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A study of some systems of nonlinear partial differential equations by using Adomian and modified decomposition methods

Mohammed E. A. Rabie[1,2]* and Tarig M. Elzaki[3]

[1]Department of Mathematics, Faculty of Education Afif, Shaqra University, Saudi Arabia.
[2]Department of Mathematics, Faculty of Science, Sudan University of Science and Technology, Sudan.
[3]Department of Mathematics, Faculty of Sciences and Arts-Alkamil, King Abdulaziz University, Jeddah-Saudi Arabia.

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In this paper, we introduce the solution of systems of nonlinear partial differential equations subject to the general initial conditions by using Adomian decomposition method (ADM) and Modified decomposition method (MDM). The proposed Adomian and Modified decomposition methods was applied to reformulated first and second order initial value problems, which leads the solution in terms of transformed variables, and the series solution will be obtained by making use of the inverse operator. The results indicate these methods to be very effective and simple.

Key words: System of nonlinear partial differential equations, Adomian decomposition method and modified decomposition method.

INTRODUCTION

The nonlinear partial differential equations was well discussed by John (2003) and systems of partial differential equations, linear or nonlinear, have attracted much concern in studying evolution equations that describe wave propagation, in investigating shallow water waves, and in examining the chemical reaction-diffusion model of Brusselator. The general ideas and the essential features of these systems are of wide applicability. The commonly used methods are the method of characteristics and the Riemann invariants among other methods. The existing techniques encountered some difficulties in terms of the size of computational work needed especially when the system involves several partial equations (Wazwaz, 2009). To avoid the difficulties that usually arise from traditional strategies, the Adomian decomposition method (Wazwaz, 2009; Wazwaz, 2006; El-Wakil et al., 2006; Ablowitz and Clarkson, 1991; Debnath, 1994; Adomian, 1984, 1986, 1994; Abassy et al., 2004, 2007; Cherruault, 1990; Lesnic, 2006, 2007; Wazwaz, 2001, 2002; Burgers, 1948; Miura, 1976) and modified decomposition method (Adomian, 1986, 1994; Ablowitz and Clarkson, 1991) will form a reasonable basis for studying systems of partial differential equations. The methods, as we have seen later, has a useful attraction in that it provides the solution in a rapidly convergent power series with elegantly computable terms. These two methods transforms the systems of partial differential equations into a set of

*Corresponding author. E-mail: mohammed.elrabie@gmail.com

recursive relations that can be easily examined (Wazwaz, 2009).

DESCRIPTION OF THE ADOMIAN DECOMPOSITION METHOD

Systems of nonlinear partial differential equations will be examined by using Adomian decomposition method. Systems of nonlinear partial differential equations arise in many scientific models such as the propagation of shallow water waves and the Brusselator model of chemical reaction-diffusion model. To achieve our goal in handling systems of nonlinear partial differential equations, we write a system in an operator form by

$$\begin{cases} L_t u + L_x v + N_1(u,v) = g_1 \\ L_t v + L_x u + N_2(u,v) = g_2 \end{cases} \tag{1}$$

With initial data

$$\begin{aligned} u(x,0) &= f_1(x), \\ v(x,0) &= f_2(x), \end{aligned} \tag{2}$$

Where L_t and L_x are considered, without loss of generality, first order partial differential operators N_1 and N_2 are nonlinear operators. In addition, g_1 and g_2 are source terms. Operating with the integral operator L_t^{-1} to the system (1) and using the initial data (2) yields

$$\begin{aligned} u(x,t) &= f_1(x) + L_t^{-1}(g_1) - L_t^{-1}L_x v - L_t^{-1}[N_1(u,v),] \\ v(x,t) &= f_2(x) + L_t^{-1}(g_2) - L_t^{-1}L_x u - L_t^{-1}[N_2(u,v),] \end{aligned} \tag{3}$$

The linear unknown functions *u*(*x*,*t*) and *v*(*x*,*t*) can be decomposed by infinite series of components

$$\begin{aligned} u(x,t) &= \sum_{n=0}^{\infty} u_n(x,t) \\ v(x,t) &= \sum_{n=0}^{\infty} v_n(x,t) \end{aligned} \tag{4}$$

However, the nonlinear operators $N_1(u,v)$ and $N_2(u,v)$ should be represented by using the infinite series of the so-called Adomian polynomials A_n and B_n as follows:

$$\begin{aligned} N_1(u,v) &= \sum_{n=0}^{\infty} A_n, \\ N_2(u,v) &= \sum_{n=0}^{\infty} B_n, \end{aligned} \tag{5}$$

