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Full Length Research Paper

Application of variational iteration method to coupled system of nonlinear partial differential physical equations

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In this article 2 examples of the coupled system of nonlinear partial differential physical equations including diffusion-reaction equation have been investigated by means of variational iteration method which is a new numerical method for solving these types of equations. The results are presented finally in comparison with the exact solution, which show a good agreement and consistency with the exact solution and introduce this method as a powerful and applicable one in this field.

Key words: Variational iteration method, physical system of nonlinear differential equation, diffusion-reaction equation.

INTRODUCTION

Nonlinear phenomena that appear in many areas of scientific fields such as solid state physic, plasma physics, fluid dynamics, mathematical biology and chemical kinematics can be modeled by partial differential equation. A broad class of analytical methods and numerical methods were used in handle these problems (Wakil and Abdou, 2007).

In this paper we consider 2 examples of the coupled system of nonlinear partial differential physical equations including diffusion-reaction equation have been investigated by means of variational iteration method (VIM) (Ganji et al, 2006; Barari et al., 2008; Momani and Abuasad, 2006; Farrokhzad et al., 2008; Omidvar et al.,2008; He, 1999, 2006, 2007, 2008; He and Wu, 2006, 2007; Xu, 2007; He and Zhang, 2007; Sweilam and Khader, 2007).

The application of VIM to the mentioned examples is investigated to compute an approximate solution to the governing equations. Finally, In order to verify the obtained results, a comparison will be made to the exact solution.

Reaction-diffusion equations describe a wide variety of nonlinear systems in physics, chemistry, ecology, biology and engineering. Reaction-diffusion equations are widely used as models for spatial effects in ecology. They support 3 important types of ecological phenomena: the existence of a minimal patch size necessary to sustain a population, the propagation of wavefronts corresponding to biological invasions and the formation of spatial pattern in the distributions of populations in homogenous environments. Reaction-diffusion equations can be analyzed by means of methods from the theory of partial differential equations and dynamical systems (Yildirim, 2009).

Basic idea of variational iteration method

To clarify the basic ideas of VIM, we consider the following differential equation:

$$Lu + Nu = g(t), \tag{1}$$

Where L is a linear operator, N is a nonlinear operator and $g(t)$ is a homogeneous term.

According to VIM, we can write down a correction functional as follows:

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda\left(Lu_n(\tau) + N\tilde{u}_n(\tau) - g(\tau)\right)d\tau \tag{2}$$

Where λ is a general lagrangian multiplier which can be

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identified optimally via the variational theory. The subscript n indicates the nth approximation and u_n is considered as a restricted variation, that is, $\delta \tilde{u}_n = 0$

Application of VIM

2 coupled system of nonlinear equations are presented here to solve using VIM.

In the first model, we shall deal with coupled system of nonlinear physical equations:

Example 1

$$\frac{\partial u(x,t)}{\partial t} = u(1-u^2-v)+u_{xx}, \quad t>0, \tag{3}$$

$$\frac{\partial v(x,t)}{\partial t} = v(1-u-v)+v_{xx}, \tag{4}$$

With initial conditions are as fallows:

$$u(x,0)=\frac{e^{kx}}{\left[1+e^{kx}\right]}, \tag{5}$$

$$v(x,0)=\frac{1+(3/4)e^{kx}}{\left[1+e^{kx}\right]^2} \tag{6}$$

and the exact solutions are as fallows:

$$u(x,t)=\frac{e^{k(x+ct)}}{\left[1+e^{k(x+ct)}\right]}, \tag{7}$$

$$v(x,t)=\frac{1+(3/4)e^{k(x+ct)}}{\left[1+e^{k(x+ct)}\right]^2} \tag{8}$$

Where k is constant.

$$u_{n+1}(x,t)=u_n(x,t)+\int_0^t \lambda(u_{n\tau}-\frac{\partial^2 u_n(x,\tau)}{\partial x^2}-u_n(x,\tau)+u_n(x,\tau)^3+ u_n(x,\tau)v_n(x,\tau))d\tau \tag{9}$$

Its stationary conditions can be obtained as follows:

$$\begin{Bmatrix} \lambda'|_{\tau=t}=0 \\ 1+\lambda|_{\tau=t}=0 \end{Bmatrix} \tag{10}$$

