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Heat absorption and chemical reaction effects on peristaltic motion of micropolar fluid through a porous medium in the presence of magnetic field

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In this paper, a study of the peristaltic motion of incompressible micropolar fluid through a porous medium in a two-dimensional channel under the effects of heat absorption and chemical reaction in the presence of magnetic field was studied. This phenomenon is modulated mathematically by a system of partial differential equations which govern the motion of the fluid. This system of equations is solved in dimensionless form under the assumption of long wave length and low Reynolds number. The expressions of the stream function, velocity, micro rotation velocity, temperature and concentration are obtained as functions of the physical parameters of the problem. The results have been discussed graphically to observe the effects of heat absorption, chemical reaction and magnetic field in the presence of micro polarity effects through a set of figures.

Key words: Heat absorption, chemical reaction, peristaltic motion, micropolar fluid, porous medium.

INTRODUCTION

Expansion and contraction of an extensible tube in a fluid generate progressive waves which spread along the length of the tube, mixing and transporting the fluid in the direction of wave propagation. This phenomenon is known as peristalsis. It is an inherent property of many tubular organs of the human body. Kavitha et al. (2011) investigated peristaltic flow of a micropolar fluid in a vertical channel with long wavelength approximation. Also, peristaltic motion of micropolor fluid in circular cylindrical tubes effect of wall properties has been done by Muthu et al. (2008).

Sreenadh et al. (2011) reported the peristaltic flow of micropolar fluid in an asymmetric channel with permeable walls. Reddy et al. (2012) dealt with the peristaltic pumping of a micropolar fluid in an inclined channel. The effect of magnetic field and wall properties on peristaltic motion of micropolar fluid was discussed by Afifi et al. (2011). The effect of dust particles on rotating micropolar fluid heated from below saturating a porous medium has been given by Reena and Rana (2011). Peristaltic transport of micropolar fluid in tubes under influence of rotation has been discussed by Abd-Alla et al. (2011). Peristaltic flow of a Newtonian fluid through a porous medium in a vertical tube under the effect of magnetic field has been studied by Vasudev et al. (2011).

The present study considered the peristaltic motion of an incompressible micropolar fluid through a porous medium in a symmetric channel under the effects of heat

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absorption and chemical reaction in the presence of magnetic field with the assumption of long wavelength and low Reynolds number. The expressions of stream function, velocity, microrotation velocity, temperature and concentration are obtained. The effects of the fluid parameters on velocity, microrotation velocity, temperature and concentration have been studied with the help of graphs.

MATHEMATICAL FORMULATION

Consider a symmetric flow of an incompressible micripolar fluid through a porous medium in a symmetric channel. The flow is generated by sinusoidal wave propagating with constant speed c along the wall. A uniform magnetic field B_o is applied in the transvers direction of the flow. The wall deformation is given by:

$$H(X,t) = a + b\sin\left(\frac{2\pi}{\lambda}(X - ct)\right) \tag{1}$$

Where a is the half width of the channel, b is the amplitude of the wave, λ is the wavelength and t is the time.

Under the assumption that the channel length is an integral multiple of the wavelength λ and the pressure difference across the ends of the channel is a constant, the flow becomes steady in the wave frame (x, y) moving with velocity c away form the fixed frame (X, Y). The transformation between these two frames is given by:

$$x = X - ct;\ y = Y;\ u(x,y) = U(X - ct, Y) - c;\ v = V(X - ct, Y).$$

Where U and V are velocity component in the fixed frame and u, v are velocity component in the wave frame.

The equations governing the two-dimensional transport of incompressible micropolar fluid through a porous medium in a channel under the effect of a magnetic field are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\rho\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial P}{\partial x} + k\frac{\partial \Omega}{\partial y} + (k+\mu)\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] - \left(\frac{\mu}{k_1} + \sigma B_o^2\right)u \tag{3}$$

$$\rho\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial P}{\partial y} + k\frac{\partial \Omega}{\partial x} + (k+\mu)\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] - \frac{\mu}{k_1}v \tag{4}$$

$$\rho J\left[u\frac{\partial \Omega}{\partial x} + v\frac{\partial \Omega}{\partial y}\right] - 2k\Omega + k\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] + \gamma_1\left[\frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2}\right] \tag{5}$$

$$\rho c_p\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = k_T\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] - Q_o(T - T_0) \tag{6}$$

$$\left[u\frac{\partial \varphi}{\partial x} + v\frac{\partial \varphi}{\partial y}\right] = D_m\left[\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}\right] + \frac{D_m k_T}{T_m}\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] - k_2(\varphi - \varphi_o) \tag{7}$$

