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Full Length Research Paper

An inventory model for deteriorating items with different constant demand rates

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In this paper, we study the inventory model for deteriorating items with different constant demand rates, that is, the demand rate is piecewise linear function. The deterioration rate is assumed as constant. We have proposed an inventory replenishment policy for this type of inventory model. The numerical solution of the model is also obtained and examined.

Key words: Inventory, deteriorating items, constant demand rate.

INTRODUCTION

In real life deterioration of goods is a common process. Food items, vegetables, fruits, medicines, drugs are a few examples of such items. Physical goods, fashions goods, pharmaceuticals, electronics components etc., undergo deterioration overtime. Therefore, the loss due to damage, decay, spoilage or deterioration can not be neglected. As inventory is defined as decay change, damage or spoilage of these items can not be used for its original purposes. The important problem for any modern organization is the control and maintenance of inventories of deteriorating items. In the classical EOQ (Economic Order Quantity) model developed in 1915, the demand of an item was assumed as constant, therefore, many researchers, considered the demand of these items as constant. Two earliest researchers, Ghare and Schrader (1963) considered the continuously decaying inventory for a constant demand. However, in real situation, the demand rate of items should vary with time. Silver and Meal (1969) suggest a simple modification of the EOQ model with varying demand. Researchers like Ritchie (1984), considered the inventory models about constant, linear and time-dependent demand. Bahari-Kashani (1989) and Goswami and Chaudhuri (1991), developed the EOQ models for deteriorating items with trended demand. The inventory model with ramp ty rate was first proposed by Hill (1995). The ramp type demand is commonly seen when some fresh fruits are brought to the market. In the case of ramp type demand rate, the demand increases linearly at the beginning and then the market grows into a stable stage such that the demand becomes a constant until the end of the inventory cycle. Hill (1995) first considered the inventory models for increasing demand followed by a constant demand. Mandal and Pal (1998) extended the inventory model with ramp type demand for deterioration items and allowing shortage. Wu and Ouyang (2000) extended the inventory model to include two different replenishment policies: (a) models starting with no shortage and (b) models starting with shortage. Deng et al. (2007) point out some questionable results of Mandal and Pal (1998) and Wu and Ouyang (2000), and then resolved the similar problem by offering a rigorous and efficient method to derive the optimal solution. Wu (2001) further investigated the inventory model with ramp type demand rate such that the deterioration followed the Weibull distribution. Giri et al. (2003) and Cheng and Wang (2009) extended the ramp type demand inventory model with a more generalized Weibull deterioration

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distribution.Various types of order-level inventory model for deteriorating items at a constant rate with a time-dependent were discussed recently.

In the following, we have worked on constant-constant-constant demand rates where the value differs with different time intervals. We assumed that the inventory system considered here has several replenishments and all the ordering cycles are of fixed length. Such type of demand pattern is generally seen in case of seasonal goods like vegetables, fruits and electronics components coming to market. We think that such types of demand are quite realistic and a useful inventory replenishment policy for such type of inventory model is also proposed.

The rest of the paper is organized as follows. Subsequently, this study describes the assumptions and notations used throughout this paper, after which it establishes the mathematical model with shortage in inventory and the necessary conditions to find an optimal solution. This is followed by the use of some numerical examples to illustrate the solution procedure. Finally, the study is summarized and some suggestions are provided for further research.

NOTATION AND ASSUMPTION

In this paper, we extend trapezoidal type demand rate type to different demand rate. The fundamental assumption and notation used in this paper are given as:

1. The replenishment rate is infinite, thus, replenishment is instantaneous.

2. The demand rate, $\mathbf{D(t)}$, which is positive and consecutive, is assumed to be a constant-constant-constant type function of time, that is:

$$\mathbf{D(t)} = \begin{cases} A_1, & t \leq \mu_1 , \\ D_0, & \mu_1 \leq t \leq \mu_2 , \\ A_2, & \mu_2 \leq t \leq T . \end{cases}$$

where A_1 is the constant distribution, $A_1 > \mathbf{0}$; $-A_2$ is the negative constant distribution, $A_2 > \mathbf{0}$; μ_1 is the time point changing from the increasing linearly demand to the constant demand and μ_2 is the time point changing from the constant demand to the decreasing linearly demand.

3. $I(t)$ is the level of inventory at time t , $0 \leq t \leq T$.

4. T is the fixed length of each ordering cycle.

5. θ is the constant deteriorating rate, $0 < \theta < 1$.

6. t_1 is the time when the inventory level reaches zero.

7. t_1^* is the optimal point.

8. A_0 is the fixed ordering cost per order.

9. c_1 is the cost of each deteriorated items.

10. c_2 is the inventory holding cost per unit per unit of time.

11. c_3 is the shortage cost per unit per unit of time.

12. S is the maximum inventory level for the ordering cycle, such that $S = I(0)$.

13. Q is the ordering quantity per cycle.

14. $C_1(t_1)$ is the average total cost per unit time under the condition $t_1 \leq \mu_1$.

15. $C_2(t_1)$ is the average total cost per unit time under the condition $\mu_1 \leq t_1 \leq \mu_2$.

16. $C_1(t_1)$ is the average total cost per unit time under the condition $\mu_2 \leq t_1 < T$.

