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Integral Tau method for fourth-order ordinary differential equations with third degree over determination

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This paper is concerned with the solution of a class of fourth order initial value problems in ordinary differential equation by the integrated formulation of the Tau method. The initial focus is the class with a maximum of third degree overdetermination. The matrix equations were constructed based on the degree of overdetermination and for purpose of automation. The automated Tau system was tested on some selected problems to validate the study numerical evidences, thus obtained, to confirm the accuracy of the method.

Key words: Tau method, variant, formulation, approximant, perturbation term.

INTRODUCTION

Lanczos proposed the Tau method techniques in 1983 for the numerical solution of ordinary differential equation with some conditions given as

$$Ly_n(x)i = \sum_{r=0}^{m}\left(\sum_{k=0}^{N} P_{rk}\ x^k\right)y^{(r)}(x) = \sum_{r=0}^{n} f_r x^r a \le x \le b \quad (1)$$

$$L * Y(x_{rk}) = \sum_{r=0}^{m-!} a_{rk}\, y^{(r)}(x_{rk}) = \alpha_k \quad k = 1(1)m \qquad (2)$$

By seeking an approximate solution of the form

$$Yn(x) = \sum_{r=0}^{n} a_r x^r \qquad (3)$$

r< $+x$ of $y(x)$ which is the exact solution of the corresponding perturbed system

$$L * Yn(x) = \sum_{r=0}^{n} f_r\, x^r Hn(x) \qquad (4)$$

$LY_n(x_{nk}) = \propto_k$ k= 1(1)m (5)

Where L is the linear differential operator $\propto_k,\ f_r,\ P_{nk,}\ {}_{-}N_M : r = 0, 1(1)m.\quad K = 0(1)N_r$ *a* and *b* are real constants, y(r) denoted the derivatives oforder r of y(x).

The perturbation term $H_n(x)$ in Equation (4) is defined by

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$$H_n(x) = \sum_{i=0}^{m+s-1} \tau_{i+1} T_{n-m+i+1}(x)$$
$$= \sum_{i=0}^{m+s-1} \tau_{i+1} \sum_{i=0}^{n-m+i+1} C_r^{(n-m+i+1)} x^r \quad (6)$$

and $C_r^{(n)}s$ are the coefficient of power of x $(that\ is\ x^r)$ $in\ the\ nth$ degree Chebyshev polynomial denoted and defined by

$$T_n(x) = \cos\left(ncos^{-1}\left[\frac{2x-a-b}{b-a}\right]\right) = \sum_{r=0}^{n} C_r^{(n)} \quad (7)$$

The $r's$ are the free Tau parameters to be determined alongside with $a_r$and S is the number of over-determination of Equation (1), which is defined by

$$S = max[N_r - r: 0 \leq r \leq m, N_r \geq r] \geq 0 \quad (8)$$

LITERATURE REVIEW

The Tau method was initially formulated as a tool for the approximation of special function of mathematical physics which could be expressed in terms of simple differential equations. It later developed into a powerful and accurate tool for the numerical solution of complex differential and functional equations. The main idea in it is to solve approximate problem. Accurate approximate polynomial solution in a linear ordinary differential equation with polynomial coefficient can be obtained by the Tau method introduced in Lanczos (1983). The method is related to the principle of economization of a differentiable function implicitly defined by a linear differential equation with polynomial coefficient. Techniques based on the Tau method have been reported in the literature with application to more general equations including non-linear ones (Onumanyi and Oritz, 1982; Oritz, 1974), while techniques based on direct Chebyshev replacement have been discussed in Fox and Parker (1968) and more recently in the work of Mousavi and Mousavi (2012). Further details on the Tau method can be found in Adeniyi (1991), Aliyu (2012), Lanczos (1956), Adeniyi and Aliyu (2011), Yisa (2012) and Adeniyi and Aliyu (2007). Because of the limitation in some of the works (Aliyu, 2012), this study seeks to extend the scope to fourth order problems with third degree overdetermination.

Integrated formulation of the Tau method

Description of the integrated formulation

Let us consider the m-th order linear differential system

$$Ly(x) := \sum_{r=0}^{m} \alpha_r(x) y^{(r)}(x) = \sum_{r=0}^{f} f_r x^r, a \leq x \leq b \quad (9)$$

$$L^* y(x_{rx}) := \sum_{r=0}^{m-1} \alpha_{rk} y^{(r)}(x_{rk}) = \alpha_k, \qquad k = 1(1)m \quad (10)$$