Where u and v are velocity component, ρ is the density of the fluid, P is the pressure, k is the micropolar viscosity, Ω is the microrotation velocity, μ is the viscosity, k_1 is the permeability of porous medium, σ is the Conductivity of the fluid, J is the microinertia constant, γ_1 is material constant, c_p is the specific heat, k_T is the thermal conductivity, T_m is the mean fluid temperature, D_m is the coefficient of mass diffusivity, Q_o is the heat absorption coefficient and k_2 is the chemical reaction parameter. Let us introduce the following dimensionless quantities as:

$$x^* = \frac{x}{\lambda},\ y^* = \frac{y}{a},\ h = \frac{H}{a},\ t^* = \frac{tc}{\lambda},\ u^* = \frac{u}{c},\ v^* = \frac{v}{c\delta},\ \delta = \frac{a}{\lambda},\ P^* = \frac{Pa^2}{\mu\lambda c},\ \Omega^* = \frac{\Omega c}{a},\ J^* = \frac{J}{a^2},\ \theta = \frac{T - T_0}{T_1 - T_0},$$
$$\varphi^* = \frac{\varphi - \varphi_o}{\varphi_1 - \varphi_o} \tag{8}$$

After substituting from Equation (8), Equations (3 to 7) can be written in dimensionless form after dropping the star mark as:

$$Re\,\delta\left[u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial P}{\partial x} + \frac{k}{\mu}\frac{\partial \Omega}{\partial y} + \frac{k+\mu}{\mu}\left[\delta^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] - \left(\frac{a^2}{k_1} + \frac{\sigma B_0^2 a^2}{\mu}\right)u \tag{9}$$

$$Re\,\delta^3\left[u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial P}{\partial y} + \delta^2\frac{k}{\mu}\frac{\partial \Omega}{\partial x} + \delta^2\frac{k+\mu}{\mu}\left[\delta^2\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] - \delta^2\frac{a^2}{k_1}v \tag{10}$$

$$Re\,\delta\,\frac{\mu J}{k}\left[u\frac{\partial \Omega}{\partial x} + v\frac{\partial \Omega}{\partial y}\right] = -2\Omega + \left[\delta^2\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] + \frac{\gamma_1}{a^2 k}\left[\delta^2\frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2}\right] \tag{11}$$

$$Re\,P_r\delta\left[u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}\right] = \left[\delta^2\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right] - \gamma P_r\theta \tag{12}$$

$$\delta\left[u\frac{\partial \varphi}{\partial x} + v\frac{\partial \varphi}{\partial y}\right] = \frac{1}{S_c}\left[\delta^2\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}\right] + S_r\left[\delta^2\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right] - S\varphi \tag{13}$$

For long wavelength that is $\delta \ll 1$ and low Reynolds number that is $Re \to 0$ the system of our Equations (9-13) can be reduced to:

$$\frac{1}{1-N}\frac{\partial^2 u}{\partial y^2} + \frac{N}{1-N}\frac{\partial \Omega}{\partial y} - n^2 u = \frac{\partial P}{\partial x} \tag{14}$$

$$\frac{\partial P}{\partial y} = 0 \tag{15}$$

$$\frac{2-N}{m^2}\frac{\partial^2 \Omega}{\partial y^2} - \frac{\partial u}{\partial y} - 2\Omega = 0 \tag{16}$$

$$\frac{\partial^2 \theta}{\partial y^2} - \gamma P_r\theta = 0 \tag{17}$$

$$\frac{1}{S_c}\frac{\partial^2 \varphi}{\partial y^2} + S_r\frac{\partial^2 \theta}{\partial y^2} - S\varphi = 0 \tag{18}$$

Where $N = \frac{k}{k+\mu}$ is the coupling number, $n^2 = a_1^2 + M^2$, $a_1^2 = \frac{a^2}{k_1}$ is the porosity parameter, $M^2 = \frac{\sigma B_0^2 a^2}{\mu}$ is the magnetic parameter, $m^2 = \frac{a^2 k(2\mu + k)}{\gamma_1(\mu + k)}$ is the micropolar parameter, $P_r = \frac{\mu c_p}{k}$ is the Prandtl number, $S_c = \frac{\mu}{\rho D_m}$ is the Schmidt number, $S_r = \frac{D_m k_T \rho (T_1 - T_o)}{T_m \mu (\varphi_1 - \varphi_o)}$ is the Soret number, $Re = \frac{\rho c a}{\mu}$ is the Reynolds number, $\gamma = \frac{Q_o a^2}{\mu c_p}$ is the coefficient of heat absorption and $S = \frac{k_2 a}{c}$ is the coefficient of chemical reaction. From Equation (15), it is clear that P is independent of y. Therefore Equation (14) can be written as:

$$\frac{\partial^2 u}{\partial y^2} - n^2(1-N)u + N\frac{\partial \Omega}{\partial y} - (1-N)\frac{dP}{dx} = 0 \qquad (19)$$

The dimensionless boundary conditions are:

$$\frac{\partial u}{\partial y} = 0\ ,\ \frac{\partial \Omega}{\partial y} = 0\ ,\ \theta = 0\ \ and\ \ \varphi = 0 \qquad at \qquad y = 0 \qquad (20)$$

$$u = -1\ ,\ \Omega = 0\ ,\ \theta = 1\ \ and\ \ \varphi = 1 \qquad at \qquad y = h \qquad (21)$$

Introducing the stream functions ψ such that:

$$u = \frac{\partial \psi}{\partial y}$$

Then Equations (16) and (19) takes the form:

$$\frac{\partial^3 \psi}{\partial y^3} - n^2(1-N)\frac{\partial \psi}{\partial y} + N\frac{\partial \Omega}{\partial y} - p_1 = 0 \qquad (22)$$

$$\frac{2-N}{m^2}\frac{\partial^2 \Omega}{\partial y^2} - \frac{\partial^2 \psi}{\partial y^2} - 2\Omega = 0 \qquad (23)$$