MATHEMATICAL FORMULATION

We consider the deteriorating inventory model with trapezoidal type demand rate. Replenishment occurs at time $t = \mathbf{0}$ when the inventory level attains its maximum. From $t = \mathbf{0}$ to t_1, the inventory level reduces due to demand and deterioration. At t_1, the inventory level achieves zero, then the shortage is allowed to occur during the time interval (t_1, T), and all of the demand during the shortage period (t_1, T) is completely backlogged. The total number of backlogged items is replaced by the next replenishment. According to the notations and assumptions mentioned above, the behavior of inventory system at any time can be described by the following differential equations:

$$\frac{dI(t)}{dt} = -\theta I(t) - D(t), \quad 0 < t < t_1 \tag{1}$$

and

$$\frac{dI(t)}{dt} = -D(t), t_1 < t < T \tag{2}$$

with the boundary condition $I(t_1) = \mathbf{0}$.

In follows, we consider three possible cases based on values of t_1, μ_1 and μ_2. These three cases are shown as follows:

Case 1. $0 < t_1 \leq \mu_1$.

Due to reasons of deteriorating items and trapezoidal type market demand, the inventory gradually diminishes during the period [0, t_1] and ultimately falls to zero at time t_1 Figure 1. Then from equation (1), we have

$$\frac{dI(t)}{dt} = -\theta I(t) - A_1 \quad 0 < t < t_1 \tag{3}$$

$$\frac{dI(t)}{dt} = -A_1 , \; t_1 < t < \mu_1 , \tag{4}$$

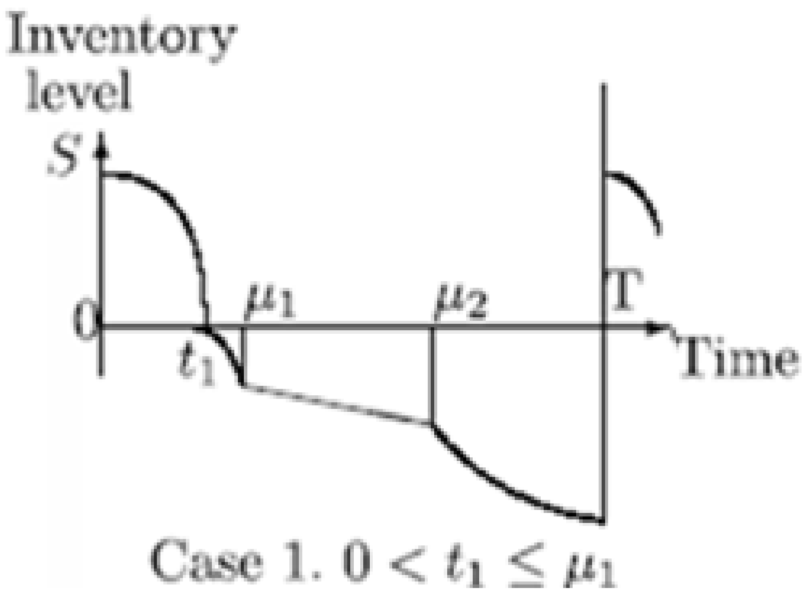


Figure 1. Graphical representation of inventory level over the cycle for case 1.

$$\frac{dI(t)}{dt} = -D_0 \quad , \mu_1 < t < \mu_2 \ , \tag{5}$$

and

$$\frac{dI(t)}{dt} = -A_2 \ , \ \mu_2 < t < T \ . \tag{6}$$

Solving the differential equations from (3) to (6) with $I(t_1) = 0$, we have:

$$I(t) = \frac{A_1}{\theta}\left(e^{\theta(t_1 - t)} - 1\right), 0 \le t \le t_1 \tag{7}$$

$$I(t) = A_1(t_1 - t), t_1 \le t \le \mu_1 \ , \tag{8}$$

$$I(t) = D_0(\mu_1 - t) + A_1(t_1 - \mu_1), \mu_1 \le t \le \mu_2 \tag{9}$$

$$I(t) = A_2(\mu_2 - t) - D_0(\mu_2 - \mu_1), + A_1(t_1 - \mu_1) \quad \mu_2 \le t \le T \ . \tag{10}$$

The beginning inventory level can be computed as:

$$S = I(0) = \frac{A_1}{\theta}\left(e^{\theta t_1} - 1\right) . \tag{11}$$

The total number of items which perish in the interval [0, t_1], say D_T , is:

$$\begin{aligned} D_T &= S - \int_0^{t_1} D(t)dt \\ &= S - \int_0^{t_1} A_1 dt \\ &= \frac{A_1}{\theta}\left(e^{\theta t_1} - 1\right) - A_1 t_1 . \end{aligned} \tag{12}$$

The total number of inventory carried during the interval [0, t_1], say H_T , is:

$$\begin{aligned} H_T &= \int_0^{t_1} I(t)dt \\ &= \int_0^{t_1} \left[\frac{A_1}{\theta}\left(e^{\theta(t_1 - t)} - 1\right)\right] dt \\ &= \frac{A_1}{\theta^2}\left(e^{\theta t_1} - \theta t_1 - 1\right) \end{aligned} \tag{13}$$