Where $u_n(x,t)$ and $u_n(x,t)$, $n \geq 0$ are the components of $u(x,t)$ and $v(x,t)$ respectively that will be recurrently determined. In addition, A_n and B_n, $n \geq 0$ are Adomian polynomials that can be generated for all forms of nonlinearity. Now substituting Equations (4) and (5) into Equation (3) gives

$$\begin{aligned} \sum_{n=0}^{\infty} u_n(x,t) &= f_1(x) + L_t^{-1}(g_1) - L_t^{-1}L_x(v_n) - L_t^{-1}\left(\sum_{n=0}^{\infty} A_n\right) \\ \sum_{n=0}^{\infty} v_n(x,t) &= f_2(x) + L_t^{-1}(g_2) - L_t^{-1}L_x(u_n) - L_t^{-1}\left(\sum_{n=0}^{\infty} B_n\right) \end{aligned} \tag{6}$$

Two recursive relations can be constructed from Equation (6) given by

$$\begin{aligned} u_0(x,t) &= f_1(x) + L_t^{-1}(g_1), \\ u_{k+1}(x,t) &= L_t^{-1}(L_x v_k) - L_t^{-1}(A_k), \quad k \geq 0, \end{aligned} \tag{7}$$

and

$$\begin{aligned} v_0(x,t) &= f_2(x) + L_t^{-1}(g_2), \\ v_{k+1}(x,t) &= L_t^{-1}(L_x u_k) - L_t^{-1}(B_k), \quad k \geq 0, \end{aligned} \tag{8}$$

It is an essential feature of the decomposition method that the zeroth components $u_0(x,t)$, and $v_0(x,t)$ are defined always by all terms that arise from initial data and from integrating the source terms. Having defined the zeroth pair (u_0, v_0), the remaining pair (u_k, v_k), $k \geq 1$ should be obtained in a recurrent manner by using Equations (7) and (8). Additional pairs for the decomposition series solutions normally account for higher accuracy. Having determined the components of $u(x,t)$ and $v(x,t)$, the solution (u,v) of the system follows immediately in the form of a power series expansion upon using Equation (4).

Example 1

Consider the nonlinear system of partial differential equations

$$\begin{aligned} u_t + v_x w_y - v_y w_x &= -u \\ v_t + w_x u_y + w_y u_x &= v \\ w_t + u_x v_y + u_y v_x &= w \end{aligned} \tag{9}$$

With initial condition

$$u(x,y.o) = e^{x+y}, v(x,y,o) = e^{x-y}, w(x,v,0) = e^{-x+y}$$

Following the analysis presented above, we obtain:

$$u(x,y,t) = e^{x+y} + L_t^{-1}(v_y w_x - v_x w_y - u)$$
$$v(x,y,t) = e^{x-y} + L_t^{-1}(v - w_x u_y - w_y u_x) \quad (10)$$
$$w(x,v,t) = e^{-x+y} + L_t^{-1}(w - u_x v_y - u_y v_x)$$

Substituting the decomposition representations for linear and nonlinear terms into Equation (10) yields:

$$\sum_{n=0}^{\infty} u_n(x,y.t) = e^{x+y} + L_t^{-1}\left(\sum_{n=o}^{\infty} A_n - \sum_{n=o}^{\infty} B_n - \sum_{n=o}^{\infty} u_n\right)$$
$$\sum_{n=0}^{\infty} v_n(x,y.t) = e^{x-y} + L_t^{-1}\left(\sum_{n=o}^{\infty} v_n - \sum_{n=o}^{\infty} C_n - \sum_{n=o}^{\infty} D_n\right)$$
$$\sum_{n=0}^{\infty} w_n(x,y.t) = e^{-x+y} + L_t^{-1}\left(\sum_{n=o}^{\infty} w_n - \sum_{n=o}^{\infty} E_n - \sum_{n=o}^{\infty} F_n\right)$$