Where $p_1 = (1-N)p, \qquad p = \frac{dP}{dx}$

Using the condition $\psi = 0$ at $y = 0$, the general solution of Equations (17), (18) (22) and (23) by using the boundary conditions (20) and (21) are given by:

$$\psi = \frac{c_1 + 2p_1 y}{b_1} + e^{b_2 y}c_2 + e^{-b_2 y}c_3 + e^{b_3 y}c_4 + e^{-b_3 y}c_5 \qquad (24)$$

$$u = \frac{2p_1}{b_1} + e^{b_2 y}b_2 c_2 - e^{-b_2 y}b_2 c_3 + e^{b_3 y}b_3 c_4 - e^{-b_3 y}b_3 c_5 + \frac{2p_1}{b_1} \qquad (25)$$

$$\Omega = \frac{e^{-y(\frac{\sqrt{2}}{\sqrt{\alpha}}+b_2+b_3)}\left(e^{y b_3} b_5 \left(e^{y b_2}\left(e^{\frac{2\sqrt{2}y}{\sqrt{\alpha}}} c_6 + c_7\right) b_4 + e^{\frac{\sqrt{2}y}{\sqrt{\alpha}}} b_2^2 \left(e^{2y b_2} c_2 + c_3\right)\right) + e^{y(\frac{\sqrt{2}}{\sqrt{\alpha}}+b_2+2b_3)} b_3^2 b_4 c_4 + e^{y(\frac{\sqrt{2}}{\sqrt{\alpha}}+b_2)} b_3^2 b_4 c_5\right)}{b_4 b_5} \qquad (26)$$

$$\theta = \frac{1}{2} e^{(h-y)\sqrt{\gamma}\sqrt{P_r}}\left(1 + e^{2y\sqrt{\gamma}\sqrt{P_r}}\right)\left(-1 + \mathrm{Coth}[h\sqrt{\gamma}\sqrt{P_r}]\right) \qquad (27)$$

$$\Omega = \frac{e^{-y(\frac{\sqrt{2}}{\sqrt{\alpha}}+b_2+b_3)}\left(e^{y b_3} b_5 \left(e^{y b_2}\left(e^{\frac{2\sqrt{2}y}{\sqrt{\alpha}}} c_6 + c_7\right) b_4 + e^{\frac{\sqrt{2}y}{\sqrt{\alpha}}} b_2^2 \left(e^{2y b_2} c_2 + c_3\right)\right) + e^{y(\frac{\sqrt{2}}{\sqrt{\alpha}}+b_2+2b_3)} b_3^2 b_4 c_4 + e^{y(\frac{\sqrt{2}}{\sqrt{\alpha}}+b_2)} b_3^2 b_4 c_5\right)}{b_4 b_5} \qquad (28)$$

Where $c_1 \rightarrow c_7$, $b_1 \rightarrow b_5$ and α are defined in the Appendix.

RESULTS AND DISCUSSION

In this study, the peristaltic motion of an incompressible micropolar fluid through a porous medium in a symmetric channel under the effects of heat absorption and chemical reaction in the presence of magnetic field was considered. In order to see the quantitative effects of the various parameters involved in the results on the pumping characteristic, the Mathematical program was used. The effect of physical parameters on the velocity distribution is indicated through Figures 2 to 6. In these figures the velocity distribution u is plotted against the coordinate y. Figures 1 and 2 illustrate the effect of the coupling number N. It is found that the velocity increases with increasing N. The effect of the micropolar parameter m is to decrease the velocity at fixed values of y with an increase in m which is shown in Figure 3. It is found from Figure 4, that the velocity u at fixed values of y decreased by increasing the pressure p. The effects of the porosity parameter a_1 and the magnetic parameter M are indicated graphically through Figures 5 and 6.

In these figures, we observed that velocity u at fixed values of y increases with the increase of a_1 and M. Figures 7 to 11 represents the effects of the above parameters on the microrotation velocity Ω when potted against y. Figures 7 and 8 are graphed to illustrate the effects of the coupling number N and the micropolar parameter m. It is seen that microrotation velocity Ω at fixed values of y decreases as N and m increases, this occurs near the center and the inverse effect will occur near the upper wall. Figure 9 illustrates the effect of the pressure p on the microrotation velocity velocity Ω at fixed values of y. We can see that the microrotation velocity Ω increases by increasing p, this occurs near the center and the inverse effect will occur near the upper wall.

Figures 10 and 11 shows the variation of the microrotation velocity Ω for different values of the porosity parameter a_1 and the magnetic parameter M. It is found that the microrotation velocity Ω at fixed values of y decreases with an increase in a_1 and M this occurs near the center and the inverse effect occurs near the upper wall. The effect of the different parameters on the temperature when plotted against y is indicated graphically through Figures 12 and 13. In these figures the temperature θ at fixed values of y decrease by

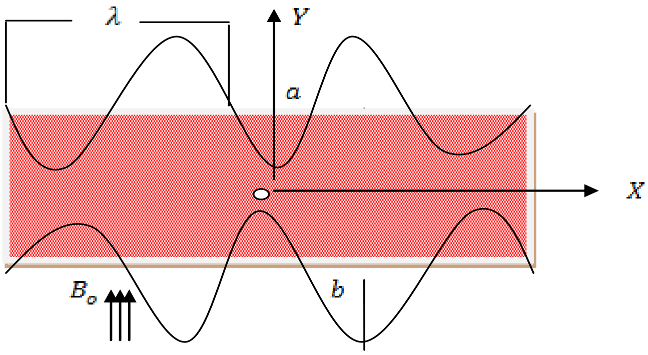

Figure 1. The physical model.