Let $\iiint \overset{i}{\cdots} \int y(x)dx$ denote the indefinite integration i times applied to the function $g(x)$ and let

$$I_L = \iiint \overset{m}{\cdots} \int L(\cdot)dx \quad (11)$$

The integral form of (11) now becomes

$$I_L(y(x)) = \iiint \overset{m}{\cdots} \int f(x)dx + C_{m-1}(x) \quad (12)$$

The Tau approximant $y_n(x)$ of Equation (16), satisfies the perturbed problem:

$$I_L(y_n(x)) = \iiint \overset{m}{\cdots} \int f(x)dx + H_{n+m+1}(x) \quad (13)$$

$$L^* y_n(x_{rk}) = \alpha_k, k = 1(1)m \quad (14)$$

where:

$$H_{n+m}(x) = \sum_{r=0}^{m+s+1} \tau_{m+s-r} T_{n-m+r+1}(x) \quad (15)$$

A CLASS OF OVERDETERMINED FOURTH ORDER DIFFERENTIAL EQUATIONS

We consider here the integrated form of the Tau method for the class of problems:

$$Ly(x) := \sum_{r=0}^{m} P_r(x) y^{(r)}(x) = f(x), \qquad a \leq x \leq b \quad (16)$$

$$L^* y_n(x_{rk}) = \sum_{r=0}^{m-1} a_{rk} y_n^{(r)}(x_{rk}) = \alpha_k, \ k = 0(1)(m-1) \quad (17)$$

$$P_r(x) = \sum_{k=0}^{N_r} p_{rk} x^k \quad (18)$$

for the case $m = 4$ and $s = 3$.

So, we now derive a fifth degree approximants for the equation. From Equation (16), the general case for $m = 4$, $s = 3$ is given by

$$Ly(x) \coloneqq (\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 + \alpha_4 x^4 + \alpha_5 x^5 + \alpha_6 x^6 + \alpha_7 x^7) y^{iv}(x) + \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4 + \beta_5 x^5 + \beta_6) y^{iii}(x) + (\gamma_0 + \gamma_1 x + \gamma_2 x^2 + \gamma_3 x^3 + \gamma_4 x^4 + \gamma_5 x^5) y^{ii}(x) (\lambda_0 + \lambda_1 x + \lambda_2 x^2 + \lambda_3 x^3 + \lambda_4 x^4) y^{i}(x) + (\mu_0 + \mu_1 x + \mu_2 x^2 + \mu_3 x^3) y(x) = f(x) = \sum_{r=0}^{n} f_r x^r \quad (19)$$

$$y(0) = \rho_0, y'(0) = \rho_1, y''(0) = \rho_2, y'''(0) = \rho_3$$

where, for convenience, we have chosen $\alpha, \beta, \gamma, \lambda$ and μ to denote $\rho_4, \rho_3, \rho_2, \rho_1$ and ρ_0 respectively; $x_{rk} = 0$ and $a = 0$. That is,

$$Ly(x) \coloneqq \int_0^x\int_0^u\int_0^t\int_0^s (\alpha_0 + \alpha_1 v + \alpha_2 v^2 + \alpha_3 v^3 + \alpha_4 v^4 + \alpha_5 v^5 + \alpha_6 v^6 + \alpha_7 v^7) y^{iv}(v) dv ds dt du + \int_0^x\int_0^u\int_0^t\int_0^s (\beta_0 + \beta_1 v + \beta_2 v^2 + \beta_3 v^3 + \beta_4 v^4 + \beta_5 v^5 + \beta_6) y'''(v) dv ds dt du + \int_0^x\int_0^u\int_0^t\int_0^s (\gamma_0 + \gamma_1 v + \gamma_2 v^2 + \gamma_3 v^3 + \gamma_4 v^4 + \gamma_5 v^5) y''(v) dv ds dt du + \int_0^x\int_0^u\int_0^t\int_0^s (\lambda_0 + \lambda_1 v + \lambda_2 v^2 + \lambda_3 v^3 + \lambda_4 v^4) y'(v) dv ds dt du + \int_0^x\int_0^u\int_0^t\int_0^s (\mu_0 + \mu_1 v + \mu_2 v^2 + \mu_3 v^3) y(v) dv ds dt du = \int_0^x\int_0^u\int_0^t\int_0^s f(V) dv ds dt du + \tau_1 T_{n+7}(x) + \tau_1 T_{n+6}(x) + \tau_2 T_{n+5}(x) + \tau_3 T_{n+4}(x) + \tau_4 T_{n+3}(x) + \tau_5 T_{n+2}(x) + \tau_6 T_{n+1}(x) \quad (20)$$