The total shortage quantity during the interval [0, t_1], say B_T , is:

$$\begin{aligned} B_T &= -\int_{t_1}^{T} I(t)dt \\ &= -\int_{t_1}^{\mu_1} [A_1(t_1 - t)]dt - \int_{\mu_1}^{\mu_2} [D_0(\mu_1 - t) + A_1(t_1 - \mu_1)]dt \\ &- \int_{\mu_2}^{T} [A_2(\mu_2 - t) - D_0(\mu_2 - \mu_1) + A_1(t_1 - \mu_1)]dt \\ &= \frac{A_1}{2}(t_1 - \mu_1)(t_1 + \mu_1 - 2T) + \frac{A_2}{2}(\mu_2 - T)^2 \\ &+ \frac{D_0}{2}(\mu_1 - \mu_2)(\mu_1 + \mu_2 - 2T) . \end{aligned} \tag{14}$$

Then, the average total cost per unit time under the condition $t_1 \le \mu_1$ can be given by

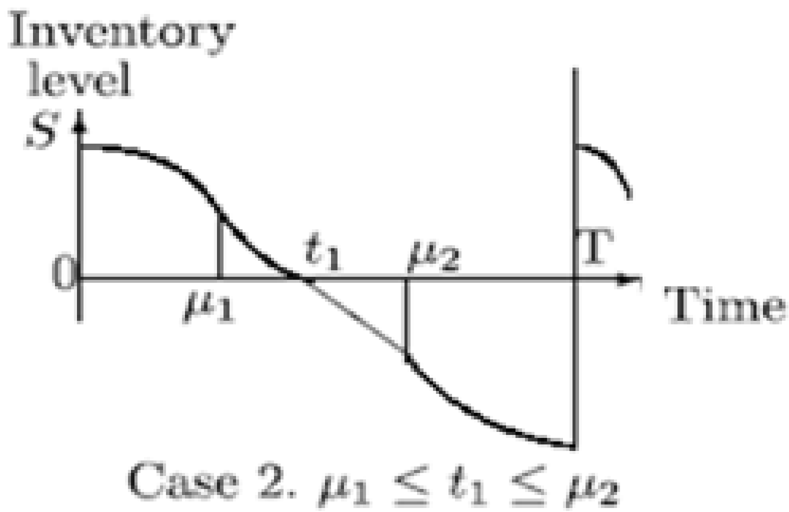


Figure 2. Graphical representation of inventry level over the cycle for case 2.

$$C_1(t_1) = \frac{1}{T}[A_0 + c_1 D_T + c_2 H_T + c_3 B_T] \tag{15}$$

The first order differential of $C_1(t_1)$ with respect to t_1 is as follows:

$$\frac{dC_1(t_1)}{dt_1} = \frac{1}{T}\left[\left(c_1 + \frac{c_2}{\theta}\right)(e^{\theta t_1} - 1) + c_3(t_1 - T)\right]A_1 \tag{16}$$

The necessary condition for $C_1(t_1)$ in (15) to be minimized is $\frac{dC_1(t_1)}{dt_1} = 0$, that is:

$$\left[\left(c_1 + \frac{c_2}{\theta}\right)(e^{\theta t_1} - 1) + c_3(t_1 - T)\right]A_1 = 0 \tag{17}$$

Let $f(t_1) = \left(c_1 + \frac{c_2}{\theta}\right)(e^{\theta t_1} - 1) + c_3(t_1 - T)$, Since $f(0) = -c_3 T < 0$, $f(T) = \left(c_1 + \frac{c_2}{\theta}\right)(e^{\theta T} - 1) > 0$ and $f'(t_1) = (\theta c_1 + c_2)e^{\theta t_1} + c_3 > 0$, it implies that $f(t_1)$ is a strictly monotone increasing function and equation (17) has a unique solution as t_1^*, for $t_1^* \epsilon (0, T)$. Therefore, we have:

Property 1

The deteriorating inventory model under the condition $0 < t_1 \le \mu_1$, $C_1(t_1)$ obtains its minimum at $t_1 = t_1^*$, where $f(t_1^*) = 0$ if $t_1^* < \mu_1$. On the other hand, $C_1(t_1)$ obtains its minimum at $t_1^* = \mu_1$ if $t_1^* \ge \mu_1$.

From property 1, we know that the total back-order amount at the end of the cycle is

$\Delta_1 = A_1(\mu_1 - t_1^*) + D_0(\mu_2 - \mu_1) + A_2(T - \mu_2)$.

Therefore, the optimal order quantity, denoted by Q^*, is $Q^* = S^* + \Delta_1$, where S^* denotes the optimal value of S.

Case 2. $\mu_1 \le t_1 \le \mu_2$.

If the time $t_1 \epsilon (\mu_1, \mu_2)$, then, the differential equations governing the inventory model can be expressed as follows Figure 2:

$$\frac{dI(t)}{dt} = -\theta I(t) - A_1, 0 < t < \mu_1 \tag{18}$$

$$\frac{dI(t)}{dt} = -\theta I(t) - D_0, \mu_1 < t < t_1 \tag{19}$$

$$\frac{dI(t)}{dt} = -D_0, t_1 < t < \mu_2, \tag{20}$$

and

$$\frac{dI(t)}{dt} = -A_2, \mu_2 < t < T. \tag{21}$$

Solving the differential equations from (18) to (21) with $I(t_1) = 0$, we have

$$I(t) = \frac{D_0}{\theta}(e^{\theta t_1} - e^{\theta \mu_1})e^{-\theta t} + \frac{A_1}{\theta}(e^{\theta(\mu_1 - t)} - 1), 0 \le t \le \mu_1 \tag{22}$$

$$I(t) = \frac{D_0}{\theta}(e^{\theta(t_1 - t)} - 1), \mu_1 \le t \le t_1 \tag{23}$$

$$I(t) = D_0(t_1 - t), t_1 \leq t \leq \mu_2, \quad (24)$$

$$I(t) = A_2(\mu_2 - t) + D_0(t_1 - \mu_2), \mu_2 \leq t \leq T \quad (25)$$

The beginning inventory level can be computed as:

$$S = I(0) = \frac{D_0}{\theta}(e^{\theta t_1} - e^{\theta \mu_1}) + \frac{A_1}{\theta}(e^{\theta \mu_1} - 1). \quad (26)$$