We obtain the lagrangian multiplier:

$$\lambda=-1 \tag{11}$$

As a result, we obtain the following iteration formula:

$$u_{n+1}(x,t)=u_n(x,t)+\int_0^t (-1)(u_{n\tau}-\frac{\partial^2 u_n(x,\tau)}{\partial x^2}-u_n(x,\tau)+u_n(x,\tau)^3+ u_n(x,\tau)v_n(x,\tau))d\tau \tag{12}$$

Now we start with an arbitrary initial approximation that satisfies the initial condition:

$$u(x,0)=\frac{e^{kx}}{\left[1+e^{kx}\right]}, \tag{13}$$

$$v(x,0)=\frac{1+(3/4)e^{kx}}{\left[1+e^{kx}\right]^2}$$

Using the above variational formula (12), we have

$$u_1(x,t)=u_0(x,t)+\int_0^t (-1)(u_{0\tau}-\frac{\partial^2 u_0(x,\tau)}{\partial x^2}-u_0(x,\tau)+u_0(x,\tau)^3+ u_0(x,\tau)v_0(x,\tau))d\tau \tag{14}$$

Substituting Eq. (13) in to Eq. (14) and after simplifIcations, we have:

$$u(x,t)=u_1(x,t)=-\frac{e^{kx}(-4-8e^{kx}-4e^{2kx}-4k^2t+4k^2e^{kx}t-5e^{kx}t}{4(1+e^{kx})^3} \tag{15}$$

And for determining $v(x,t)$, we consider the fallowing equation:

$$v_{n+1}(x,t)=v_n(x,t)+\int_0^t \lambda(v_{n\tau}-\frac{\partial^2 v_n(x,\tau)}{\partial x^2}-v_n(x,\tau)+v_n(x,\tau)^2+ u_n(x,\tau)v_n(x,\tau))d\tau \tag{16}$$

Its stationary conditions can be obtained as follows:

$$\begin{Bmatrix} \lambda'|_{\tau=t}=0 \\ 1+\lambda|_{\tau=t}=0 \end{Bmatrix} \tag{17}$$

We obtain the lagrangian multiplier:

$$\lambda=-1 \tag{18}$$

As a result, we obtain the following iteration formula:

$$v_{n+1}(x,t)=v_n(x,t)+\int_0^t (-1)(v_{n\tau}-\frac{\partial^2 v_n(x,\tau)}{\partial x^2}-v_n(x,\tau)+v_n(x,\tau)^2+ u_n(x,\tau)v_n(x,\tau))d\tau \tag{19}$$

Now we start with an arbitrary initial approximation that satisfies the initial condition:

$$u(x,0)=\frac{e^{kx}}{\left[1+e^{kx}\right]}, \tag{20}$$

$$v(x,0)=\frac{1+(3/4)e^{kx}}{\left[1+e^{kx}\right]^2}$$

Using the above variational formula (19), we have

$$v_1(x,t)=v_0(x,t)+\int_0^t(-1)(v_{0\tau}-\frac{\partial^2 v_0(x,\tau)}{\partial x^2}-v_0(x,\tau)+v_0(x,\tau)^2+ u_0(x,\tau)v_0(x,\tau))d\tau \tag{21}$$

Substituting Eq. (20) in to Eq. (21) and after simplifications, we have:

$$v(x,t)=v_1(x,t)=-\frac{1}{16(1+e^{kx})^4}(-16-44e^{kx}-40e^{2kx}-12e^{3kx} +20k^2e^{kx}t-16k^2e^{2kx}t-12e^{3kx}k^2t-4e^{kx}t-3e^{2kx}t) \tag{22}$$

And so on. In the same way the rest of the components of the iteration formula can be obtained, but we show that using the one iteration, we obtained good results which they are very close to the results of the exact solution.

