_3, E_4) = 1$$

And

$$S\left(E_1, E_3\right) = 0.$$

Thus the scalar curvature is given by

$$r = \sum_{i}^{4} g\left(E_i, E_i\right) S\left(E_i, E_i\right) = 5 - 5e^{2u}.$$

On the other hand, by direct calculations, we can derive

$$\left(\nabla_{E_i} R\right)\left(E_j, E_k, E_l, E_t,\right) = 0, \; 1 \le i, j, k, l, t \le 4.$$

So the manifold in Example 1 is weakly symmetric, but is not $\eta-$parallel because:

$$\left(\nabla_{E_2} S\right)\left(E_1, E_1\right) = 2e^{u} - 2e^{3u} \neq 0.$$

By direct calculations, we have:

$$\tau\left(E_i\right) = \begin{cases} 0, & i = 1,2,3 \\ \dfrac{-2e^{2u}}{e^{2u} - 1}, & i=4 \end{cases}$$

$$\beta\left(E_i\right) = \begin{cases} 0, & i = 1,2,3 \\ \dfrac{e^{2u}}{e^{2u} - 1}, & i=4 \end{cases}$$

$$\gamma\left(E_i\right) = \begin{cases} 0, & i = 1,2,3 \\ 1, & i=4 \end{cases}$$

$$\delta\left(E_i\right) = \begin{cases} 0, & i = 1,2,3 \\ \dfrac{e^{2u}}{e^{2u} - 1}, & i=4 \end{cases}$$

$$\sigma\left(E_i\right) = \begin{cases} 0, & i = 1,2,3 \\ \dfrac{e^{2u}}{e^{2u} - 1}, & i=4 \end{cases}$$

In this example, we can infer $\tau(\xi) + \beta(\xi) + \delta(\xi) = \lambda(\xi) + \mu(\xi) + \sigma(\xi) = 0$ which verifies the statement of Theorem 1.

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