Where $A_n, B_n, C_n, D_n, E_n, and\ F_n$ are Adomision polynomials for the nonlinear terms $v_y w_x, v_x w_y, w_x u_y, w_y u_x, u_x v_y\ and\ u_y v_x$ respectively . Three recursive relations can be constructed from Equation (10) given by:

$$u_o = e^{x+y}$$
$$u_{k+1} = L_t^{-1}(A_k - B_k - u_k), k \geq 0 \quad (11)$$

$$v_o = e^{x-y}$$
$$v_{k+1} = L_t^{-1}(v_k - C_k - D_k), k \geq 0 \quad (12)$$

and

$$w_0 = e^{-x+y}$$
$$w_{k+1} = L_t^{-1}(w_k - E_k - F_k), k \geq 0 \quad (13)$$

We list the first three Adomian polynomials as follows for $v_y w_x$we find:

$$A_0 = (v_0)_y (w_0)_x$$
$$A_1 = (v_1)_y (w_0)_x + (v_0)_y (w_1)_x$$
$$A_2 = (v_2)_y (w_0)_x + (v_1)_y (w_1)_x + (v_0)_y (w_2)_x$$

For $v_x w_y$ we find:

$$B_0 = (v_0)_x (w_0)_y$$
$$B_1 = (v_1)_x (w_0)_y + (v_0)_x (w_1)_y$$
$$B_2 = (v_2)_x (w_0)_y + (v_1)_x (w_1)_y + (v_0)_x (w_2)_y$$

For $w_x u_y$ we find:

$$C_0 = (w_0)_x (u_0)_y$$
$$C_1 = (w_1)_x (u_0)_y + (w_0)_x (u_1)_y$$
$$C_2 = (w_2)_x (u_0)_y + (w_1)_x (u_1)_y + (w_0)_x (u_2)_y$$

For $w_y u_x$ we find:

$$D_0 = (w_0)_y (u_0)_x$$
$$D_1 = (w_1)_y (u_0)_x + (w_0)_y (u_1)_x$$
$$D_2 = (w_2)_y (u_0)_x + (w_1)_y (u_1)_x + (w_0)_y (u_2)_x$$

For $u_x v_y$ we find:

$$E_0 = (v_0)_y (u_0)_x$$
$$E_1 = (v_0)_y (u_1)_x + (v_1)_y (u_0)_x$$
$$E_2 = (v_0)_y (u_2)_x + (v_1)_y (u_1)_x + (v_2)_y (u_0)_x$$

For $u_y v_x$ we find:

$$F_0 = (v_0)_x (u_0)_y$$
$$F_1 = (v_0)_x (u_1)_y + (v_1)_x (u_0)_y$$
$$F_2 = (v_0)_x (u_2)_y + (v_1)_x (u_1)_y + (v_2)_x (u_0)_y$$

Using the derived Adomian polynomials into Equations (11), (12) and (13), we obtain:

$$u_0 = e^{x+y} \quad , v_0 = e^{x-y} \quad , w_0 = e^{-x+y}$$
$$u_1 = L_t^{-1}(A_0 - B_0 - u_0)$$
$$= L_t^{-1}\left((-e^{x-y})(-e^{-x+y}) - (e^{x-y})(e^{-x+y}) - e^{x+y}\right) = -te^{x+y}$$
$$v_1 = L_t^{-1}(v_0 - C_0 - D_0)$$
$$= L_t^{-1}\left(e^{x-y} - (-e^{-x+y})(e^{x+y}) - (e^{-x+y})(e^{x+y})\right) = te^{x-y}$$
$$w_1 = L_t^{-1}(w_0 - E_0 - F_0)$$
$$= L_t^{-1}\left(e^{-x+y} - (e^{x+y})(-e^{x-y}) - (e^{x-y})(e^{x+y})\right) = te^{-x+y}$$
$$u_2 = L_t^{-1}(A_1 - B_1 - u_1) = -\frac{t^2}{2} e^{x+y}$$
$$v_2 = L_t^{-1}(v_1 - C_1 - D_1) = \frac{t^2}{2} e^{x-y}$$
$$w_2 = L_t^{-1}(w_1 - E_1 - F_1) = \frac{t^2}{2} e^{-x+y}$$
$$u_3 = -\frac{t^3}{3!} e^{x+y}, \ v_3 = \frac{t^3}{3!} e^{x-y} \text{ and } \ w_3 = \frac{t^3}{3!} e^{-x+y}$$