After simplifying and equating the corresponding coefficient powers of x, we have the recurrence relation:

$$\alpha_0 a_0 - \tau_1 C_0^{n+7} - \tau_2 C_0^{n+6} - \tau_3 C_0^{n+5} - \tau_4 C_0^{n+4} - \tau_5 C_0^{n+3} - \tau_6 C_0^{n+2} - \tau_7 C_0^{n+1} = \alpha_0 \rho_0 \quad (21)$$

$$\beta_0 a_0 + \alpha_0 a_1 - \tau_1 C_1^{n+7} - \tau_2 C_1^{n+6} - \tau_3 C_1^{n+5} - \tau_4 C_1^{n+4} - \tau_5 C_1^{n+3} - \tau_6 C_1^{n+2} - \tau_7 C_1^{n+1} = \alpha_1 \rho_1 - 2\alpha_0 \rho_1 - 2\beta_0 \rho_0 \quad (22)$$

$$\frac{1}{2}[(3\gamma_0 - 4\beta_1 - 7\alpha_2) a_0 - (\alpha_1 + \beta_0) a_1 - 2\alpha_0 a_0] - \tau_1 C_2^{n+7} - \tau_2 C_2^{n+6} - \tau_3 C_2^{n+5} - \tau_4 C_2^{n+4} - \tau_5 C_2^{n+3} - \tau_6 C_2^{n+2} - \tau_7 C_2^{n+1} = \frac{\alpha_0 \rho_1 + 2\alpha_2 \beta_2 - \beta_0 \gamma_0 - 3\gamma_0 \rho_0 - 4\beta_1 \rho_0 - 7\alpha_2 \rho_0 + \alpha_1 \rho_1 + 2\alpha_0 \rho_2}{2} \quad (23)$$

$$\frac{1}{6}[(30\alpha_3 + 12\beta_2 + 5\gamma_1 + 2\lambda_0) a_0 + (8\alpha_2 + 5\beta_1 + 3\gamma_0) a_1 - 2(2\alpha_1 + \beta_0) a_2 + \alpha_0 a_3] - \tau_1 C_3^{n+7} - \tau_2 C_3^{n+6} - \tau_3 C_3^{n+5} - \tau_4 C_3^{n+4} - \tau_5 C_3^{n+3} - \tau_6 C_3^{n+2} - \tau_7 C_3^{n+1} = \frac{181\alpha_3 \rho_0 - 3\alpha_1 \rho_2 + 6\alpha_2 \rho_1 - 3\beta_0 \rho_2 + 10\beta_2 \rho_0 + 6\gamma_1 \rho_0 + 2\gamma_0 \rho_1 + \lambda_0 \rho_0 - 5\alpha_0 \rho_3 + 3\rho_1 + 5\beta_1 \rho_1}{6} \quad (24)$$

$$\frac{1}{24}[(116\alpha_4 + 30\beta_3 + 8\gamma_2 + \lambda_1 + \mu_0) a_0 + (42\alpha_3 + 14\beta_2 + 4\gamma_1 + 2\lambda_0) a_1 + (18\alpha_2 + 12\beta_1 + 6\gamma_0) a_2 - (18\alpha_1 + 6\beta_0) a_3 + 24\alpha_0 a_4] - \tau_1 C_4^{n+7} - \tau_2 C_4^{n+6} - \tau_3 C_4^{n+5} - \tau_4 C_4^{n+4} - \tau_5 C_4^{n+3} - \tau_6 C_4^{n+2} - \tau_7 C_4^{n+1} = 0 \quad (25)$$

$$\frac{1}{120}[(720\alpha_5 + 168\beta_4 + 368\gamma_3 + 6\lambda_2 + \mu_1) a_0 + (264\alpha_4 + 96\beta_3 + 24\gamma_4 + 3\lambda_1 + \mu_0) a_1 + 2(42\alpha_3 + 34\beta_2 + 11\gamma_1 + 2\lambda_0) a_2 + (60\alpha_2 + 42\beta_1 + 18\gamma_0) a_3 + 24(4\alpha_1 + \beta_0) a_4 + 120\alpha_0 a_5] - \tau_1 C_5^{n+7} - \tau_2 C_5^{n+6} - \tau_3 C_5^{n+5} - \tau_4 C_5^{n+4} - \tau_5 C_5^{n+3} - \tau_6 C_5^{n+2} - \tau_7 C_5^{n+1} = 0 \quad (26)$$

$$\frac{1}{360}[(1440\alpha_6 + 900\beta_5 + 48\gamma_4 + 24\lambda_3 + \mu_2) a_0 + (1240\alpha_5 + 300\beta_4 + 60\gamma_3 + 8\lambda_2 + \mu_1) a_1 + (432\alpha_1 - 198\beta_3 - 46\gamma_2 - 5\lambda_1 + \mu_0) a_2 + (72\alpha_3 + 144\beta_2 + 42\gamma_1 + 6\lambda_0) a_3 + 120(11\alpha_2 - 8\beta_1 - 3\gamma_0) a_4 + 60(5\alpha_1 + \beta_0) a_5 + 360\alpha_0 a_6] - \tau_1 C_6^{n+7} - \tau_2 C_6^{n+6} - \tau_3 C_6^{n+5} - \tau_4 C_6^{n+4} - \tau_5 C_6^{n+3} - \tau_6 C_6^{n+2} - \tau_7 C_6^{n+1} = 0 \quad (27)$$

