Conceptual networks: The role of concept lattices in knowledge management

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Concepts are defined as couples \((O, A)\) of sets \(O\) and \(A\): the object \(O\) (a set of none, or one or more elements) is assigned to the set \(A\) of these elements’ (common) attributes. The objects change according to the sequence of attributes. Only couples of objects and attributes, that is concepts, are adequate for our world. The connections and links we need in databases and multimedia are expressed, naturally, by concepts, since concepts have been proved to dispose the order of a lattice (more complex than linear and hierarchical ones). The lattice can be created by two algebraic operations: "intersection" as the multiplication and "symmetric-difference (!)" as the addition (!). There are, also, two other operations: the "union" and the "complement of a concept". Intersection and union (which cannot play the role neither of the addition nor of the multiplication) express similarities, while the other two operations express dissimilarities. The operation “complement of a concept” expresses the different, the uncommon, the variety. The symmetric-difference of two concepts has been proved to be a "distance" between them (in the mathematical sense!). This research results reveal that concept lattices serve as the deep knowledge for decision-making, virtual reality, databases and multimedia, without any restriction or standardization.

Key words: Knowledge, concept, lattice, network, distance.

INTRODUCTION

Mathematical structure of concepts

Definition 1: Concept is every assignment of a prototype to an icon, whatever may be the prototype and the icon. We call the prototype “object” and the icon “attributes”. We symbolize a concept with a couple whose left part is the object and right part the attributes.

Definition 2: \((O_1, A_1) \cup (O_2, A_2) = (O_1 \cup O_2, A_1 \cap A_2)\), where \(\cup\) and \(\cap\) are the usual operations between sets, union and intersection, respectively. We call this operation “union of concepts”.

Definition 3: \((O_1, A_1) \cap (O_2, A_2) = (O_1 \cap O_2, A_1 \cup A_2)\). We call this operation "intersection of concepts". With the above two operations, for every two concepts, there exist a “higher” and a “lower” concept.

Definition 4: \((O_1, A_1) \subseteq (O_2, A_2) \iff (O_1 \subseteq O_2 \text{ and } A_1 \supseteq A_2)\), where \(\subseteq\) and \(\supseteq\) denote the usual subset and superset, respectively. The subordinated concept \((O_1, A_1)\) are the species and the superordinated concept \((O_2, A_2)\) is the genus.

Definition 5: Two concepts \((O_1, A_1)\) and \((O_2, A_2)\) are equivalent if, and only if, \(A_1 A_2\). In symbols, \((O_1, A_1) = (O_2, A_2) \iff A_1 = A_2\).

Proof that \(\approx\) is an equivalence relation among concepts:

1. \(\approx\) is reflexive. Indeed, \((O, A) \approx (O, A) \iff A = A\), which is true because of the reflexivity of the equality relation = for sets.
2. ≈ is symmetric. Indeed, \((O_1, A_1) \approx (O_2, A_2) \iff A_1 = A_2 \iff A_2 = A_1\) (which is true because of the symmetric property of the equality relation for sets) \(\iff (O_2, A_2) = (O_1, A_1)\).

3. ≈ is transitive. Indeed, \((O_1, A_1) = (O_2, A_2) \iff A_1 = A_2\) and \((O_2, A_2) = (O_3, A_3) \iff A_2 = A_3\). These two equalities together are \(\iff A_1 = A_3\) (which is true because of the transitivity of the equality relation for sets) \(\iff (O_1, A_1) \approx (O_3, A_3)\). Since ≈ is an equivalence relation, we have classes of concepts. As we know, the classes are disjoint sets and their union makes the set of reference. So, in our case, we have a partitioning of the set \(C\) of all concepts (neither of the set \(\Omega\) of isolated objects nor of the potential set \(P(\Omega)\) of objects).

**Definition 6.** The complement of the concept \((O, A)\) is the concept \((O^C, A^C)\), where \(O^C\) and \(A^C\) are the usual set-theoretic complements of \(O\) and \(A\), respectively.

**Definition 7.** The symmetric-difference of two concepts \((O_1, A_1)\) and \((O_2, A_2)\), is the concept \(D = (O_1 + O_2, (A_1 + A_2)^C)\), where \(O_1 + O_2\), \(A_1 + A_2\) are the usual set-theoretic symmetric-differences of \(O_1\) and \(O_2\), \(A_1\) and \(A_2\), respectively.