And so on,

The solutions $u(x,y,t), v(x,y,t) and\ w(x,y,t)$ in a series form are given by:

$$u(x,y,t) = e^{x+y} - te^{x+y} - \frac{t^2}{2!} e^{x+y} - \frac{t^3}{3!} e^{x+y} - \cdots$$

$$v(x,y,t) = e^{x-y} + te^{x-y} + \frac{t^2}{2!}e^{x-y} + \frac{t^3}{3!}e^{x-y} + \cdots \quad (14)$$

$$w(x,y,t) = e^{-x+y} + te^{-x+y} + \frac{t^2}{2!}e^{-x+y} + \frac{t^3}{3!}e^{-x+y} + \cdots$$

And in a closed form by:

$$u(x,y,t) = e^{x+y-t}$$
$$v(x,y,t) = e^{x-y+t}$$
$$w(x,y,t) = e^{-x+y+t}$$

Which are the exact solutions

ANALYSIS OF THE MODIFIED DECOMPOSITION METHOD

Consider the systems of nonlinear partial differential Equations (1) and (3) with the initial data (2) the procedure of the modified decomposition method assume that the linear unknown functions $u(x,t)$ and $v(x,t)$ in addition the inhomogeneous terms $g_1(x,t)\, and\ \ g_2(x,t)$ should be decomposed by infinite series

$$u(x,t) = \sum_{m=0}^{\infty} a_m(x)t^m \ , \ v(x,t) = \sum_{m=0}^{\infty} b_m(x)t^m \quad (15)$$

$$g_1 = \sum_{m=0}^{\infty} r_m(x)\, t^m \ and \ \ g_2 = \sum_{m=0}^{\infty} s_m(x)\, t^m \quad (16)$$

However, the nonlinear terms $N_1(u,v)$ and $N_2(u,v)$ should be represented by using the infinite series as follows:

$$N_1(u,v) = \sum_{m=0}^{\infty} A_m t^m ,$$
$$N_2(u,v) = \sum_{m=0}^{\infty} B_m t^m , \quad (17)$$

Where $a_m(x)$ and $b_m(x)$, $m \geq 0$ are the coefficients of $u(x,t)$ and $v(x,t)$ respectively that will be recurrently determined, and A_m and B_m, $m \geq 0$ can be generated for all forms of nonlinearity. Substituting Equations (15), (16) and (17) into Equation (3) gives

$$\sum_{m=0}^{\infty} a_m(x)t^m = f_1(x) + L_t^{-1}\left(\sum_{m=0}^{\infty} r_m(x)t^m\right) - L_t^{-1}L_x\left(\sum_{m=0}^{\infty} b_m(x)t^m\right) - L_t^{-1}\left(\sum_{m=0}^{\infty} A_m t^m\right) \quad (18)$$
$$\sum_{m=0}^{\infty} b_m(x)t^m = f_2(x) + L_t^{-1}\left(\sum_{m=0}^{\infty} s_m(x)t^m\right) - L_t^{-1}L_x\left(\sum_{m=0}^{\infty} a_m(x)t^m\right) - L_t^{-1}\left(\sum_{m=0}^{\infty} B_m t^m\right)$$

Integrating the right hand side of Equation (21) gives

$$\sum_{m=0}^{\infty} a_m(x)t^m = f_1(x) + \frac{1}{m+1}\left[\left(\sum_{m=0}^{\infty} r_m(x)t^{m+1}\right) - L_x\left(\sum_{m=0}^{\infty} b_m(x)t^{m+1}\right) - \left(\sum_{m=0}^{\infty} A_m t^{m+1}\right)\right] \quad (19)$$
$$\sum_{m=0}^{\infty} b_m(x)t^m = f_2(x) + \frac{1}{m+1}\left[\left(\sum_{m=0}^{\infty} s_m(x)t^{m+1}\right) - L_x\left(\sum_{m=0}^{\infty} a_m(x)t^{m+1}\right) - \left(\sum_{m=0}^{\infty} B_m t^{m+1}\right)\right]$$

Substituting m by $m-1$ in the right hand sides of Equation (19) and equating the coefficients of like power of t, two recursive relations can be constructed, given by

$$a_0 = f_1(x)$$
$$a_m = \frac{1}{m}\left[r_{m-1}(x) - \frac{\partial}{\partial x}(b_{m-1}(x)) - A_{m-1}\right] \quad m \geq 1 \quad (20)$$

and

$$b_0 = f_2(x)$$
$$b_m = \frac{1}{m}\left[s_{m-1}(x) - \frac{\partial}{\partial x}(a_{m-1}(x)) - B_{m-1}\right] \quad m \geq 1 \quad (21)$$

Having determined the coefficients $a_m(x)$ and $b_m(x)$, the solution (u,v) of the system follow immediately.