The total number of items which perish in the interval [0, t_1] is:

$$\begin{aligned} D_T &= S - \int_0^{t_1} D(t)dt \\ &= S - \left[\int_0^{\mu_1} A_1 dt + \int_{\mu_1}^{t_1} D_0 dt\right] \\ &= \frac{D_0}{\theta}(e^{\theta t_1} - e^{\theta \mu_1}) + \frac{A_1}{\theta}(e^{\theta \mu_1} - 1) - D_0 t_1. \end{aligned} \quad (27)$$

The total number of inventory carried during the interval [0, t_1] is:

$$\begin{aligned} H_T &= \int_0^{t_1} I(t)dt \\ &= \int_0^{\mu_1}\left[\frac{D_0}{\theta}(e^{\theta t_1} - e^{\theta \mu_1})e^{-\theta t} + \frac{A_1}{\theta}(e^{\theta(\mu_1 - t)} - 1)\right]dt \\ &\quad + \int_{\mu_1}^{\mu_2}\left[\frac{D_0}{\theta}(e^{\theta(t_1 - t)} - 1)\right]dt \\ &= \frac{D_0}{\theta^2}(e^{\theta t_1} - e^{\theta \mu_1}) + \frac{A_1}{\theta^2}(e^{\theta \mu_1} - 1) - \frac{D_0}{\theta} t_1. \end{aligned} \quad (28)$$

The total shortage quantity during the interval $[t_1, T]$ is:

$$\begin{aligned} B_T &= -\int_{t_1}^{T} I(t)dt \\ &= -\int_{t_1}^{\mu_2}[D_0(t_1 - t)]dt - \int_{\mu_2}^{T}[A_2(\mu_2 - t) + D_0(t_1 - \mu_2)]dt \\ &= \frac{D_0}{2}(t_1 - \mu_2)(t_1 + \mu_2 - 2T) + \frac{A_2}{2}(\mu_2 - T)^2. \end{aligned} \quad (29)$$

Then, the average total cost per unit under the condition $\mu_1 \leq t_1 \leq \mu_2$, can be given by:

$$C_2(t_1) = \frac{1}{T}[A_0 + c_1 D_T + c_2 H_T + c_3 B_T]. \quad (30)$$

The first order differential of $C_2(t_1)$ with respect to t_1 is as follows:

$$\frac{dC_2(t_1)}{dt_1} = \frac{D_0}{T}\left[\left(c_1 + \frac{c_2}{\theta}\right)(e^{\theta t_1} - 1) + c_3(t_1 - T)\right]. \quad (31)$$

The necessary condition for $C_2(t_1)$ in (30) to be minimized is $\frac{dC_2(t_1)}{dt_1} = 0$, that is:

$$\left[\left(c_1 + \frac{c_2}{\theta}\right)(e^{\theta t_1} - 1) + c_3(t_1 - T)\right] = 0. \quad (32)$$

Similar to the first case, we have:

Property 2

The inventory model under the condition $\mu_1 < t_1 \leq \mu_2$, $C_2(t_1)$ obtains its minimum at $t_1 = t_1^*$, where $f(t_1^*) = 0$ if $\mu_1 < t_1^* < \mu_2$; $C_2(t_1)$ obtains its minimum at $t_1^* = \mu_1$ if $t_1^* < \mu_1$; and $C_2(t_1)$ obtains its minimum at $t_1^* = \mu_2$ if $\mu_2 < t_1^*$.

From property 2, we know that the total back-order amount at the end of the cycle is $\Delta_2 = D_0(\mu_2 - t_1^*) + A_2(T - \mu_2)$. Therefore, the optimal order quantity, denoted by Q^*, is $Q^* = S^* + \Delta_2$, where S^* denotes the optimal value of S.

Case 3. $\mu_2 \leq t_1 \leq T$

If the time $t_1 \in [\mu_2, T)$, then, the differential equations governing the inventory model can be expressed as follows Figure 3:

$$\frac{dI(t)}{dt} = -\theta I(t) - A_1, \quad 0 < t < \mu_1, \quad (33)$$

$$\frac{dI(t)}{dt} = -\theta I(t) - D_0, \quad \mu_1 < t < \mu_2 \quad (34)$$

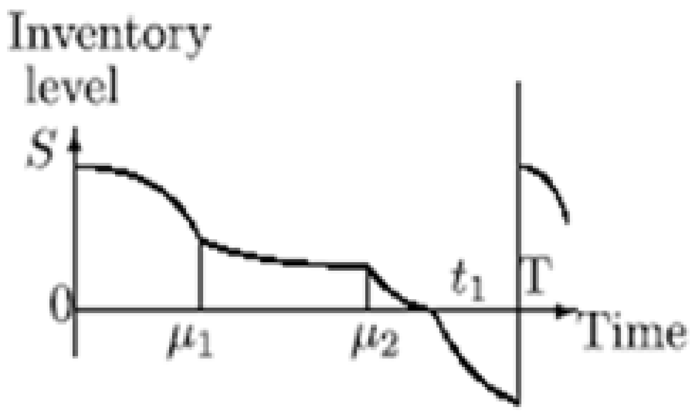
$$\frac{dI(t)}{dt} = -\theta I(t) - A_2, \quad \mu_2 < t < t_1, \quad (35)$$

