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Full Length Research Paper

Minimal strongly balanced changeover designs with first residuals

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Assuming the presence of first order residual effects, besides periods, experimental units and direct effects of treatments, a class of minimal strongly balanced changeover designs has been proposed. For v treatments, the designs require 2v experimental units for v/2 periods. It is seen that in terms of variances of estimated elementary contrasts in treatment effects, these designs are partially balanced based on the circular association scheme. The efficiency factors for the estimation of various treatment effects of the proposed designs relative to the orthogonal designs have been tabulated for v ≤ 24.

Key words: Minimal strongly balanced designs, changeover design, crossover designs, repeated measurements designs, residual effects, partially balanced changeover designs.

INTRODUCTION

In many fields of research, experiments are conducted in which several treatments are applied to experimental units over a number of periods and observations are recorded in each period. Statistical designs used in such experiments have been given several names in the literature such as changeover designs, crossover designs, repeated measurements designs, designs involving sequences of treatments. A distinctive feature of these experiments is that the treatments have carryover effects in the periods following the periods of their direct application. The carryover effects may be of different magnitudes. The carryover effects that persist only for one period after the period of treatments application are called first order carryover (residual) effects. Here, we shall consider the presence of first order residual effects of the treatments. These designs have been extensively studied in the literature from various angles (Jones and Kenward, 2003; Afsarinejad, 1990; Bailey and Kunert, 2006; Hedayat and Yang, 2005; Varghese et al., 2002).

Hedayat and Afsarinejad (1975) presented a class of minimal balanced change over designs that require 2v experimental units and (v+1)/2 periods for v odd number of treatments and gave a method of construction for prime or prime power values of v. Constantine and Hedayat (1982) constructed balanced minimal changeover designs with number of periods, p < v and gave divisibility conditions for existence of such designs. These designs are essentially partially variance balanced for the estimation of contrasts in direct effects as well as first residual effects. Subsequently, Sharma et al. (2003) constructed minimal balanced changeover designs for all odd values of v with p = (v+1)/2 using 2v experimental units.

Here, we present a new class of minimal strongly balanced changeover designs for even number of treatments. The designs require v/2 periods and 2v experimental units for v treatments. These designs are seen to be partially balanced following the circular association scheme in terms of variances of estimated contrasts in direct or first order residual effects.

We first give some definitions that will be used in the subsequent sections.

DEFINITIONS

Pre-period

In usual changeover designs, the observations in the first

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period do not contain the residual effects of the experimental treatments and may contain the residual effects of some unknown treatments received by the experimental units before the start of the experiment. In view of this, anduration as the other periods, called pre-period, is introduced just before the start of the experiment during which the treatments are applied to the experimental units but the observations are not recorded or if recorded, are not taken into account while analyzing the data. As a result of this, observations in the first period also contain the residual effect of the treatments included in the experiment.

Minimal strongly balanced changeover (MSBCO) designs: A design is said to be strongly balanced, if

(a) every treatment occurs equally often in each period, say λ_1 times, where λ_1 is a positive integer, and
(b) each treatment is preceded by every treatment including itself equally frequently, say λ_2 times; λ_2 being a positive integer.

Evidently, for such a design, $n = \lambda_1 v$ and $np = \lambda_2 v^2$. For given v and p, a strongly balanced changeover design is said to be minimal if its parameter λ_1 is the smallest integer such that $\lambda_1 p \equiv 0 \pmod v$. Thus a minimal strongly balanced changeover design consists of the minimum number of experimental units and contains each ordered pair of treatments once.

The designs studied by Quenouille (1953), Berenblut (1964) and Patterson (1973) require v^2 experimental units and 2v periods for v treatments and are strongly balanced. However, these designs are not minimal as each ordered pair of treatments occurs 2v times in the design. The circular designs obtained from Sharma (1981) require v experimental units for 2v periods and are Strongly balanced and orthogonal for the estimation of various effects. But these designs are also not minimal as each ordered pair of treatments occurs two times in the design. A good feature of these classes of designs is that the designs are variance balanced and have high efficiency for the estimation of treatment effects.