$$[\frac{[\alpha_1 k-\alpha_1+\beta_0]}{k}a_{k-1}+\frac{[\alpha_2(k-1)-(2\alpha_2+\beta_1)(k-1)-3(2\alpha_2+\beta_1+\gamma_0)]}{k(k-1)}a_{k-2}+[\alpha_3(k-2)$$

$$\frac{(k-1)k-(3\alpha_3+\beta_2)(k-2)(k-1)-3(6\alpha_3+2\beta_2+\gamma_1)(k-2)-2(6\alpha_3+2\beta_2+\gamma_1-\lambda_0)]}{(k-2)(k-1)k}a_{k-3}$$

$$[\alpha_4(k-3)(k-2)(k-1)k-(4\alpha_4+\beta_3)(k-3)(k-2)(k-1)-3(12\alpha_4+3\beta_3+\gamma_2)$$

$$\frac{(k-3)(k-2)-2(24\alpha_4+6\beta_3+2\gamma_2-\lambda_1)(k-3)+(24\alpha_4+6\beta_3+2\gamma_2+\lambda_1+\mu_0)]}{(k-3)(k-2)(k-1)k}a_{k-4}$$

$$[\alpha_5(k-3)(k-2)(k-1)k-(5\alpha_5+\beta_4)(k-3)(k-2)(k-1)-3(20\alpha_5+4\beta_4+\gamma_3)$$

$$\frac{(k-3)(k-2)-2(60\alpha_5+12\beta_4+3\gamma_3-\lambda_2)(k-3)-(120\alpha_5+24\beta_4+6\gamma_3+2\lambda_2+\mu_1)]}{(k-3)(k-2)(k-1)k}a_{k-5}$$

$$[\alpha_6(k-3)(k-2)(k-1)k-(6\alpha_6+\beta_5)(k-3)(k-2)(k-1)-3(30\alpha_6+5\beta_5+\gamma_4)$$

$$\frac{(k-3)(k-2)-2(120\alpha_6+120\beta_5+4\gamma_4+\lambda_3)(k-3)+(360\alpha_6+60\beta_5+12\gamma_4+3\lambda_3+\mu_2)]}{(k-3)(k-2)(k-1)k}a_{k-6}$$

$$[\alpha_7(k-3)(k-2)(k-1)k-(7\alpha_7+\beta_6)(k-3)(k-2)(k-1)-3(42\alpha_7+6\beta_6+\gamma_5)$$

$$\frac{(k-3)(k-2)-2(210\alpha_7+30\beta_6+5\gamma_5+\lambda_4)(k-3)+(840\alpha_7+120\beta_6+20\gamma_5+4\lambda_4+\mu_3)]}{(k-3)(k-2)(k-1)k}a_{k-7}$$

$$\tau_1 C_K^{n+7}-\tau_2 C_K^{n+6}-\tau_3 C_K^{n+5}-\tau_4 C_K^{n+4}-\tau_5 C_K^{n+3}-\tau_6 C_K^{n+2}-\tau_7 C_K^{n+1}]$$

$$=\frac{f_{k-7}}{(k)(k-1)(k-2)(k-3)(k-4)(k-5)(k-6)} \qquad (28)$$

$$\frac{1}{(n-2)(n-1)(n)(n+1)}[[(\alpha_7(n-2)[n^3-4n^2-120n-294]+840\alpha_7)-(\beta_6(n-2)[n^2+17n+62]-$$

$$120\beta_6)-(\gamma_5(n-2)(n-1)]-20\gamma_5)-(2\lambda_4(n-4)+\mu_3)a_{n-6}]+[(\alpha_6(n-2)(n^3-6n^2-85n-150)+360\alpha_6)$$

$$-(\beta_5(n-2)[n^2+14n+225]-60\beta_5)-\gamma_4(n-2)[3n+5]-12\gamma_4)-2\lambda_3(n-17)+\mu_2)]a_{n-5}+[(\alpha_5(n-2)$$

$$[n^3-5n^2-56n-60]-120\alpha_5)-(\beta_4(n-2)(n^2+11n+12)-24\beta_4)-\gamma_3(n-2)(3n+3)+6\gamma_3)+\lambda_2 n+\mu_1)]$$

$$a_{n-4}+[\alpha_4(n-2)[-35n^3+20n^2+15n)+24\alpha_4)-(\beta_3(n-2)[n^2-10n-3]+6\beta_3)+(\gamma_2(n-2)(-3n-1)$$