and

$$\frac{dI(t)}{dt} = -A_2, \quad t_1 < t < T \quad (36)$$

Solving the differential equations from (33) to (36) with $I(t_1) = 0$, we have

$$\begin{aligned} I(t) &= \frac{A_1}{\theta}(e^{\theta(\mu_1 - t)} - 1) \\ &+ \left[\frac{D_0}{\theta}(e^{\theta \mu_2} - e^{\theta \mu_1}) + \frac{A_2}{\theta}(e^{\theta t_1} - e^{\theta \mu_2})\right]e^{-\theta t}, \end{aligned}$$



Case 3. $\mu_2 \le t_1 < T$

Figure 3. Graphical representation of inventry level over the cycle for case 3.

$$0 \le t \le \mu_1 \tag{37}$$

$$I(t) = \frac{D_0}{\theta}\left(e^{\theta(\mu_2 - t)} - 1\right) + \left[\frac{A_2}{\theta}\left(e^{\theta t_1} - e^{\theta \mu_2}\right)\right] e^{-\theta t},$$
$$\mu_1 \le t \le \mu_2, \tag{38}$$

$$I(t) = \frac{A_2}{\theta}\left(e^{\theta(t_1 - t)} - 1\right), \mu_2 \le t \le t_1 \tag{39}$$

$$I(t) = A_2(t_1 - t), t_1 \le t \le T \tag{40}$$

The beginning inventory level can be computed as

$$S = I(0) = \frac{A_1}{\theta}\left(e^{\theta \mu_1} - 1\right) + \frac{D_0}{\theta}\left(e^{\theta \mu_2} - e^{\theta \mu_1}\right) + \frac{A_2}{\theta}\left(e^{\theta t_1} - e^{\theta \mu_2}\right) \tag{41}$$

The total number of items which perish in the interval [0, t_1], say D_T, is:

$$\begin{aligned} D_T &= S - \int_0^{t_1} D(t)dt \\ &= S - \left[\int_0^{\mu_1} A_1 dt + \int_{\mu_1}^{\mu_2} D_0 dt + \int_{\mu_2}^{t_1} A_2 dt\right] \\ &= \frac{D_0}{\theta}\left(e^{\theta \mu_2} - e^{\theta \mu_1}\right) + \frac{A_1}{\theta}\left(e^{\theta \mu_1} - 1\right) \\ &+ \frac{A_2}{\theta}\left(e^{\theta t_1} - e^{\theta \mu_2}\right) - A_2 t_1 \end{aligned} \tag{42}$$

The total number of inventory carried during the interval [0, t_1], say H_T, is:

$$\begin{aligned} H_T &= \int_0^{t_1} I(t)dt \\ &= \int_0^{\mu_1}\left[\frac{A_1}{\theta}\left(e^{\theta(\mu_2 - t)} - 1\right) + \left[\frac{D_0}{\theta}\left(e^{\theta \mu_2} - e^{\theta \mu_1}\right) + \frac{A_2}{\theta}\left(e^{\theta t_1} - e^{\theta \mu_2}\right)\right] e^{-\theta t}\right] dt \\ &+ \int_{\mu_1}^{\mu_2}\left[\frac{D_0}{\theta}\left(e^{\theta(\mu_2 - t)} - 1\right) + \left[\frac{A_2}{\theta}\left(e^{\theta t_1} - e^{\theta \mu_2}\right)\right] e^{-\theta t}\right] dt \\ &+ \int_{\mu_2}^{t_1}\left[\frac{A_2}{\theta}\left(e^{\theta(t_1 - t)} - 1\right)\right] dt \\ &= \frac{D_0}{\theta^2}\left(e^{\theta \mu_2} - e^{\theta \mu_1}\right) + \frac{A_1}{\theta^2}\left(e^{\theta \mu_1} - 1\right) + \frac{A_2}{\theta}\left(e^{\theta t_1} - e^{\theta \mu_2}\right) - \frac{A_2}{\theta} t_1 \end{aligned} \tag{43}$$

The total shortage quantity during the interval [0, t_1], say B_T, is:

$$\begin{aligned} B_T &= -\int_{t_1}^{T} I(t)dt \\ &= -\int_{t_1}^{T} [A_2(t_1 - t)]dt \\ &= \frac{A_2}{2}(t_1 - T)^2 \end{aligned} \tag{44}$$

Then, the average total cost per unit time under the condition $\mu_2 \le t_1 \le T$, can be given by:

$$C_3(t_1) = \frac{1}{T}[A_0 + c_1 D_T + c_2 H_T + c_3 B_T]. \tag{45}$$

The first order differential of $C_1(t_1)$ with respect to t_1 is as follows:

$$\frac{dC_3(t_1)}{dt_1} = \frac{A_2}{T}\left[\left(c_1 + \frac{c_2}{\theta}\right)\left(e^{\theta t_1} - 1\right) + c_3(t_1 - T)\right]. \quad (46)$$

The necessary condition for $C_1(t_1)$ in (45) to be minimized is $\frac{dC_3(t_1)}{dt_1} = 0$, that is:

$$A_2\left[\left(c_1 + \frac{c_2}{\theta}\right)\left(e^{\theta t_1} - 1\right) + c_3(t_1 - T)\right] = 0. \quad (47)$$

Similar to the first case, we have:

Property 3

The inventory model under the condition $\mu_2 \le t_1 < T$, $C_3(t_1)$ obtains its minimum at $t_1 = t_1^*$, where $f(t_1^*) = 0$ if $\mu_2 < t_1^*$. On the other hand, $C_3(t_1)$ obtains its minimum at $t_1^* = \mu_2$ if $t_1^* < \mu_2$.