Example 2

Consider the system of nonlinear partial differential equations

$$u_t + vu_x + u = 1$$
$$v_t - uv_x - v = 1 \quad (22)$$

With the initial conditions

$$u(x,0) = e^x , \qquad v(x,0) = e^{-x} \quad (23)$$

Following the analysis of modified decomposition method, we obtain:

$$\sum_{m=0}^{\infty} a_m(x)t^m = e^x + \sum_{m=0}^{\infty} r_m(x)\frac{t^{m+1}}{m+1} - \sum_{m=0}^{\infty} a_m(x)\frac{t^{m+1}}{m+1} - \sum_{m=0}^{\infty} A_m(x)\frac{t^{m+1}}{m+1} \quad (24)$$

$$\sum_{n=0}^{\infty} b_n(x)t^n = e^{-x} + \sum_{n=0}^{\infty} s_n(x)\frac{t^{n+1}}{n+1} - \sum_{n=0}^{\infty} b_n(x)\frac{t^{n+1}}{n+1} + \sum_{n=0}^{\infty} B_n(x)\frac{t^{n+1}}{n+1}$$

Where

$$u=\sum_{m=0}^{\infty} a_m\ (x)t^m\ ,v=\sum_{n=0}^{\infty} b_n\ (x)t^n$$

$$1=\sum_{m=0}^{\infty} r_m\ (x)t^m=\sum_{n=0}^{\infty} s_n\ (x)t^n$$

$$uv_x=\left[\sum_{m=0}^{\infty} a_m\ (x)t^m\right]\left[\sum_{n=0}^{\infty} \mathrm{b}'_{\mathrm{n}}\ (x)t^n\right]=\sum_{n=0}^{\infty} t^n\sum_{m=0}^{n} \mathrm{b}'_{\mathrm{n-m}}\ (x).a_m(x)$$

$$=\sum_{n=0}^{\infty} B_n(x)t^n$$

$$\mathrm{b}'_{\mathrm{n}}\ (x)=\frac{d}{dx}\left(b_n(x)\right)\ and\ B_n(x)=\sum_{m=0}^{n} \mathrm{b}'_{\mathrm{n-m}}\ (x).a_m(x)$$

Similarly

$$vu_x=\sum_{m=0}^{\infty} t^m\sum_{n=0}^{m} \mathrm{a}'_{\mathrm{m-n}}\ (x).b_n(x)=\sum_{m=0}^{\infty} A_m(x)\,t^m$$

Where

$$A_m(x)=\sum_{n=0}^{m} \mathrm{a}'_{\mathrm{m-n}}\ (x).b_n(x)$$

$$B_n(x)=\sum_{m=0}^{n} \mathrm{b}'_{\mathrm{n-m}}\ (x).a_m(x)$$

Let $m=m-1$ and $n=n-1$ in the right side of the above system, we obtain the recurrence relation.

$$a_0(x)=\ e^x$$

$$a_m(x)\ =\ \frac{1}{m}[r_{m-1}\,(x)-a_{m-1}\,(x)-A_{m-1}\,(x)];$$

$m\geq 1$

and

$$a_0(x)=e^{-x}$$

$$b_n(x)=\frac{1}{n}\ [s_{n-1}(x)+b_{n-1}(x)+\ B_{n-1}(x)],\ \ n\geq 1$$

That gives

$$A_0=\mathrm{a}'_0\ .\mathrm{b}_0=1$$

$$B_0=\mathrm{b}'_0\ .\mathrm{a}_0=-1$$

Then, we find:

$$a_1=[1-\ e^x-1\ ]=-e^x$$
$$b_1=[1+\ e^{-x}-1\ ]=\ e^{-x}$$

By the same way, we find:

$$A_1=0\quad,\ a_2=\frac{1}{2}e^x$$

$$B_1=0\quad,\ b_2=\frac{1}{2}e^{-x}$$

Then,

$$A_2=0\quad,\ a_3=-\frac{1}{6}e^x$$

$$B_2=0\quad,\ b_3=\frac{1}{6}e^{-x}$$

And so on, the solution $u(x,t)$ and $v(x,t)$ in a series form are given by:

$$u(\,x,t)=e^x-\ te^x+\frac{t^2}{2!}e^x-\frac{t^3}{3!}e^x+\ ...=\ e^{x-t}$$
$$v(x,t)=e^{-x}+te^{-x}+\frac{t^2}{2!}e^{-x}+\frac{t^3}{3!}e^{-x}+\cdots=e^{-x+t}$$