$$+2\gamma_2)+(\lambda_1(5-2n)+\mu_0)]a_{n-3}+[(\alpha_3(n-2)(n^3-3n^2-19n+6)]+\beta_2[2n^2+10n)+\gamma_1(1-3n)-2\lambda_0)]$$

$$a_{n-2}+[(\alpha_2(n-2)(n^2-7n+6)]-\beta_1[n+3)-3\gamma_0]a_{n-1}+[\alpha_1(n-2)(n^2-n)(n+\beta_0)]a_n]$$

$$-\tau_1 C_{n+1}^{n+7}-\tau_2 C_{n+1}^{n+6}-\tau_3 C_{n+1}^{n+5}-\tau_4 C_{n+1}^{n+4}-\tau_5 C_{n+1}^{n+3}-\tau_6 C_{n+1}^{n+2}-\tau_7 C_{n+1}^{n+1}]x^{n+1}$$

$$=\frac{f_{n-6}}{(n+1)(n)(n-1)(n-2)(n-3)(n-4)(n-5)} \qquad (29)$$

$$\frac{1}{(n-1)(n)(n+1)(n+2)}[[(\alpha_7(n-1)[n^3-4n^2-131n-420]+840\alpha_7)-(\beta_6(n-1)[n^2+19n+60]-$$

$$120\beta_6)-(\gamma_5(n-1)(3n+10)]-20\gamma_5)-(2\lambda_4(n-3)+\mu_3)a_{n-5}]+[(\alpha_6(n-1)(n^3-3n^2-86n-240)+$$

$$360\alpha_6)-(\beta_5(n-1)[n^2+16n+240)-60\beta_5)-\gamma_4(n-1)[3n+8]-12\gamma_4)-2\lambda_3(n+6)+\mu_2)]a_{n-4}+[(\alpha_5$$

$$(n-1)[n^3-2n^2-63n-120]-120\alpha_5)-(\beta_4(n-1)(n^2+13n+24)+24\beta_4)-\gamma_3(n-1)(3n+6)+6\gamma_3)-$$

$$2\lambda_2 n-\mu_1)]a_{n-3}+[\alpha_4(n+1)[n^3-39n^2+38n-48)-24\alpha_4)-(\beta_3(n-1)[n^2-2n)+6\beta_3)-(\gamma_2(n-1)$$

$$(3n+4)-2\gamma_2)+(\lambda_1(2n+3)+\mu_0)]a_{n-2}+[(\alpha_3(n-1)(n^3-19n)-12\alpha_3)-\beta_2(n^2-7n)-4\beta_2)$$

$$(\gamma_1(3n+2)-2\lambda_0)]a_{n-1}+[(\alpha_2(n-1)(n^2-7n)-\beta_1[n+4)-3\gamma_0]a_n-\tau_1 C_{n+2}^{n+7}-\tau_2 C_{n+2}^{n+6}-\tau_3 C_{n+2}^{n+5}$$

$$-\tau_4 C_{n+2}^{n+4}-\tau_5 C_{n+2}^{n+3}-\tau_6 C_{n+2}^{n+2}=\frac{f_{n-5}}{(n+2)(n+1)(n)(n-1)(n-2)(n-3)(n-4)} \qquad (30)$$

$$\frac{1}{(n)(n+1)(n+2)(n+3)}[[(\alpha_7 n[n^3-n^2-136n-554)+840\alpha_7)-(\beta_6 n)[n^2+21n+80]-$$
$$120\beta_6)-(\gamma_5 n(3n-7)]-20\gamma_5)-2\lambda_4(n-2)+\mu_3)]a_{n-4}]+[(\alpha_6 n(n^3-85n-336)+$$
$$360\alpha_6)-(\beta_5 n(n^2+18n+25)-60\beta_5)-(\gamma_4 n(3n-5)-12\gamma_4)-2\lambda_3(n-15)+\mu_2)]a_{n-3}+[(\alpha_5 n$$
$$[n^3+n^2-64n-184]-120\alpha_5)-(\beta_4 n(n^2+15n+38)+24\beta_4)-\gamma_3 n(3n+9)+6\gamma_3)-2\lambda_2(n+1)-\mu_1)]$$
$$a_{n-2}+[\alpha_4 n[n^3+2n^2-37n-86)+24\alpha_4)-(\beta_3 n(n^2+12n+23)-(\gamma_2 n(3n+7)-2\gamma_2)$$
$$-(\lambda_1(2n-1)+\mu_0)]a_{n-1}+[(\alpha_3 n(n^3+3n^2-16n-30)-\beta_2 n(n+9)-12\beta_2)-(\gamma_1(3n+5)-2\lambda_0)]a_n$$
$$-\tau_1 C_{n+3}^{n+7}-\tau_2 C_{n+3}^{n+6}-\tau_3 C_{n+3}^{n+5}-\tau_4 C_{n+3}^{n+4}-\tau_5 C_{n+3}^{n+3}$$
$$=\frac{f_{n-4}}{(n+3)(n+2)(n+1)(n)(n-1)(n-2)(n-3)} \quad (31)$$