In the following section, we present a method of constructing MSBCO designs with a pre-period and having the parameters v = 2m, p = m and n = 4m ($m \geq 3$). The underlying additive fixed effects model with usual assumptions includes a general mean, direct and first residual effects of treatments besides periods and experimental unit effects.

CONSTRUCTION OF MSBCOD (2m, m, 4m)

Let the v = 2m treatments be denoted by the symbols 0, 1, 2,…, 2m-1.

The following two initial sequences of m+1 elements each,

For m odd:
Sequence 1: {0(≡2m), 0, 2m-1, 1, 2m-2, 2, 2m-3,…, (3m+1) / 2, (m-1) /2}
Sequence 2: {(3m-1)/2, (m-1)/2, (3m+1)/2, (m-3)/2, (3m+3)/2,…, (2m-2),1, (2m-1), 0}.

For m even:
Sequence 1: {0(≡2m), 0, 2m-1, 1, 2m-2, 2, 2m-3, …, m/2-1, 3m/2}
Sequence 2: {m/2, 3m/2, m/2-1, 3m/2+1, m/2-2, …, 2m-2, 1, 2m-1, 0}.

When developed, mod (2m) give rise to two rectangular arrays each consisting of 2m rows and m+1 columns. Number the rows of the first array from 1 to 2m and that of the second from 2m+1 to 4m and number the columns of both arrays, from 0 to m. If the rows represent the experimental units and columns, the periods, then both arrays together form a MSBCOD (2m, m, 4m) with oth period as the pre-period.

Example 1

Let v = 6 (= 2 × 3). Here, m = 3. The MSBCOD (6, 3, 12) is given in Table 1 with the initial sequence in bold figures and oth period representing the pre-period.

Example 2

Let v = 8 (= 2 × 4). Here, m = 4. The MSBCOD (8, 4, 16) is given in Table 2 with the initial sequences in bold figures; the oth period being the pre-period.

The following proof ensures that the method always yields the MSBCOD (2m, m, 4m):

Proof: Every treatment occurs in each period exactly twice as the initial sequences have been developed mod (2m). Besides, if x_i and y_i (i = 1, 2, …, m+1) denote the ith element of the Sequences 1 and 2, respectively, then the set {x_i - x_{i+1}, y_i - y_{i+1}, mod (2m); i = 1, 2, …, m} contains each element of mod (2m) including zero exactly once. This implies that each ordered pair of treatment symbols occurs precisely once in the design. Thus the method always ensures existence of MSBCOD (2m, m, 4m).

Remarks

It can be easily seen that with regards to the estimation of elementary contrasts in direct or first order residual effects, the MSBCOD (2m, m, 4m) is a partially variance balanced design with m-associate classes based on the following circular association scheme:

Circular association scheme: Arrange v = 2m treatment

Table 1. Minimal strongly balanced changeover design (v = 6, p = 3, n = 12).

Experimental units	Periods 0	1	2	3
1	**0**	**0**	**5**	**1**
2	1	1	0	2
3	2	2	1	3
4	3	3	2	4
5	4	4	3	5
6	5	5	4	0
7	**4**	**1**	**5**	**0**
8	5	2	0	1
9	0	3	1	2
10	1	4	2	3
11	2	5	3	4
12	3	0	4	5

Table 2. Minimal strongly balanced changeover design (v = 8, p = 4, n = 16).

Experimental units	Periods 0	1	2	3	4
1	**0**	**0**	**7**	**1**	**6**
2	1	1	0	2	7
3	2	2	1	3	0
4	3	3	2	4	1
5	4	4	3	5	2
6	5	5	4	6	3
7	6	6	5	7	4
8	7	7	6	0	5
9	**2**	**6**	**1**	**7**	**0**
10	3	7	2	0	1
11	4	0	3	1	2
12	5	1	4	2	3
13	6	2	5	3	4
14	7	3	6	4	5
15	0	4	7	5	6
16	1	5	0	6	7

symbols on the circumference of a circle. Treatment symbol β is the i^{th} associate of α, if it lies (i = 1, 2,…,m) distance apart from α on either side of it. The parameters of the association scheme are $v = 2m$, $n_i = 2$ (i = 1, 2,…,m-1), $n_m = 1$ and the association matrices, $\mathbf{P}_i$ (i = 1, 2, …,m) of order m are:

$$\mathbf{P}_1 = (p^1_{\alpha\beta}), \qquad (1)$$

where

$p^1_{\alpha\beta} = 1$, if α = i and β = i+1 or α = i+1 and β = i (i = 1,2,…,m-1)= 0, otherwise

$$\mathbf{P}_2 = (p^2_{\alpha\beta}) \qquad (2)$$

where

$p^2_{\alpha\beta} = 1$ if α = i and β = i+2 or α = i+2 and β = i (i = 1,2,…,m-2) or $\alpha = \beta$ = 1 or m-1 = 0, otherwise etc. and

$$\mathbf{P}_m = \left(p^m_{\alpha\beta}\right), \qquad (3)$$

where

$$p^m_{\alpha\beta} = 2, \text{ if } \alpha = i \text{ and } \beta = m - i \ (i = 1,2,..., m-1)$$
$$= 0, \text{ otherwise.}$$

If residual effects are ignored and experimental units are taken as blocks, then MSBCOD (2m, m, 4m) reduces to a PBIB design with repeated blocks and has the parameters $v = 2m$, $b = 4m$, $r = 2m$, $k = m$, $\lambda_i = 2(m-i)$, i = 1, 2, …, m.

In Example 1, the parameters of the association scheme are: $v = 6$, $n_1 = n_2 = 2$, $n_3 = 1$ and

$$\mathbf{P}_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{P}_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \text{and } \mathbf{P}_3 = \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

and the parameters of the associated PBIB design are: $v = 6$, $b = 12$, $r = 6$, $k = 3$, $\lambda_1 = 4$, $\lambda_2 = 2$, $\lambda_3 = 0$.

The parameters of the association scheme for Example 2 are: $v = 8$, $n_1 = n_2 = n_3 = 2$, $n_4 = 1$ and

$$\mathbf{P}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \mathbf{P}_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \mathbf{P}_3 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \mathbf{P}_4 = \begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

The parameters of the related PBIB design are: $v = 8$, $b = 16$, $r = 8$, $k = 4$, $\lambda 1 = 6$, $\lambda 2 = 4$, $\lambda 3 = 2$, $\lambda 4 = 0$.

Table 3. Efficiency factors of the MSBCODs relative to the orthogonal designs.

S/No.	v	p	n	E_d	E_r	E_t
1	6	3	12	0.4976	0.4526	0.7434
2	8	4	16	0.5941	0.5623	0.8095
3	10	5	20	0.6585	0.6372	0.8480
4	12	6	24	0.7047	0.6882	0.8732
5	14	7	28	0.7397	0.7271	0.8913
6	16	8	32	0.7672	0.7569	0.9048
7	18	9	36	0.7894	0.7810	0.9152
8	20	10	40	0.8077	0.8006	0.9236
9	22	11	44	0.8230	0.8170	0.9304
10	24	12	48	0.8360	0.8309	0.9362

v= no. of treatments; p= no. of periods; n= no. of units; E_d, E_r, and E_t denote respectively, the efficiency factors for direct, first residual and treatments effects ignoring residuals.

EFFICIENCY FACTORS

The efficiency factor is defined as

$$E = \frac{\overline{V}_o}{\overline{V}} \tag{4}$$

where $\overline{V}$ is the average variance of the estimated elementary contrasts of treatment effects for the proposed design and $\overline{V}_o$ that for an orthogonal design using the same number of observations. It is assumed that the error variance is the same in both the designs. It can be seen that the efficiency factor given at (4) reduces to

$$E = \frac{\delta}{\bar{r}} \tag{5}$$

where δ is the harmonic mean of the non-zero characteristic roots of the C-matrix for treatment effects and $\bar{r}$ is the average number of replications of the treatments in the proposed designs. In case of these designs $\bar{r}$ = 2m for various treatment effects. The efficiency factors of the designs have been computed for direct effects, residual effects and treatment effects ignoring residual effects with number of treatments ≤ 24 and are given in Table 3. If first residual effects are not found to be significant, then efficiency of treatment effects ignoring residual effects becomes important. These efficiencies have also been presented in Table 3.

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