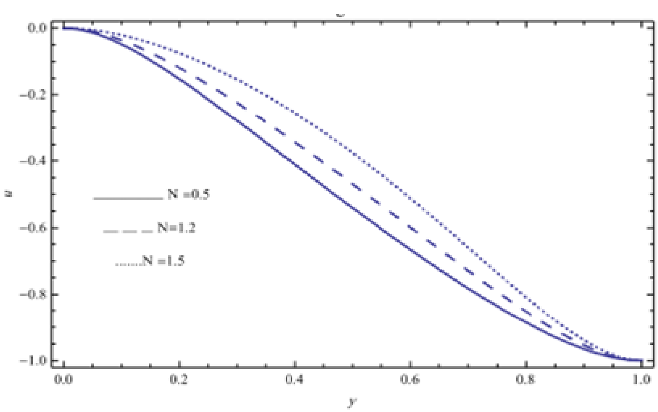

Figure 2. The velocity distribution u is plotted against y for different values of N when $m = 8\,, p = 1$, a_1 =0.1, M =0.3, $\epsilon = 0.5$, $x = \pi, t = \pi$.

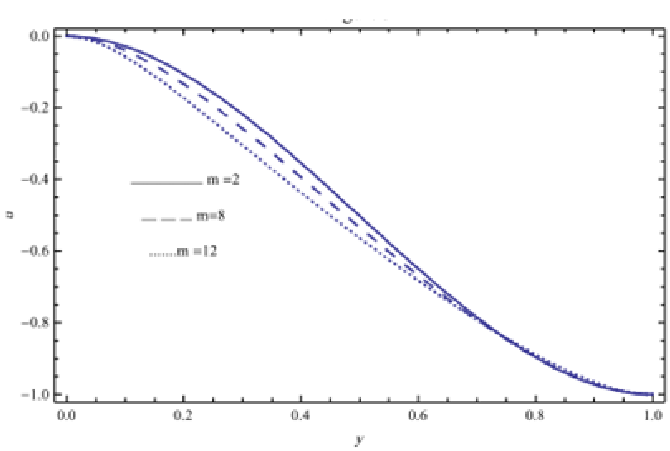

Figure 3. The velocity distribution u is plotted against y for different values of m when $N = 0.2\,, p = 1$, a_1 =0.1, M =0.3, $\epsilon = 0.5$, $x = \pi, t = \pi$.

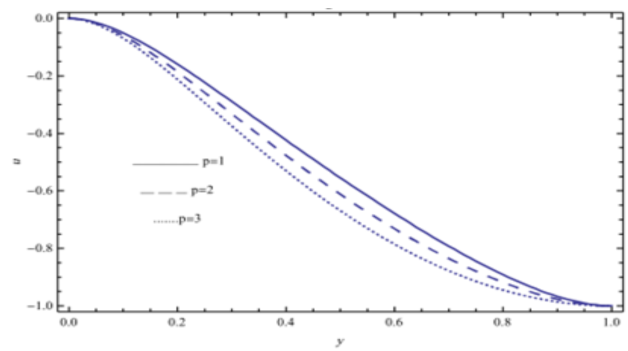

Figure 4. The velocity distribution u is plotted against y for different values of p when $m = 8\,, N = 0.2$, a_1 =0.1, M =0.3, $\epsilon = 0.5$, $x = \pi, t = \pi$.

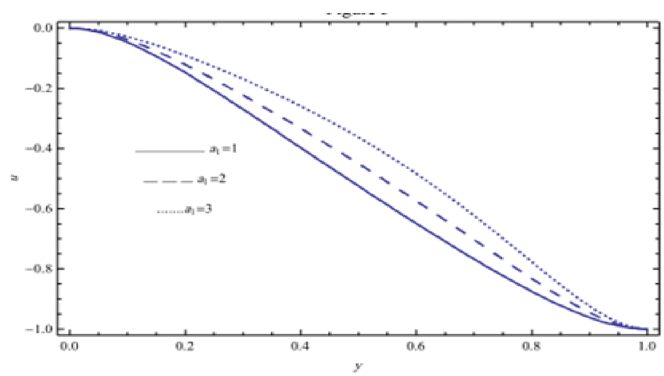

Figure 5. The velocity distribution u is plotted against y for different values of a_1 when $m = 8\,, N = 0.2$, $p = 1$, M =0.3, $\epsilon = 0.5$, $x = \pi, t = \pi$.