$$\frac{1}{(n+1)(n+2)(n+3)(n+4)}[[(\alpha_7(n+1)[n^3+2n^2-135n-688]+840\alpha_7)-(\beta_6(n+1)(n^2+23n+102)$$
$$-120\beta_6)-(\gamma_5(n+1)(3n+16)]-20\gamma_5)-(2\lambda_4(n-1)+\mu_3)a_{n-3}]+[(\alpha_6(n+1)(n^3-9n^2-94n-282)+$$
$$360\alpha_6)-(\beta_5(n+1)(n^2+20n+276)-60\beta_5)-\gamma_4(n+1)[3n+14]-12\gamma_4)-2\lambda_3(n-14)+\mu_2)]a_{n-2}+$$
$$[(\alpha_5(n+1)[n^3+4n^2-59n-234]-120\alpha_5)-(\beta_4(n+1)(n^2+17n+54)+24\beta_4)-\gamma_3(n+1)(3n+36)+6\gamma_3)$$
$$-2\lambda_2(n+2)-\mu_1)]a_{n-1}+[\alpha_4(n+1)[n^3+5n^2-30n-120)+24\alpha_4)-(\beta_3(n+1)[n^2-4n-24)-6\beta_3)-$$
$$(\gamma_2(n+1)(3n+2)-2\gamma_2)-(\lambda_1(2n+1)+\mu_0)]a_n-\tau_1 C_{n+4}^{n+7}-\tau_2 C_{n+4}^{n+6}-\tau_3 C_{n+4}^{n+5}-\tau_4 C_{n+4}^{n+4}$$
$$=\frac{f_{n-3}}{(n+4)(n+3)(n+2)(n+1)(n)(n-1)(n-2)} \quad (32)$$

$$\frac{1}{(n+2)(n+3)(n+4)(n+5)}[[(\alpha_7(n+2)[n^3+5n^2-128n-822]+840\alpha_7)-(\beta_6(n+2)(n^2+25n+12)$$
$$-120\beta_6)-(\gamma_5(n+2)(3n+19)]-20\gamma_5)-(2\lambda_4 n+\mu_3)]a_{n-2}+[(\alpha_6(n+2)(n^3+6n^2-85n+520)+$$
$$360\alpha_6)-(\beta_5(n+2)(n^2+22n+297)-60\beta_5)-\gamma_4(n+2)(3n-5)-12\gamma_4)-2\lambda_3(n-13)+\mu_2)]a_{n-1}+$$
$$[(\alpha_5(n+2)[n^3+7n^2-48n-300]-120\alpha_5)-(\beta_4(n+2)(n^2+19n+72)+24\beta_4)-\gamma_3(n+2)(3n+15)+6\gamma_3)$$
$$-2\lambda_2(n+3)-\mu_1)]a_n-\tau_1 C_{n+5}^{n+7}-\tau_2 C_{n+5}^{n+6}-\tau_3 C_{n+5}^{n+5}=\frac{f_{n-2}}{(n+5)(n+4)(n+3)(n+2)(n+1)(n)(n-1)} \quad (33)$$

$$\frac{1}{(n+3)(n+4)(n+5)(n+6)}[[(\alpha_7(n+3)[n^3+8n^2-11n-944]+840\alpha_7)-(\beta_6(n+3)(n^2+27n+152)$$
$$-120\beta_6)-(\gamma_5(n+3)(n+14)]-20\gamma_5)-(2\lambda_4(n+1)+\mu_3)]a_{n-1}+[(\alpha_6(n+3)(n^3+9n^2+70n-600)+$$
$$360\alpha_6)-(\beta_5(n+3)(n^2+24n+320)-60\beta_5)-\gamma_4(n+3)(3n+20)-12\gamma_4)-2\lambda_3(n-12)+\mu_2)]a_n+$$
$$-\tau_1 C_{n+6}^{n+7}-\tau_2 C_{n+6}^{n+6}=\frac{f_{n-1}}{(n+6)(n+5)(n+4)(n+3)(n+2)(n+1)(n)} \quad (34)$$

$$\frac{1}{(n+4)(n+5)(n+6)(n+7)}[[(\alpha_7(n+4)[n^3+11n^2+58n-630]+840\alpha_7)-(\beta_6(n+4)$$
$$(n^2+29n+180)120\beta_6)-(\gamma_5(n+4)(3n+25)-20\gamma_5)-(2\lambda_4(n+2)+\mu_3)]a_n$$
$$-\tau_1 C_{n+7}^{n+7}=\frac{f_n}{(n+7)(n+6)(n+5)(n+4)(n+3)(n+2)(n+1)} \quad (35)$$