The set \(C\) of all concepts, with the two operations intersection \((\cap)\) and symmetric-difference \(D\) is proved to have the order of Lattice, which is equivalent to the structure of a Boolean Algebra. The symmetric-difference shows us the dissimilarities and the intersection the similarities among our objects (Figure 1) where \(D\) is the symmetric-difference of \((O_1, A_1)\) and \((O_2, A_2)\). The arrows give the subordinated concepts. \((O_1, A_1), (O_2, A_2)\) and \(D\) are located on the same level and, consequently, there is no order among them.

**CONCEPTUAL DISTANCE**

**Definition 8.** We call distance \(d(X, Y)\) of two sets \(X\) and \(Y\), the non-negative integer expressing the number of elements of their symmetric-difference \(X + Y\) (in symbols: \(n(X + Y)\)). So, \(d(X, Y) = n(X + Y)\). The three known properties of a distance hold:

1. \(d(X, X) = n(X + Y) \geq 0\) and \(d(X, Y) = n(X + Y) = n(Y + X)\) \(\iff (\Phi) = 0\)
2. \(d(X, Y) = n(X + Y) = n(X + Y) = d(Y, X)\), since \(X + Y = Y + X\)
3. \(d(X, Y) + d(Y, Z) = n(X + Y) + n(Y + Z)\).

We observe that \((X + Y) + (Y + Z) = X + Z\). Generally, \(A \cup B = (A \cap B)\) - \((A \cap B)\), but \(n(A + B) = [n(A) + n(B) - n(A \cap B)] - n(A \cap B) = n(A) + n(B) - 2n(A \cap B)\). So, \(n(X + Z) = n(X + Y) + n(Y + Z) - 2n((X + Y) \cap (Y + Z)) \Rightarrow n(X + Y) + n(Y + Z) \geq n(X + Z)\). Consequently, the third property is valid. Let's go, now, to the concepts. We can take \(d(O_1, O_2) = n(O_1 + O_2)\), which is a distance between objects, but it does not say many things, since it is quantitative but not qualitative: two sets of objects may have many different elements, coming from the same homogenous population (biometry, psychometry, students' and teachers' evaluation ...). Besides, we are not working with objects or attributes, but with both of them, that is concepts. The symmetric-difference \(O_1 + O_2\) of the objects, has the icon \((A_1 + A_2)^C\). So, if we want the real distance of \(O_1\) and \(O_2\), we must check \((A_1 + A_2)^C\).

\[d(A_1, A_2) = n(A_1 + A_2) = n(\Omega) - n((A_1 + A_2)^C),\] where \(\Omega\) is the set of all attributes (in our certain application). So, \(n((A_1 + A_2)^C) = n(\Omega) - d(A_1, A_2)\). \(n(\Omega)\) is a constant.

Consequently, if the distance of the attributes is increasing, \(n((A_1 + A_2)^C)\) is decreasing and the distance of the objects is, accordingly, decreasing. The explanation comes naturally: if we have a large range of attributes, this range can fit only to a small range of objects (larger intensity, smaller extension). It is the same with statistical analysis by zones.

How can we succeed to have small distance between two objects? When \(d(A_1, A_2)\) is large, or, equivalently, when \(n(A_1 + A_2)\) is large, which refers to the intersection \((A_1 \cap A_2)\) and «the area out of \(A_1 \cup A_2\)», that is the set \((A_1 \cap A_2)^C\). This second set is a fuzzy factor in the definition or comparison of concepts (Sotiropoulos 2010 and 2011). If it is large, then \(d(O_1, O_2)\) is small, maybe \(d(O_1, O_2) = 0\), which means, not that \(O_1 = O_2\) (on the contrary...), but that \(O_1\) and \(O_2\) can be easily confused and considered as equal! In many applications, we do not know exactly \(O_1\), \(O_2\), \(A_1\), \(A_2\), but only their common elements and their differences. Instead of «fishing» (stochastically...) in the «area» of \((A_1 \cap A_2)^C\), it is better to try to maximize \(A_1 \cap A_2\) (except if we take the risk ...). The more attributes two objects have in common, the more they are alike, or almost equal, that is \(d(O_1, O_2) = 0\). In the same time, we try to minimize or, at least,
keep below a certain level, the «area» \((A_1 \cup A_2)^c\) (since, we can never extinguish this fuzzy factor). It is exactly what happens in Statistics with the mistakes of I or II kind and the levels of significance.