Example 2

A second interactive model is a coupled system of diffusion-reaction equation:

$$\frac{\partial u(x,t)}{\partial t}=u(1-u-v)+u_{xx},\quad t>0, \tag{23}$$

$$\frac{\partial v(x,t)}{\partial t}=v_{xx}-uv, \tag{24}$$

The initial condition is as fallows:

$$u(x,0)=\frac{e^{kx}}{\left[1+e^{0.5kx}\right]^2},$$

(25) and (26)

$$v(x,0)=\frac{1}{\left[1+e^{0.5kx}\right]},$$

And the exact solutions are as fallows:

$$u(x,t)=\frac{e^{k(x+ct)}}{\left[1+e^{0.5k(x+ct)}\right]^2},$$

(27) and (28)

$$v(x,t)=\frac{1}{\left[1+e^{0.5k(x+ct)}\right]},$$

Where k is constant.

$$u_{n+1}(x,t)=u_n(x,t)+\int_0^t\lambda(u_{n\tau}-\frac{\partial^2 u_n(x,\tau)}{\partial x^2}-u_n(x,\tau)+u_n(x,\tau)^2+ u_n(x,\tau)v_n(x,\tau))d\tau \tag{29}$$

Its stationary conditions can be obtained as follows:

$$\begin{Bmatrix}\lambda'\big|_{\tau=t}=0\\ 1+\lambda\big|_{\tau=t}=0\end{Bmatrix} \tag{30}$$

We obtain the lagrangian multiplier:

$$\lambda=-1 \tag{31}$$

As a result, we obtain the following iteration formula:

$$u_{n+1}(x,t)=u_n(x,t)+\int_0^t(-1)(u_{n\tau}-\frac{\partial^2 u_n(x,\tau)}{\partial x^2}-u_n(x,\tau)+u_n(x,\tau)^2+ u_n(x,\tau)v_n(x,\tau))d\tau \tag{32}$$

Now we start with an arbitrary initial approximation that satisfies the initial condition:

$$u(x,0)=\frac{e^{kx}}{\left[1+e^{.5kx}\right]^2},$$

$$v(x,0)=\frac{1}{\left[1+e^{.5kx}\right]} \tag{33}$$

Using the above variational formula (32), we have

$$u_1(x,t)=u_0(x,t)+\int_0^t(-1)(u_{0\tau}-\frac{\partial^2 u_0(x,\tau)}{\partial x^2}-u_0(x,\tau)+u_0(x,\tau)^2+ u_0(x,\tau)v_0(x,\tau))d\tau \tag{34}$$

Substituting Eq. (33) in to Eq. (34) and after simplifications, we have:

$$u(x,t)=u_1(x,t)=\frac{1}{(1+e^{0.5kx})^4}\begin{bmatrix}(0.5)(2e^{kx}+4e^{1.5kx}+2e^{2kx}+\\ 2k^2e^{kx}t-k^2e^{1.5kx}t+2e^{1.5kx}t\end{bmatrix} \tag{35}$$

And for determining $v(x,t)$, we consider the fallowing equation:

$$v_{n+1}(x,t)=v_n(x,t)+\int_0^t\lambda(v_{n\tau}-\frac{\partial^2 v_n(x,\tau)}{\partial x^2}+u_n(x,\tau)v_n(x,\tau))d\tau \tag{36}$$

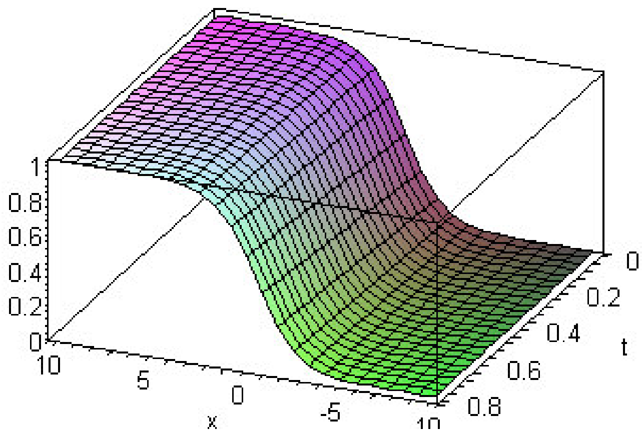

Figure 1a. The numerical solution of by VIM for different values of time. (k = 1) (Example 1).

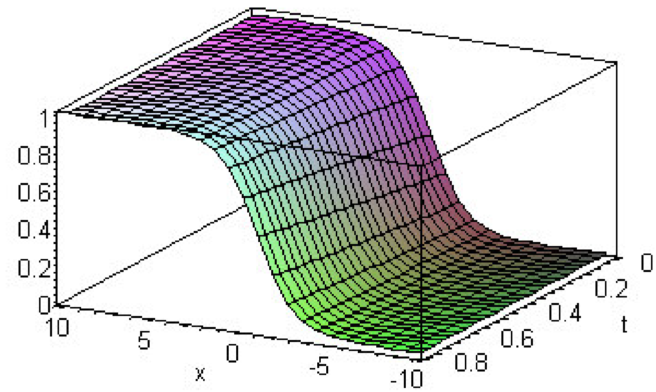

Figure 2a. The exact wave front solution of $u(x,t)$ with fixed values of $k=c=1$ (Example 1).