Table 1. Numerical results for Example 1 (Case $n = 8,9,10$).

x	$y_8(x)$	$y_9(x)$	$y_{10}(x)$	$\lvert y_9(x) - y_8(x)\rvert$	$\lvert y_{10}(x) - y_9(x)\rvert$
0	1.0000000000	1.0000000000	1.0000000000	$0.0000\,e+00$	$0.0000\,e+00$
0.1	1.1004999990	1.1005000000	1.1005000000	$7.8449\,e-10$	$5.7548\,e-11$
0.2	1.2039999820	1.2039999830	1.2039999830	$1.5551\,e-09$	$3.3891\,e-11$
0.3	1.3134997050	1.3134997000	1.3134997000	$5.2749\,e-09$	$3.8674\,e-10$
0.4	1.4319976580	1.4319976490	1.4319976490	$9.1879\,e-09$	$2.2208\,e-10$
0.5	1.5624882710	1.5624882780	1.5624882770	$7.3319\,e-09$	$1.0258\,e-09$
0.6	1.7079560510	1.7079560760	1.7079560770	$2.5509\,e-08$	$4.2041\,e-10$
0.7	1.8713648410	1.8713648410	1.8713648430	$3.8179\,e-10$	$1.9375\,e-09$
0.8	2.0556398720	2.0556398270	2.0556398250	$4.5653\,e-08$	$1.4648\,e-09$
0.9	2.2636397740	2.2636397940	2.2636397930	$1.9539\,e-08$	$1.1165\,e-09$
1.0	2.4981152240	2.4981151530	2.4981151490	$7.0892\,e-08$	$4.1212\,e-09$

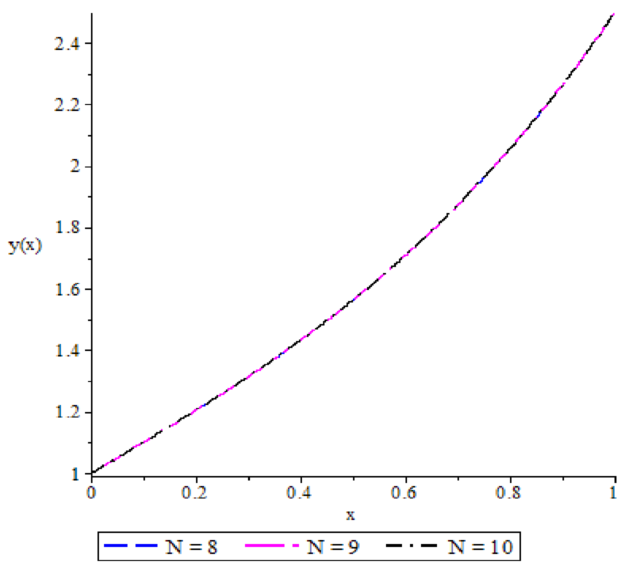

Figure 1. Grahical representation of numerical results for Example 1 (Case $n = 8,9,10$).

A NUMERICAL EXPERIMENT

We consider here the following problems for experiment with our preceding results. The exact error is defined by

$$\varepsilon *= \max_{0\le x\le 1}[|Y(x_k) - Y_n(x_k)|], 0 \le x \le 1, [x_k] = [0.01k], k = 0(1)100$$

Example 1

$$Ly(x) \equiv y^{iv}(x) + x^3 y(x) = 0$$

$$y(0) = 1, y'(0) = 1, y''(0) = 0, y'''(0) = 3$$

$m = 4$, $s = 3$

The set of linear equations obtained were solved by Maple 18 package as shown in Table 1 and Figure 1..

Example 2

$$Ly(x) \equiv 3y^{iv}(x) + x^4 y'(x) = 0$$

Table 2. Numerical results for Example 2 ($n = 8,9,10$).