At this point, I would like to emphasize the connection of the fuzzy nature of concepts with what Professor Ivar Ekeland declares in his book «Mathematics and the unexpected» (The University of Chicago Press, 1988) «the limits of the capabilities of computations», «the separation of the quantitative estimation from the qualitative understanding», «even inside the most exact and ambitious mathematical models, there exists enough place for the unexpected», «a new type of model that will show us the possibilities of the future, without foreseeing just one that will happen», «curves that looked like clearly and exactly sketched, can be resolved in a fuzzy area», «we have passed from identity to resemblance», «a deterministic system can show random behaviour, if part of the information is unknown» and many others of the same kind.

**KNOWLEDGE NETWORKS**

The proposed algebraic structure is in complete accordance with the General Theory of Terminology (Felber, 1984; Sotiropoulos, 2009). It gives, also, acceptable results from the view of Cognition and Learning (Sotiropoulos; 1998). For example, all the situations, “real” or “imaginary” can be expressed (Figure 2). Let’s take an isolated object \(O_1\). As soon as we discover an isolated attribute \(a^1\) we have the concept \((O^1, a^1)\). With a second isolated attribute \(a^2\) we have:

\[
(O^1, a^1) \cup (O^1, a^2) = (O^1 \cup O^1, a^1 \cup a^2) = (O^1, a^1 \cup a^2) = (O^1, \emptyset)
\]

(because we suppose that \(a^1\) and \(a^2\) are different attributes) and

\[
(O^1, a^1) \cap (O^1, a^2) = (O^1 \cap O^1, a^1 \cap a^2) = (O^1, a^1 \cap a^2).
\]

So, we have created a new concept: \((O^1, a^1 \cup a^2)\). This means that the isolated object \(O^1\) has both attributes \(a^1\) and \(a^2\). With a third isolated attribute \(a^3\) we take three new concepts, \((O^1, a^1 \cup a^3)\), \((O^1, a^2 \cup a^3)\) and \((O^1, a^1 \cup a^2 \cup a^3)\). We proceed, till we create the concept \((O^1, A_1)\), where \(A_1\) is the set of all the attributes of \(O^1\). So, for every object, we go to deeper levels of order as we discover more attributes of it. The meaning is that for every new attribute, the object is classified to an always thinner classification. It may seem strange that, though we add attributes, we go to subordinate concepts but, really this is an advantage and very logical: the more attributes we add, the more we specify the object, the “closer” we go to it (the conceptual distance from it becomes always smaller). On the other hand, there are concepts with no order between them. For example \((O^1, a^1)\) and \((O^2, a^2)\) are located in the same level: neither superordinated nor subordinated the one to the other, but not equal. Also, the symmetric-difference \(D\) (new concept) of any two given concepts...Suppose we have the concepts \((O^1, A_1), (O^2, A_2)\) and \((O^3, A_3)\), where \(O^1, O^2, O^3\) are isolated objects and \(A_1, A_2, A_3\) their sets of attributes, respectively. This means that every one of them stands at the top of figure like figure 3. We begin to take unions (\(\cup\)) and intersections (\(\cap\)) between them and afterwards between them and the first results and so on (Sotiropoulos 1995 and 1999). From the unions we find: \((O^1 \cup O^2, A_1 \cap A_2), (O^1 \cup O^3, A_1 \cap A_3), (O^2 \cup O^3, A_2 \cap A_3)\) and \((O^1 \cup O^2 \cup O^3, A_1 \cap A_2 \cap A_3)\). The intersections gives us always the empty set in the left part of every couple because the isolated objects are supposed to be distinct. In this way, \((\emptyset, A)\) means that the empty set has all the attributes of the set \(A\). There is no problem to accept such a thing because, to the empty set, we may assign every attribute. Moreover, a concept like \((\emptyset, A)\) can be useful in the structure because, for example, \((\emptyset, A) \cup (B, C) = (\emptyset \cup B, A \cap C) = (B, A \cap C)\), which is a new result. The attributes \(A\) may correspond to an object not yet discovered (like the case of some planets, comets and atomic particles) or to an object that should be created or named (terminological-didactical work), so that we make an extension of our system. The cycle in Figure 4 represents Figure 3.