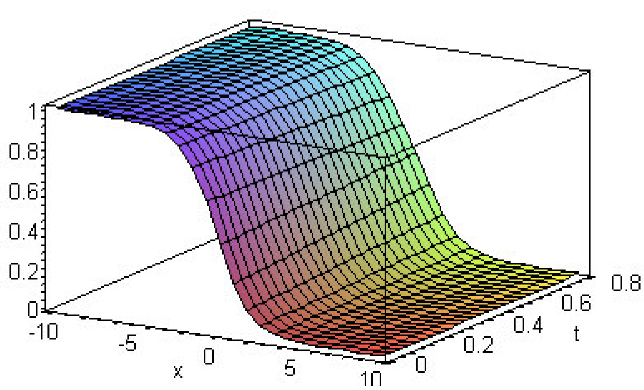

Figure 1b. The numerical solution of $v(x,t)$ by VIM for different values of time. (k = 1) (Example 1).

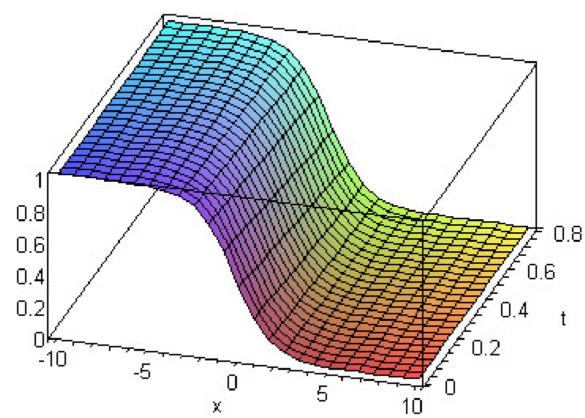

Figure 2b. The exact wave front solution of with fixed values of (Example 1).

Its stationary conditions can be obtained as follows:

$$\begin{Bmatrix} \lambda'\big|_{\tau=t} = 0 \\ 1+\lambda\big|_{\tau=t} = 0 \end{Bmatrix} \tag{37}$$

We obtain the lagrangian multiplier:

$$\lambda = -1 \tag{38}$$

As a result, we obtain the following iteration formula:

$$v_{n+1}(x,t) = v_n(x,t) + \int_0^t (-1)(v_{n\tau} - \frac{\partial^2 v_n(x,\tau)}{\partial x^2} + u_n(x,\tau)v_n(x,\tau))d\tau \tag{39}$$

Now we start with an arbitrary initial approximation that satisfies the initial condition

$$\begin{Bmatrix} u(x,0) = \dfrac{e^{kx}}{\left[1+e^{0.5kx}\right]^2}, \\ \\ v(x,0) = \dfrac{1}{\left[1+e^{0.5kx}\right]}, \end{Bmatrix} \tag{40}$$

Using the above variational formula (39), we have

$$v_1(x,t) = v_0(x,t) + \int_0^t (-1)(v_{0\tau} - \frac{\partial^2 v_0(x,\tau)}{\partial x^2} + u_0(x,\tau)v_0(x,\tau))d\tau \tag{41}$$

Substituting Eq. (40) in to Eq. (41) and after simplifications, we have:

$$v(x,t) = v_1(x,t) = \frac{(0.25)(4+8e^{0.5kx}+4e^{kx}+k^2e^{kx}t-k^2e^{0.5kx}t-4e^{kx}t}{(1+e^{0.5kx})^3} \tag{42}$$

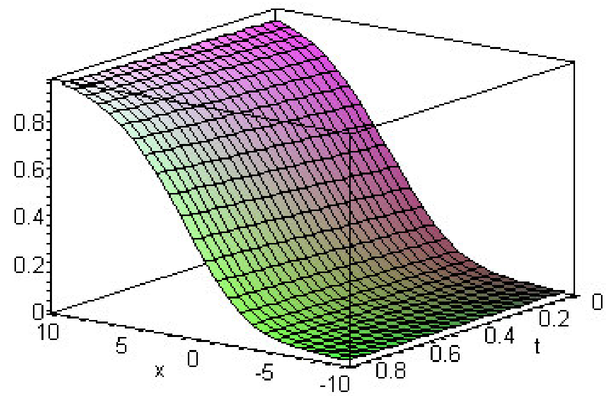

Figure 3a. The numerical solution of $u(x,t)$ by VIM for different values of time. (k = 2/3) (Example 2).