From Property 3, we know that the total back-order amount at the end of the cycle is $\Delta_3 = A_2(T - t_1^*)$. Therefore, the optimal order quantity, denoted by Q^*, is $Q^* = S^* + \Delta_3$, where S^* denotes the optimal value of S.

Remark 1: The previous analysis shows that Equations (16), (31) and (46) can be expressed as $\frac{dC_j(t)}{dt_1} = \frac{D_{j[(t_1)]}}{T} f(t_1), j = 1,2,3.$ which denotes the total marginal cost function under different demand rate conditions, respectively.

Combining the above properties, we know that $C_1(\mu_1) = C_2(\mu_1)$, and $C_2(\mu_2) = C_3(\mu_2)$. Therefore, we can derive the following result.

Theorem 1: For the deteriorating inventory model with trapezoidal type demand rate, the optimal replenishment time is t_1^* and $C_1(t_1)$ obtains its minimum at $t_1 = t_1^*$, if and only If $t_1^* < \mu_1$. On the other hand, $C_2(t_1)$ obtains its minimum at t_1^* if and only if $\mu_1 < t_1^* < \mu_2$ and $C_{\downarrow}3\ (t_{\downarrow}1$) obtains its minimum at t_1^* if and only if $\mu_2 < t_1^*$, where t_1^* is the unique solution of the equation $f(t_1) = 0$.

NUMERICAL EXAMPLES

In this section, we provide several numerical examples to illustrate the above theory.

Example 1

The parameter values are given as follows: $T = 15$ weeks, $\mu_1 = 5$ weeks, $\mu_2 = 10$ weeks, $A_1 = 150$ units, $A_2 = 210$ units, $\theta = 0.2$, $A_0 = \$200$, $c_1 = \$2$ per unit, $c_2 = \$4$ per unit, $c_3 = \$3$ per unit, $D_0 = \$360$. Based on the solution procedure as aforementioned, we have $f(\mu_1) = 7.8022 > 0$, then it yields that the optimal replenishment time $t_1^* = 4.4556227$ weeks, the optimal order quantity, Q^*, is 4010.06 unit and the minimum cost $C_1(t_1^*) = 4007.51$

Example 2

The parameter values are given as follows: $T = 15$ weeks, $\mu_1 = 2$ weeks, $\mu_2 = 10$ weeks, $A_1 = 150$ units, $A_2 = 210$ units, $\theta = 0.2$, $A_0 = \$200$, $c_1 = \$2$ per unit, $c_2 = \$4$ per unit, $c_3 = \$3$ per unit, $D_0 = \$360$. Based on the solution procedure as aforementioned, we have $f(\mu_1) = -28.1799 < 0$ and $f(\mu_2) = 125.559 > 0$, then it yields that the optimal replenishment time $t_1^* = 4.4556227$ weeks, the optimal order quantity, Q^*, is 5117.73 unit and the minimum cost $C_2(t_1^*) = 4326.95$.

Example 3

The parameter values are given as follows:

$T = 15$ weeks, $\mu_1 = 2$ weeks, $\mu_2 = 4$ weeks, $A_1 = 150$ units, $A_2 = 210$ units, $\theta = 0.2$, $A_0 = \$200$, $c_1 = \$3$ per unit, $c_2 = \$2$ per unit, $c_3 = \$5$ per unit, $D_0 = \$360$. Based on the solution procedure as above, we have $f(\mu_2) = -30.4892 < 0$, then it yields that the optimal replenishment time $t_1^* = 5.922857$ weeks, the optimal order quantity, Q^*, is 4691.69 unit and the minimum cost $C_3(t_1^*) = 4952.73$

SENSITIVITY ANALYSIS

We study the effects of changes in the system parameter T, μ_1, μ_2, A_1, A_2, θ, A_0, c_1, c_2, c_3, and D_0 on the optimal values of t_1, the optimal order quantity Q^* and the minimal optimal costs $C_1(t_1^*)$, $C_2(t_1^*)$ and $C_3(t_1^*)$ on the three tables. The sensitivity analysis is performed by changing each of the parameters by +25, +10, -10 and -25% on taking one parameter at a time and keeping the remaining parameters unchanged. The results are shown in Tables 1, 2 and 3.

On the basis of the results in Table 1, the following observations are taken into account:

Table 1. Based on Example 1.