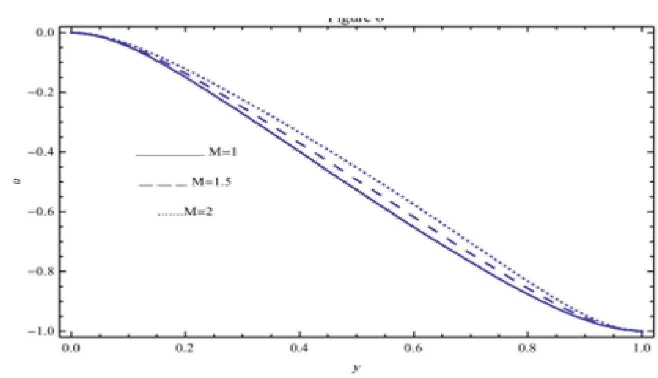

Figure 6. The velocity distribution u is plotted against y for different values of M when $m = 8\,, N = 0.2$, $p = 1$, a_1 =0.1, $\epsilon = 0.5$, $x = \pi, t = \pi$.

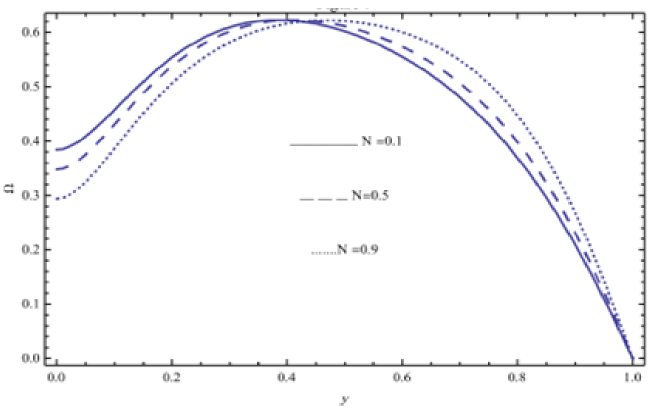

Figure 7. The microrotation velocity Ω is plotted against y for different values of N when $m = 8, p =1$, a_1 =0.1, M =0.3, $\epsilon = 0.5$, $x = \pi, t = \pi$.

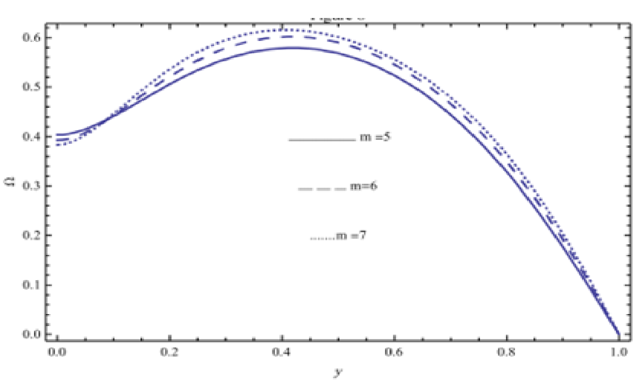

Figure 8. The microrotation velocity Ω is plotted against y for different values of m when $N = 0.2, p = 1$, a_1 =0.1, M =0.3, $\epsilon = 0.5$, $x = \pi, t = \pi$.

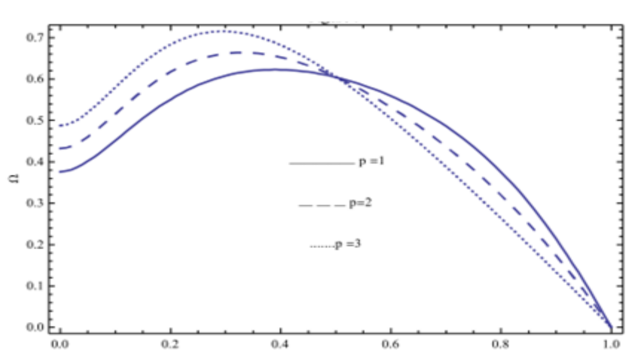

Figure 9. The microrotation velocity Ω is plotted against y for different values of p when $m = 8, N = 0.2$, a_1 =0.1, M =0.3, $\epsilon = 0.5$, $x = \pi, t = \pi$.

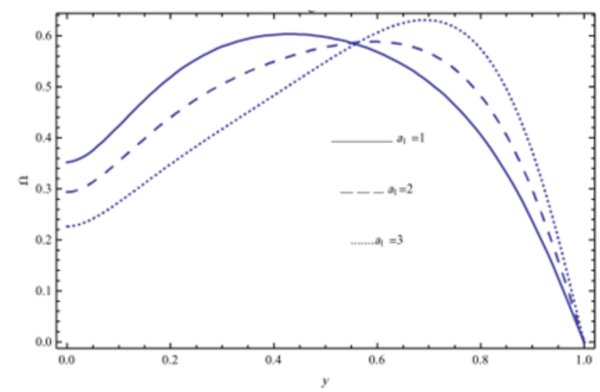

Figure 10. The microrotation velocity Ω is plotted against y for different values of a_1 when $m = 8, N = 0.2$, $p = 1$, M =0.3, $\epsilon = 0.5$, $x = \pi, t = \pi$.