x	$y_8(x)$	$y_9(x)$	$y_{10}(x)$	$\|y_9(x)-y_8(x)\|$	$\|y_{10}(x)-y_9(x)\|$
0	1.0000000000	1.0000000000	1.0000000000	$0.0000\,e+00$	$0.0000\,e+00$
1	0.9951666667	0.9951666667	0.9951666667	$2.7786\,e-11$	$1.5337\,e-11$
2	0.9813333333	0.9813333334	0.9813333334	$3.8048\,e-11$	$2.5758\,e-11$
3	0.9595000021	0.9595000018	0.9595000020	$2.4147\,e-10$	$1.2258\,e-10$
4	0.9306666926	0.9306666922	0.9306666921	$3.4552\,e-10$	$1.1643\,e-10$
5	0.8958335163	0.8958335167	0.8958335163	$3.8585\,e-10$	$3.5358\,e-10$
6	0.8560009096	0.8560009107	0.8560009109	$1.0810\,e-09$	$2.1331\,e-10$
7	0.8121701801	0.8121701800	0.8121701807	$1.2106\,e-10$	$6.9196\,e-10$
8	0.7653445797	0.7653445778	0.7653445772	$1.9916\,e-09$	$6.0242\,e-10$
.9	0.7165311731	0.7165311740	0.7165311736	$9.3978\,e-10$	$3.6296\,e-10$
1.0	0.6667438262	0.6667438231	0.6667438215	$3.1295\,e-09$	$1.5560\,e-09$

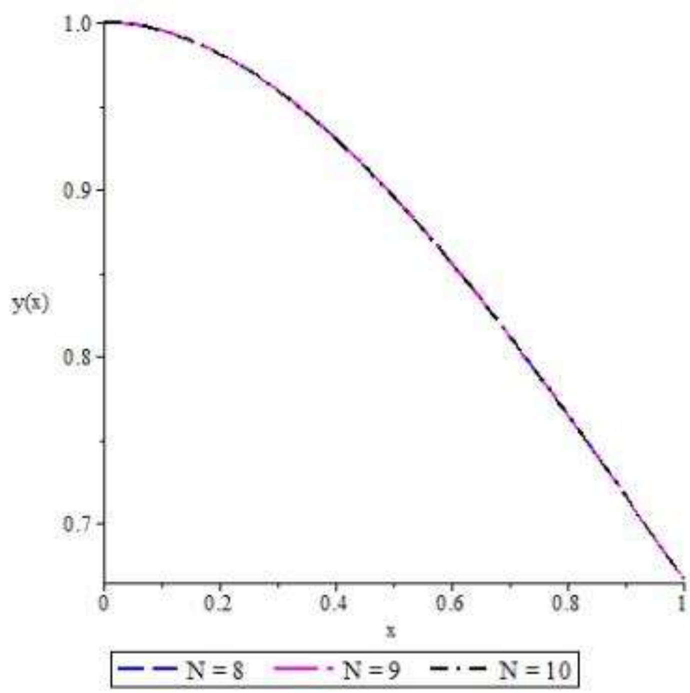

Figure 2. Grahical representation of numerical results for Example 2 ($n = 8,9,10$).

$$y(0)=1,\; y'(0)=0,\; y''(0)=-1,\; y'''(0)=1$$

$$m = 4, \quad S = 3$$

The linear equations obtained were solved by Maple package as shown in Table 2 and Figure 2.

Conclusion

A method for the solution of the class of fourth order differential equations with third degree overdetermination by the integral method has been presented. The integral Tau method closely approximates the analytic solution.

Numerical evidences obtained from some selected problems show that the method is accurate and effective.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

REFERENCES

Adeniyi RB (1991). On The Method of Tau Method for Numerical Solution of Ordinary Differential Equations. Doctoral Thesis, University of Ilorin, Ilorin, Nigeria.

Adeniyi RB, Aliyu AI (2007). Maali on the integrated formulation of numerical schemes for a class of Non-overdetermined second order Differential Equations. Nigeria Mathematical Society 29:111-128.

Adeniyi RB, Aliyu AI (2011). Malli on the Integrated Formulation of the Tau Method involving at most Two Parameters. International Journal on Research and Development. Book of Proceedings 3:104-114.

Aliyu IMA (2012). Computational Error Estimate of Integrated Formulation of the Tau Method for some class of ODEs. Doctoral Thesis, University of Ilorin, Ilorin, Nigeria.

Fox L, Parker IB (1968). Chebyshev Polynomials in Numerical Analysis. Oxford University Press Oxford.

Lanczos C (1956). Applied Analysis. Prentice Hall, New Jersey.

Lanczos C (1983). Trigonometry Interpolationof Empirical and Analytical Function. Math and Physics 17(1-4):123-199.

Mousavi S, Mousavi W (2012). New Application of Tau Method for Solving Differential Equations of the lan Emden type. World Applied Science Journal 18(5):642-644.

Onumanyi P, Oritz EL (1982). Numerical Solution of Higher Order Boundary Value Problems for Ordinary Differnetial Equations with an Estimation of the Error. International Journal for Numerical Methods in Engineering 18(5):775-781.

Oritz EL (1974). Canomical Polynomial in the Lanczos Tau method, Studies in Numerical Analysis (edited by Scaife B.K.P), Academic Press, New York.

Yisa BM (2012). The recursive formulation of the Tau method for non-over determined ODEs, Doctoral Thesis, University of Ilorin, Ilorin, Nigeria.