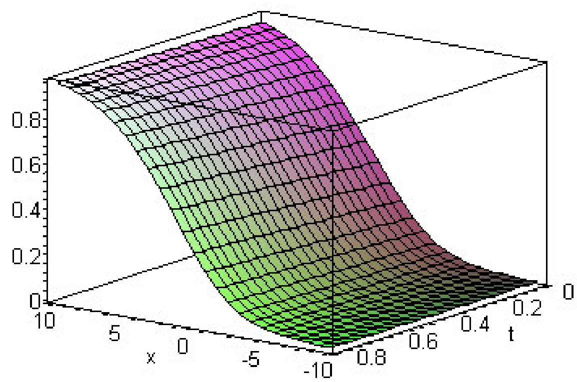

Figure 3a. The numerical solution of $u(x,t)$ by VIM for different values of time. (k = 2/3) (Example 2).

And so on. In the same way, the rest of the components of the iteration formula can be obtained.

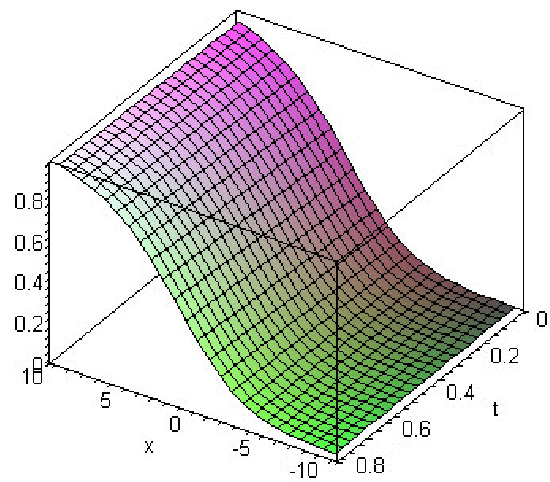

Figure 4a. The exact wave front solution of $u(x,t)$ with fixed values of $k = 2/3, c = 1$ (Example.2).

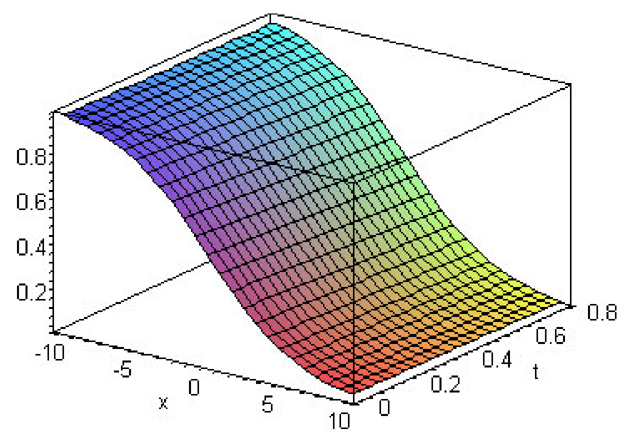

Figure 4b. The exact wave front solution of $v(x,t)$ with fixed values of $k = 2/3, c = 1$. (Example 2)

RESULTS AND DISCUSSIONS

To verify obtained results, we assume $k = c = 1$ and therefore, Figures (1 - 4) show comparison of obtained results $u(x,t)$ and $v(x,t)$ by VIM with the exact solution. The results clearly illustrate that using the one iteration, we obtained good results which they are very close to the results of the exact solution.

Conclusion

In this letter we applied variational iteration method, for solving coupled system of nonlinear partial deferential equations. The results obtained here were compared with the exact solution. The results revealed that the variational iteration method is a powerful mathematical tool for solutions of nonlinear equations in terms of accuracy and efficiency. Besides, in all examples the convergence of the VIM method are faster than ADM method (Wakiland Abdou, 2007) which introduces this method as an efficient method for solving nonlinear partial deferential equation.

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