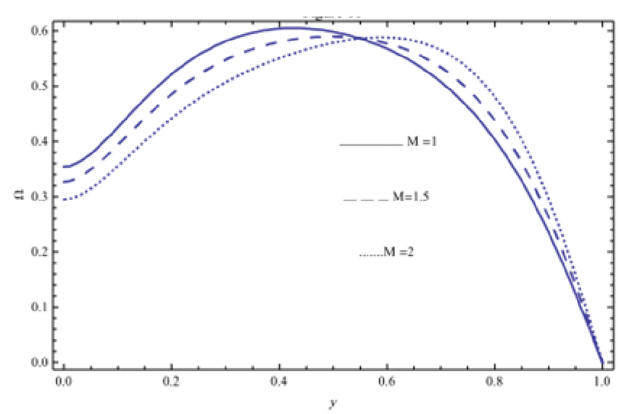

Figure 11. The microrotation velocity Ω is plotted against y for different values of M when $m = 8, N = 0.2$, $p = 1$, a_1 =0.1, $\epsilon = 0.5$, $x = \pi, t = \pi$.

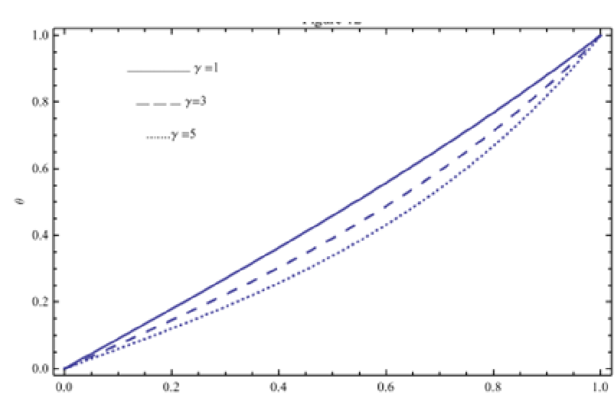

Figure 12. The temperature θ is plotted against y for different values of γ when $\Pr = .71$, $\epsilon = 0.5$, $x = \pi, t = \pi$.

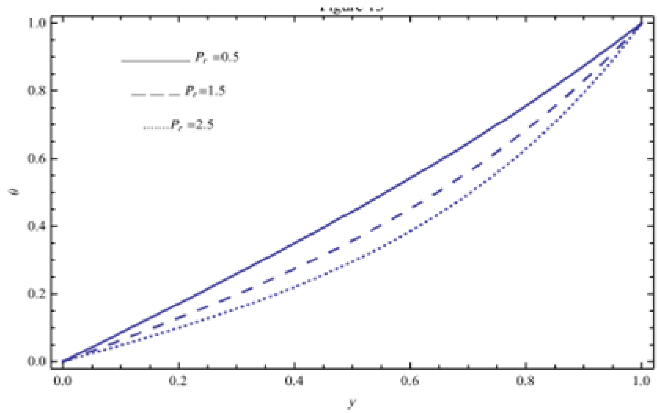

Figure 13. The temperature θ is plotted against y for different values of P_r when $\epsilon = 0.5$, $x = \pi, t = \pi$, $\gamma = 0.5$.

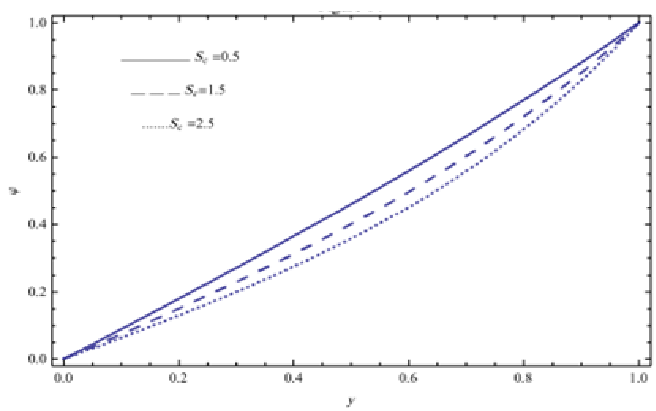

Figure 14. The concentration φ is plotted against y for different values of S_C when $\gamma = 2, S_r = 0.5, S = 2$.

increasing the coefficient of heat absorption γ and the Prandtl number P_r. Figure 14 illustrates the effect of the Schmidt number S_c on the concentration φ when plotted against y. It is clear that φ decrease by increasing S_c. In Figures 15 and 16 the concentration φ increased by increasing Soret number S_r and the coefficient of heat absorption γ. Figure 17 represents the effect of the coefficient of chemical reaction S on the concentration φ. It is observed that φ decreased by increasing S.

CONCLUSION

In this paper, we modeled the effects of heat absorption and chemical reaction in the presence of magnetic field on the peristaltic flow of a viscous incompressible

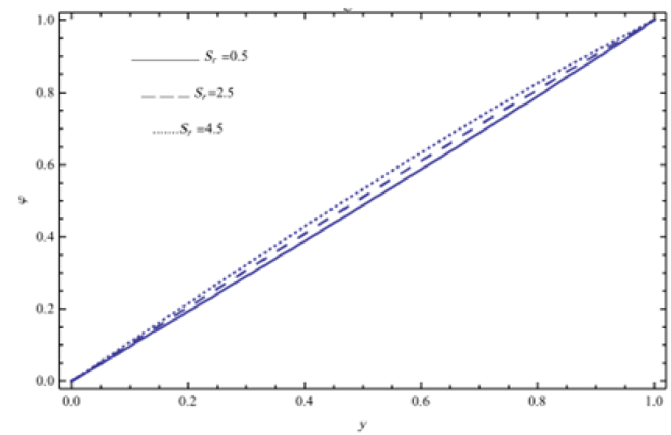

Figure 15. The concentration φ is plotted against y for different values of S_r when $= 2, S_c = 0.5, S = 2$.