Figure 4 gives us another possibility: all the concepts involving the same object \(O^i\), form another sublattice (class). So, we have all the possible scenarios concerning this object. 4 shows us the same result with 2, but with a whole set \(A_i\) of attributes, instead of the isolated attribute \(a^i\). This means that we can impose a whole set of conditions. The access to information in not linear, but follows the structure of the lattice. Of course, real applications may give a sublattice of \(A\), or, if we are unlucky, no order among the concepts. In every case, if we, afterwards, create all the possible combinations (by using \(\cap\), \(\cup\) and the symmetric-differences), we find a complete lattice. Links are a main characteristic of lattices. Conceptual lattices are created naturally from the attributes of their objects. For example, in 2, from the top to the bottom there are two paths concerning the object \(O^i\) and the attribute \(a^i\). The smallest sublattice is the “elementary rhomb” of Figure 1.

**APPLICATIONS**

**Geometry**

*The structure is not a tree: a rectangle with equal sides*
becomes a square, but, also, a rhomb, with equal angles becomes a square. A square can be considered as a descendant of the rectangle, but also, of the rhomb. The four concepts (parallelogram, rectangle, rhomb, square) form a lattice: \( \{ \text{rectangle} \} \cup \{ \text{rhomb} \} = \{ \text{parallelogram} \} \) and \( \{ \text{rectangle} \} \cap \{ \text{rhomb} \} = \{ \text{square} \} \) (Figure 5).

**Vehicles**

(i) A submarine can move on the surface of the sea, but, also, under the surface (or rest on the bottom of the sea!). Is it a ship...?

(ii) “Columbia” space vehicle, when coming back to the Earth, moves as a usual airplane...

(iii) What about “flying dolphins”, “hovercrafts” and so on? They move on the surface of the sea (ships), but their movement obeys, partly, the laws of physics obeyed by the airplanes! So:

- a ship has sails or/and propeller, moves only on the sea surface.
- a submarine has propeller, moves on the sea surface and under the surface.
- a flying dolphin has propeller, moves on the sea surface and slides with wings on it - a wind-surfer has sail, slides on the sea surface.
It is obvious that the relations among these concepts do not form an hierarchy (a tree) for example, one could say that a submarine is a special ship (that is, a ship capable of moving underwater), but the truth is that a submarine is something different than a ship (despite the fact that it can move, also, on the sea surface as the ships do). Besides, there are ships moving with sails or with propeller and sails together (while the submarines use only propeller). Conclusion: objects and attributes are both necessary for a concept to be formed. A knowledge database should include all these concepts, without trying to separate objects into distinct (however, artificial!) categories (Sotiropoulos, 2009).

**Medicine**

(i) “Reading” an X-ray picture is a very difficult task. For
cancer of lungs? To a certain extent, the two pictures are alike.
(ii) Fever and cough may lead to several different diseases...
(iii) Some substances can serve as a poison or a medicine (like women)! That is why, in Greece, we say “pharmako” = medicine and “pharmaki” = poison (Kitsos and Sotiropoulos, 2009).

Everyday life
(i) In German language, women use the expression “meine Man”. But, it not clear what they mean by the term “Man”: their husband, their boy-friend or something else...?
(ii) A famous Greek painter has expressed the following opinion: “you are what you declare to be”! This is a great truth! According to the set of attributes you present each time, you belong to the corresponding class (=concept) of people...
(iii) Virtual reality is based, exactly, on the set of attributes we present each time. Women know something more about that, by using make-up etc.
(iv) “It is so, if you consider it so” (Luigi Pirantello).