Parameter changing	Change (%)	t_1^*	Q^*	$C_1(t_1^*)$	Percentage change in $C_1(t_1^*)$
T	+25	-	-	-	-
	+10	4.775294	4397.83	4479.1	+11.7677
	-10	4.11914	3626.538	3519.72	-12.1719
	-25	3.579408	3060.082	2741.2	-31.5984
μ_1	+25	4.4556227	3747.56	3515.33	-12.2814
	+10	4.4556227	3905.06	3802.75	-5.10941
	-10	4.4556227	4115.06	4222.75	+5.37092
	-25	4.4556227	4272.56	4565.33	+13.9194
μ_2	+25	4.4556227	4385.06	4288.75	+7.01782
	+10	4.4556227	4160.06	4142.51	+3.36868
	-10	4.4556227	3860.06	3842.51	-4.11727
	-25	4.4556227	3635.06	3538.75	-11.697
A_1	+25	4.4556227	4300.07	4199.81	+4.79849
	+10	4.4556227	4126.06	4084.43	+1.9194
	-10	4.4556227	3894.052	3930.59	-1.9194
	-25	4.4556227	3720.042	3815.22	-4.79824
A_2	+25	4.4556227	4272.56	4138.75	+3.27485
	+10	4.4556227	4115.06	4060.01	+1.31004
	-10	4.4556227	3905.06	3955.01	-1.31004
	-25	4.4556227	3747.56	3876.25	-3.27535
θ	+25	4.13413	4066.47	4069.09	+1.53662
	+10	4.321147	4034.15	4033.01	+0.636305
	-10	4.598762	3983.71	3980.73	-0.668245
	-25	4.831456	3939.43	3938.01	-1.73424
A_0	+25	4.4556227	4010.06	4010.85	+0.0833435
	+10	4.4556227	4010.06	4008.85	+0.0334372
	-10	4.4556227	4010.06	4006.18	-0.0331877
	-25	4.4556227	4010.06	4004.18	-0.083094
c_1	+25	4.403952	3999.01	4020.99	+0.336368
	+10	4.434786	4005.58	4012.95	+0.135745
	-10	4.476688	4014.62	4002.01	-0.137242
	-25	4.5087249	4021.53	3993.65	-0.345851
c_2	+25	3.9940306	3918.064	4127.82	+3.00211
	+10	4.256916	3968.62	4059.34	+1.29332
	-10	4.677329	4059.70	3949.63	-1.44429
	-25	-	-	-	-
c_3	+25	4.9807361	4133.76	4834.66	+20.64
	+10	4.677329	4059.70	4343.26	+8.37802
	-10	4.2154887	3960.345	3664.46	-8.56018
	-25	3.8135331	3886.023	3134.38	-21.7873
D_0	+25	4.4556227	4460.06	4692.51	+17.0929
	+10	4.4556227	4055.06	4277.51	+6.73735
	-10	4.4556227	3830.06	3737.51	-6.73735
	-25	4.4556227	3560.06	3332.51	-16.8434

Table 2. Based on Example 2.

Parameter changing	Change (%)	t_1^*	Q^*	$C_2(t_1^*)$	Percentage change in $C_2(t_1^*)$
T	25	5.2263	6359.1	5461.8	26.228
	10	4.7753	5607.4	4791.4	10.735
	-10	4.1191	4638.3	3824.9	-11.6
	-25	3.5794	3940.3	2962.8	-31.53
µ1	25	4.4556	4953	4085.3	-5.584
	10	4.4556	5053.8	4233.2	-2.167
	-10	4.4556	5179.2	4417	2.0818
	-25	4.4556	5266.8	4533.6	4.7754
µ2	25	4.4556	5492.7	4608.2	6.5002
	10	4.4556	5267.7	4462	3.12
	-10	4.4556	4967.7	4162	-3.813
	-25	4.4556	4742.7	3858.2	-10.83
A1	25	4.4556	5210	4467.5	3.2489
	10	4.4556	5154.6	4381.1	1.2503
	-10	4.4556	5080.8	4272.9	-1.25
	-25	4.4556	5025.5	4191.7	-3.126
A2	25	4.4556	5380.2	4458.2	3.0336
	10	4.4556	5222.7	4379.5	1.2133
	-10	4.4556	5012.7	4274.5	-1.213
	-25	4.4556	4855.2	4195.7	-3.033
θ	25	4.1341	5224.6	4578.3	5.8083
	10	4.3211	5164.4	4435.7	2.5142
	-10	4.5988	5065.4	4203.7	-2.848
	-25	4.8315	4974.8	3981.5	-7.984
A0	25	4.4556	5117.7	4330.3	0.077
	10	4.4556	5117.7	4328.3	0.0307
	-10	4.4556	5117.7	4325.6	-0.031
	-25	4.4556	5117.7	4323.6	-0.077
c_1	25	4.404	5091.2	4342.1	0.3499
	10	4.4348	5107	4333.1	0.1426
	-10	4.4767	5128.7	4320.6	-0.146
	-25	4.5087	5145.5	4310.9	-0.371
c_2	25	3.994	4896.9	4443.5	2.6941
	10	4.2569	5018.3	4382.5	1.2836
	-10	4.6773	5236.9	4284.3	-0.986
	-25	5.0642	5466.6	4117.8	-4.835
c_3	25	4.9807	5414.6	5183.4	19.793
	10	4.6773	5236.9	4681.3	8.1885
	-10	4.2155	4998.4	3955.1	-8.593
	-25	3.8135	4820	3360.2	-22.34
D_0	25	4.4556	6042.4	5138.9	18.764
	10	4.4556	5487.6	4651.7	7.5055
	-10	4.4556	4747.8	4002.2	-7.506
	-25	4.4556	4193	3515	-18.76

Table 3. Based on Example 3.