Consequently the exact solution of the system of non linear partial differential equations is given by:

$$(u,v)=(e^{x-t},\ \ e^{-x+t})$$

Example 3

Consider the nonlinear system of partial differential Equations (9) and (10) and write:

$$u=\sum_{m=0}^{\infty} a_m(x,y)t^m,v=\sum_{m=0}^{\infty} b\ _m(x,y)t^m,w=\sum_{m=0}^{\infty} c_m(x,y)t^m$$

Therefore:

$$v_yw_x=\left[\sum_{m=0}^{\infty} b_{m_y}t^m\right]\left[\sum_{m=0}^{\infty} c_{m_x}t^m\right]=\sum_{m=0}^{\infty} t^m\sum_{n=0}^{m}[b_{m-n}]_y.c_{n_x}=\sum_{m=0}^{\infty} A_m t^m$$

Similarly,

$$v_x w_y = \sum_{m=0}^{\infty} t^m . \sum_{n=0}^{m} [b_{m-n}]_x . c_{n_y} = \sum_{m=0}^{\infty} B_m t^m$$
$$w_x u_y = \sum_{m=0}^{\infty} t^m . \sum_{n=0}^{m} [C_{m-n}]_x . a_{n_y} = \sum_{m=0}^{\infty} S_m t^m$$
$$w_y u_x = \sum_{m=0}^{\infty} t^m . \sum_{n=0}^{m} [C_{m-n}]_y . a_{n_x} = \sum_{m=0}^{\infty} D_m t^m$$
$$u_x v_y = \sum_{m=0}^{\infty} t^m . \sum_{n=0}^{m} [a_{m-n}]_x . b_{n_y} = \sum_{m=0}^{\infty} E_m t^m$$
$$u_y v_x = \sum_{m=0}^{\infty} t^m . \sum_{n=0}^{m} [a_{m-n}]_y . b_{n_X} = \sum_{m=0}^{\infty} F_m t^m$$

Where,

$$A_m = \sum_{n=0}^{m} [b_{m-n}]_y . c_{n_x} \quad , B_m = \sum_{n=0}^{m} [b_{m-n}]_x . c_{n_y}$$
$$S_m = \sum_{n=0}^{m} [c_{m-n}]_x . a_{n_y} \quad , D_m = \sum_{n=0}^{m} [c_{m-n}]_y . a_{n_x}$$
$$E_m = \sum_{n=0}^{m} [a_{m-n}]_x . b_{n_y} \quad , E_m = \sum_{n=0}^{m} [a_{m-n}]_y . b_{n_x}$$

The substituting leads to:

$$\sum_{m=0}^{\infty} a_m t^m = e^{x+y} + \sum_{m=0}^{\infty} A_m \frac{t^{m+1}}{m+1} - \sum_{m=0}^{\infty} B_m \frac{t^{m+1}}{m+1} - \sum_{m=0}^{\infty} a_m \frac{t^{m+1}}{m+1}$$
$$\sum_{m=0}^{\infty} b_m t^m = e^{x-y} + \sum_{m=0}^{\infty} b_m \frac{t^{m+1}}{m+1} - \sum_{m=0}^{\infty} S_m \frac{t^{m+1}}{m+1} - \sum_{m=0}^{\infty} D_m \frac{t^{m+1}}{m+1}$$
$$\sum_{m=0}^{\infty} C_m t^m = e^{-x+y} + \sum_{m=0}^{\infty} C_m \frac{t^{m+1}}{m+1} - \sum_{m=0}^{\infty} E_m \frac{t^{m+1}}{m+1} - \sum_{m=0}^{\infty} F_m \frac{t^{m+1}}{m+1} \qquad (25)$$

Let $m = m - 1$ on the right sides of Equation (25) and equate the coefficients of like power of "t" on both sides, we obtain the recurrence relation

$$a_0 = e^{x+y}$$
$$a_m = \frac{1}{m} [A_{m-1} - B_{m-1} - a_{m-1}] , m \geq 1 \qquad (26)$$
$$b_0 = e^{x-y}$$

$$b_m = \frac{1}{m} [b_{m-1} - S_{m-1} - D_{m-1}] , m \geq 1 \qquad (27)$$

and,

$$c_0 = e^{-x+y}$$
$$C_m = \frac{1}{m} [C_{m-1} - E_{m-1} - F_{m-1}] , m \geq 1 \qquad (28)$$