Multimedia
Objects and attributes cannot be absolutely defined. Our world is relative and dynamic, not sure and static. An object does not exist by itself, but by its attributes. That is why we say that an object «is equal» to its attributes. Other set of attributes, other object. So, objects are evolving from the one to the other. The question, now, is how? Is there any connection, any rule? The answer is yes: obeying the laws of thought, which means not only the laws of formal (...) logic, but something much more: the laws of concepts (Bakaoukas and Sotiropoulos, 2008). Everything in our mind is concepts, so everything in our world is concepts. Do objects have memory? Of course: they can be expressed with pre-existing concepts by using the operations on concepts. Is it possible for new objects to emerge? Again, by the operations on concepts (s 1 to 6). So, concepts are being transformed continuously in our mind. An algebra of concepts is being carried out every time, every second in our mind. Exactly, as in multimedia!

Files, icons, sounds are being connected continuously, in the one way or the other, inside the computer. What is really important is the connection, how it is carried out and what sequence is followed. The question is: what comes after what? What is connected to what? Are there many choices, a few or one? It is obvious, now, that the «multimedia way of thinking» is just a matter of concepts. Old and new concepts are connected, there is an order among them, not an hierarchical one but the more complex of a lattice. The connections are not the classic ones superordinated - subordinated, but there are, also, dissimilarities, varieties, fantasy. There are links and freedom of choice - exactly as in multimedia happens (Figure 6).

Is there an optimum path in order to treat a subject we are interested in? We can go «surfing» but, in any case, inside the frame of the multimedia design of this subject. Files, icons and sounds are connected «properly» when the connection has a meaning, which is equivalent to the
conceptual context of the subject. Concepts are connected by their nature, from their «birth». The sequence (or, better, net) of files, icons and sounds has the meaning that is imposed by the concepts - only concepts give a meaning to the plentiness of them. The conceptual sublattice, created by the concepts of this subject, is the optimum path! ...(Sotiropoulos, 1999).

Conclusion

Everything in our structure comes from the operations. We do not make assumptions, we do not impose restrictions (except of those coming from the nature of every problem). The four operations are easily implemented in the computer. By computer programming we can express the creation of superordinated concepts from subordinated ones (and vice versa). We go «from bottom to top» (and vice versa) and we find the concepts successively. Moreover, we are not obliged to store in the memory of the computer every concept, but only the ones the author is interested in. We can create the others whenever we want, taking advantage of the classes and of the fact that, every concept creates (is) a classification.

All concept lattices are not equivalent (Wille, 1997). My operations, which are original, construct a lattice "richer" than others. “Rich” means that they are consisted of more concepts, more possible situations. All concepts are accepted, not only the standardized. So, we are free to construct classes as thin as possible. All the above tools are really new. More than one concept lattices can be constructed, but the point is which one corresponds to human logic and real life. The proposed structure is not a cartesian product of sets. It is not just subsets of spaces spanned by attributes (Sowa, 1983). It is something more knowledge, which can be extracted from sets of objects and from sets of attributes by using only the operations.

At the end of 1985, when this work on concept lattices started, Professor. R. Wille and his works (Wille, 1982 and 1984) were not known. But, even afterwards, when I got to know the papers of his team (Wille, 1997), I continued working on my own way of thinking which permits in real data freedom of choice, imagination and saving of time and size by the use of classes of concepts (not of objects, which, do not, really, exist). We use couples in order to define concepts, because, in this way, we express the relativity of the objects. Professor R. Wille takes as concepts only the standardized ones (Sotiropoulos, 2009). Obviously, this fits well with robots, but not with human beings. Freedom and fantasy can give birth to many partly different or absolutely new objects—do not have the right to exclude them. Our proposed system of concepts is not closed but open. Open to new concepts (like union, intersection and symmetric-difference of concepts), open to probability, open to fuzziness (it exists anyway!), open to not predefined situations, open to new didactical purposes (methods). Obviously, another mentality … But, real life is full of differences and we are not allowed to normalize human thought! People are taught to think with trees – this is the simple and not annoying way…But, real life is full of complexities and lattice is an adequate instrument to express them. In a tree, the child comes from only one parent (...), while in the lattice every child has two parents. Hierarchies are not the unique kind of order (...).

Knowledge databases should have the structure of Lattice, because Lattice is the structure of knowledge itself... (Sotiropoulos, 1992).

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