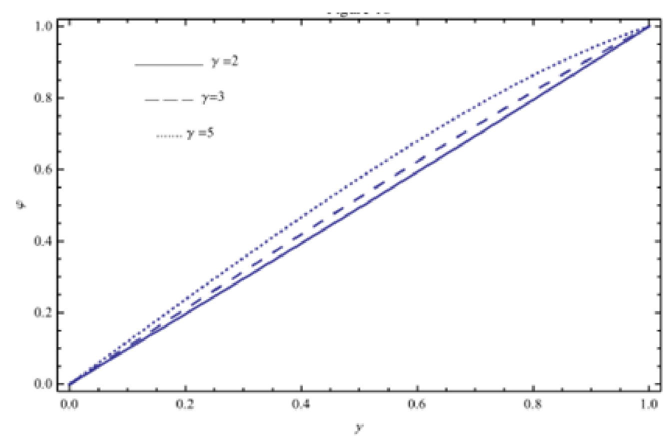

Figure 16. The concentration φ is plotted against y for different values of γ when $S_r = 0.5,\ S_c = 0.5, S = 2$.

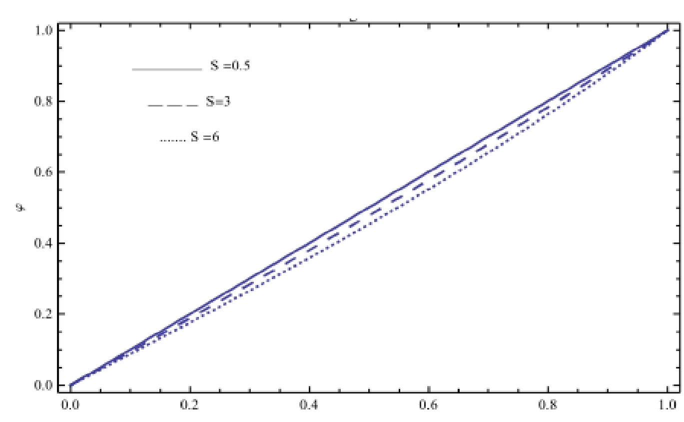

Figure 17. The concentration φ is plotted against y for different values of S when $S_r = 0.5,\ S_c = 0.5, \gamma = 2$.

micropolar fluid through a porous medium in a symmetric channel. The resulting equation which controls the motion of a micropolar fluid are solved by using the Mathematical program in the case of long wave length and low Reynolds number. The stream function, velocity, microrotation velocity, temperature and concentration of micropolar fluid are obtained. The effects of the problem parameters such as the coupling number N, the micropolar parameter m, the pressure p, the coefficient of heat absorption γ, the porosity parameter a_1 , the coefficient of chemical reaction S, the Schmidt number S_c, Soret number S_r and the magnetic parameter M on these distributions were discussed using a set of graphs.

Finally we came to the conclusion that the micropolar parameters, magnetic parameter, gravity parameter and permeability have a deep effect on the onset of the convection in porous medium. It is believed that the present work will serve for understanding more complex problems including the various physical effects investigated in the present problem.

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APPENDIX

$$c_1 = \frac{2(b_2^2-b_3^2)(2e^{h(b_2+b_3)}b_1(\text{Sinh}[hb_2]b_2-\text{Sinh}[hb_3]b_3)+(-1+e^{hb_3})k_1((-1-e^{hb_2})(-1+e^{hb_3})b_2+b_3k_1k_2)p_1)}{b_2b_3(-2(1+e^{2hb_2}+e^{2hb_3}-4e^{h(b_2+b_3)}+e^{2h(b_2+b_3)})b_2b_3+b_2^2k_5k_6+b_3^2k_5k_6)}$$

$$c_2 = \frac{b_3(b_1((e^{hb_2}-2e^{hb_3}+e^{h(b_2+2b_3)})b_2-e^{hb_2}b_3k_6)+2k_2((-1-e^{hb_3})b_3k_1+b_2k_2k_3)p_1)}{b_1b_2(-2(1+e^{2hb_2}+e^{2hb_3}-4e^{h(b_2+b_3)}+e^{2h(b_2+b_3)})b_2b_3+b_2^2k_5k_6+b_3^2k_5k_6)}$$

$$c_3 = \frac{e^{hb_2}b_3(-b_1((1+e^{2hb_3}-2e^{h(b_2+b_3)})b_2+b_3k_6)+2((-1-e^{hb_2})b_2k_2^2+b_3k_1k_6)p_1)}{b_1b_2(-2(1+e^{2hb_2}+e^{2hb_3}-4e^{h(b_2+b_3)}+e^{2h(b_2+b_3)})b_2b_3+b_2^2k_5k_6+b_3^2k_5k_6)}$$