Parameter changing	Change (%)	t_1^*	Q^*	$C_3(t_1^*)$	Percentage Change in $C_3(t_1^*)$
T	+25	6.888121	6007.47	6158.35	+24.3425
	+10	6.3256856	5210.11	5421.12	+9.45721
	-10	5.4951019	4185.06	4506.83	-9.00312
	-25	4.80104	3449.87	3891.70	-21.4229
μ_1	+25	5.922857	4526.95	4733.08	-4.43493
	+10	5.922857	4627.76	4867.49	-1.72107
	-10	5.922857	4753.11	5034.62	+1.65343
	-25	5.922857	4840.75	5151.48	+4.01294
μ_2	+25	5.922857	5061.25	5445.47	+9.94886
	+10	5.922857	4830.71	5138.09	+3.74258
	-10	5.922857	4563.36	4781.62	-3.45486
	-25	5.922857	4389.12	4549.3	-8.14561
A_1	+25	5.922857	4783.91	5075.69	+2.48267
	+10	5.922857	4728.58	5001.91	+0.992988
	-10	5.922857	4654.80	4903.54	-.99319
	-25	5.922857	4599.47	4829.77	-2.48267
A_2	+25	5.922857	5442.22	5624.39	+13.5614
	+10	5.922857	4991.90	5221.39	+5.42448
	-10	5.922857	4391.48	4684.06	-5.42468
	-25	5.922857	3941.16	4281.07	-13.5614
θ	+25	5.244098	4811.16	5337.94	+7.77773
	+10	5.6332	4743.12	5111.73	+3.21035
	-10	6.240075	4633.23	4786.61	-3.35411
	-25	6.774499	4527.33	4523.89	-8.65866
A_0	+25	5.922857	4691.69	4956.06	+0.0672356
	+10	5.922857	4691.69	4954.06	+0.0268539
	-10	5.922857	4691.69	4951.4	-0.0268539
	-25	5.922857	4691.69	4949.4	-0.0672356
c_1	+25	5.6287167	4557.34	5198.08	+4.95383
	+10	5.8006196	4634.45	5053.57	+2.03605
	-10	6.051904	4754.34	4847.89	-2.11681
	-25	6.259809	4860.24	4682.29	-5.46042
c_2	+25	5.6287167	4557.34	5198.08	+4.95383
	+10	5.8006196	4634.45	5053.57	+2.03605
	-10	6.051904	4754.34	4847.89	-2.11681
	-25	6.259809	4860.24	4682.29	-5.46042
c_3	+25	6.48752	4983.52	5628.25	+13.6393
	+10	6.16304	4810.17	5233.43	+5.66758
	-10	5.659581	4570.9	4656.04	-5.99043
	-25	5.2109858	4385.64	4176.26	-15.6776
D_0	+25	5.922857	5021.86	5392.96	+8.88863
	+10	5.922857	4823.76	5128.82	+3.55541
	-10	5.922857	4559.62	4776.64	-3.55541
	-25	5.922857	4361.52	4512.5	-8.88863

--- indicates the solutions are infeasible.

1) With increase in the value of parameter T and c_3 : t_1 , the optimal values of Q^* and $C_1(t_1^*)$ increase.
2) With increase in the value of parameter c_1 and c_2 : t_1 and the optimal values of Q^* decrease and $C_1(t_1^*)$ increase.
3) With increase in the value of parameter: t_1 decreases and the optimal values of Q^* and $C_1(t_1^*)$ increase.
4) With increase in the value of parameter μ_2 , A_1 , A_2 and D_0 : t_1 remains constant and the optimal values of Q^* and $C_1(t_1^*)$ increase.
5) With increase in the value of parameter μ_1 : t_1 remains constant and the optimal values of Q^* and $C_1(t_1^*)$ decrease.
6) With increase in the value of parameter A_0 : t_1 and the optimal values of Q^* remains constant and $C_1(t_1^*)$ increase.

Similar results are obtained from Tables 2 and 3:

1) The changes in values of t_1^* , Q^* and $C_i(t_1^*)$ where i = 1, 2 and 3 more seen when we increase the percentage of μ_1 . So μ_1 is more sensitive.
2) With increase in the value of parameter μ_2 , A_1 , A_2 , θ , A_0 , c_1 , c_2 , and D_0 ,the optimal values of t_1^* , Q^* and $C_i(t_1^*)$ where $i = 1$, 2 and 3 are moderately sensitive.
3) With increase in the value of parameter T and c_3 , the optimal values of t_1^* , Q^* and $C_i(t_1^*)$ where i = 1, 2 and 3 are less sensitive.

CONCLUDING REMARKS

In this paper, we study the inventory model for deteriorating items with constant-constant-constant type demand rate, that is, the demand rate is a piecewise constant function. We proposed an inventory replenishment policy for this type of inventory model. Such type of demand pattern is generally seen in case of seasonal goods like vegetables, fruits and electronics components coming to market. We think that such types of demand are quite realistic and a useful inventory replenishment policy for such type of inventory model is also proposed. From the market information, we can find that this type of demand rate model is some applicable than trapezoidal type demand rate model in the stage of product life cycle. This paper provides an interesting topic for the future study of such kind of important inventory models and we extend to quadratic demand, stock-dependent demand and time-dependent demand. We also extend constant deterioration rate to variable deterioration rates, Weibull two parameter deterioration rates, Weibull three parameters deterioration rate and also Gamma deterioration rate.

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