For the nonlinear terms A_m, B_m, S_m, D_m, E_m and F_m we find

$$A_0 = b_{0_y} C_{0_X} = (-e^{x-y})(-e^{-x+y}) = 1$$
$$B_0 = b_{0_x} C_{0_y} = (e^{x-y})(e^{-x+y}) = 1$$
$$S_0 = C_{0_X} a_{0_y} = (-e^{-x+y})(e^{x+y}) = -e^{2y}$$

Similarly,

$$D_0 = C_{0_y} a_{0_x} = e^{2y}$$
$$E_0 = a_{0_x} b_{0_y} = -e^{2X}$$
$$F_0 = a_{0_y} b_{0_x} = e^{2X}$$

and,

$$A_1 = b_{1_y} C_{0_x} + b_{o_y} C_{1_x}$$
$$B_1 = b_{1_X} C_{0_y} + b_{o_x} C_{1_y}$$
$$S_1 = C_{1_X} a_{0_y} + c_{o_x} a_{1_y}$$
$$D_1 = C_{1_y} a_{0_x} + c_{o_y} a_{1_x}$$
$$E_1 = a_{1_x} b_{0_y} + a_{o_x} b_{1_y}$$
$$F_1 = a_{1_y} b_{0_x} + a_{o_y} b_{1_x}$$

And so on, using the derived above into Equations (26), (27) and (28), we obtain,

$$a_0 = e^{x+y}$$
$$a_1 = [1 - 1 - e^{x+y}] = -e^{x+y}$$
$$a_2 = \frac{1}{2} [2 - 2 + e^{x+y}] = \frac{1}{2!} e^{x+y}$$
$$a_3 = \frac{1}{3} \left[3 - 3 - \frac{1}{2} e^{x+y}\right] = -\frac{1}{3!} e^{x+y}$$

and,

$$b_0 = e^{x-y}$$
$$b_{1=} [e^{x-y} + e^{2y} - e^{2y}] = e^{x-y}$$
$$b_{2=} \frac{1}{2} [e^{x-y} + 0 - 0] = \frac{1}{2} e^{x-y}$$
$$b_{3=} \frac{1}{3!} e^{x-y}$$

and,

$$c_0 = e^{-x+y}$$
$$c_1 = [e^{-x+y} + e^{2x} - e^{2x}] = e^{-x+y}$$
$$c_2 = \frac{1}{2}[e^{-x+y} + 0 - 0] = \frac{1}{2}\, e^{-x+y}$$
$$c_3 = \frac{1}{3!}\, e^{-x+y}$$

Accordingly, the solutions $u, v \; and \; w \; in$ a series form are given by:

$$u(x,y,t) = e^{x+y} - te^{x+y} + \frac{t^2}{2!}e^{x+y} - \frac{t^3}{3!}e^{x+y} + \ldots$$
$$v(x,y,t) = e^{x-y} + te^{x-y} + \frac{t^2}{2!}e^{x-y} + \frac{t^3}{3!}e^{x-y} + \ldots$$
$$w(x,y,t) = e^{-x+y} + te^{-x+y} + \frac{t^2}{2!}e^{-x+y} + \frac{t^3}{3!}e^{-x+y} + \ldots$$

And in a closed form by:

$$u(x,y,t) = e^{x+y}\left(1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \cdots\right) = e^{x+y}.e^{-t} = e^{x+y-t}$$
$$v(x,y,t) = e^{x-y}\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots\right) = e^{x-y}.e^{+t} = e^{x-y+t}$$
$$w(x,y,t) = e^{-x+y}\left(1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots\right) = e^{-x+y}.e^{+t} = e^{-x+y+t}$$

Thus the exact solution of the system (9) is given by:

$$(u, v, w) = (\, e^{x+y-t}, e^{x-y+t}, e^{-x+y+t})$$

Which are the same solutions founded by ADM.

CONCLUSION

In this paper, we introduced systems of nonlinear partial differential equations, and solved them by using Adomian and modified decomposition methods. These methods are very effective and accelerate the convergent of solution.

Conflict of Interest

The authors have not declared any conflict of interest.

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