$$c_4 = \frac{b_2(b_1((-2e^{hb_2}+e^{hb_3}+e^{h(2b_2+b_3)})b_3-e^{hb_3}b_2k_5)+2k_1((-1-e^{hb_2})b_2k_2+b_3k_1k_4)p_1)}{b_1b_3(-2(1+e^{2hb_2}+e^{2hb_3}-4e^{h(b_2+b_3)}+e^{2h(b_2+b_3)})b_2b_3+b_2^2k_5k_6+b_3^2k_5k_6)}$$

$$c_5 = \frac{e^{hb_3}b_2(-b_1((1+e^{2hb_2}-2e^{h(b_2+b_3)})b_3+b_2k_5)+2((-1-e^{hb_3})b_3k_1^2+b_2k_2k_5)p_1)}{b_1b_3(-2(1+e^{2hb_2}+e^{2hb_3}-4e^{h(b_2+b_3)}+e^{2h(b_2+b_3)})b_2b_3+b_2^2k_5k_6+b_3^2k_5k_6)}$$

$$c_6 = \frac{1}{\sqrt{2}(1+e^{\frac{2\sqrt{2}h}{\sqrt{\alpha}}})(-2+\alpha b_2^2)(-2+\alpha b_3^2)} e^{-h(b_2+b_3)}(-e^{h(b_2+b_3)}\sqrt{\alpha}b_2^3(-2+\alpha b_3^2)(c_2-c_3)+2e^{hb_2}b_3^2(e^{hb_3}\sqrt{\alpha}b_3(c_4-c_5)+\sqrt{2}e^{\frac{\sqrt{2}h}{\sqrt{\alpha}}}(e^{2hb_3}c_4+c_5))-b_2^2(\sqrt{2}e^{h(\frac{\sqrt{2}}{\sqrt{\alpha}}+2b_2+b_3)}(-2+\alpha b_3^2)c_2+\sqrt{2}e^{h(\frac{\sqrt{2}}{\sqrt{\alpha}}+b_3)}(-2+\alpha b_3^2)c_3+e^{hb_2}\alpha b_3^2(\sqrt{2}e^{\frac{\sqrt{2}h}{\sqrt{\alpha}}+2hb_3}c_4+e^{hb_3}\sqrt{\alpha}b_3(c_4-c_5)+\sqrt{2}e^{\frac{\sqrt{2}h}{\sqrt{\alpha}}}c_5)))$$

$$c_7 = \frac{1}{\sqrt{2}(1+e^{\frac{2\sqrt{2}h}{\sqrt{\alpha}}})(-2+\alpha b_2^2)(-2+\alpha b_3^2)} e^{-h(b_2+b_3)}(e^{h(\frac{2\sqrt{2}}{\sqrt{\alpha}}+b_2+b_3)}\sqrt{\alpha}b_2^3(-2+\alpha b_3^2)(c_2-c_3)+2e^{h(\frac{\sqrt{2}}{\sqrt{\alpha}}+b_2)}b_3^2(-e^{h(\frac{\sqrt{2}}{\sqrt{\alpha}}+b_3)}\sqrt{\alpha}b_3(c_4-c_5)+\sqrt{2}(e^{2hb_3}c_4+c_5))-e^{\frac{\sqrt{2}h}{\sqrt{\alpha}}}b_2^2(-e^{h(\frac{\sqrt{2}}{\sqrt{\alpha}}+b_2+b_3)}\alpha^{3/2}b_3^3(c_4-c_5)+\sqrt{2}(e^{h(2b_2+b_3)}(-2+\alpha b_3^2)c_2+e^{hb_3}(-2+\alpha b_3^2)c_3+e^{hb_2}\alpha b_3^2(e^{2hb_3}c_4+c_5))))$$

$$b_1 = 2(-1+a)n^2$$

$$b_2 = \sqrt{\left(-\frac{m^2}{-2+N}+\frac{Nm^2}{2(-2+N)}-\frac{n^2}{-2+N}+\frac{3Nn^2}{2(-2+N)}-\frac{N^2n^2}{2(-2+N)}-\frac{\sqrt{(2m^2-Nm^2+2n^2-3Nn^2+N^2n^2)^2-4(-2+N)(-2m^2n^2+2Nm^2n^2)}}{2(-2+N)}\right)}$$

$$b_3 = \sqrt{\left(-\frac{m^2}{-2+N}+\frac{Nm^2}{2(-2+N)}-\frac{n^2}{-2+N}+\frac{3Nn^2}{2(-2+N)}-\frac{N^2n^2}{2(-2+N)}+\frac{\sqrt{(2m^2-Nm^2+2n^2-3Nn^2+N^2n^2)^2-4(-2+N)(-2m^2n^2+2Nm^2n^2)}}{2(-2+N)}\right)}$$

$b_4 = -2+\alpha b_2^2$, $b_5 = -2+\alpha b_3^2$, $\alpha = \frac{2-N}{m^2}$

$k_1 = -1+e^{hb_2}$, $k_2 = -1+e^{hb_3}$, $k_3 = 1+e^{hb_2}$, $k_4 = 1+e^{hb_3}$, $k_5 = -1+e^{2hb_2}$, $k_6 = -1+e^{2hb_3}$