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Short Communication

4th-Step implicit formula for solution of initial value problems of second order ordinary differential equations

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This paper proposes a direct method of solution for second order initial value problems of ordinary differential equations. Its derivation adopts Taylor series expansion techniques and Dahlquist stability test model. It was revealed from the results obtained that the method is consistent, convergent and zero-stable. Numerical examples are solved to ensure accuracy of the method on both linear and non-linear initial value problems of second order ordinary differential equations.

Key words: Discrete, consistency, zero-stability, error constant, IVPs and convergence.

INTRODUCTION

In this article, second order initial value problems of ordinary differential equations of the form

$$y'' = f(x, y, y'),\ y(a) = \eta_0,\ y'(a) = \eta_1 \qquad \text{........(1)}$$

Is proposed for even step-number k=4. This class of problem has a lot of applications in areas of Science and Engineering, most especially in mechanical systems without dissipation, electrical network, radioactive process, and motion of projectiles. In literature, problems of the form (1) are conventionally tackled by reducing the differential system to first order equations, this approach is inefficient, a lot of computer time and human efforts were wasted on implementation due to the dimension of problem (1). Some eminent scholars such as Lambert (1973, 1976, 1991), Fatunla (1988), Sarafyan (1990), Wright et al. (1991), Lambert et al. (1976), Henrici (1962), Awoyemi (2002, 2003), Yakubu (2003), Areo et al. (2008) and Owolabi et al. (2010) to mention a few, have contributed immensely in this area of research.

THE METHOD DERIVATION

Four-step numerical formula of the form

$$y_{n+4} = \alpha_0 y_n + \alpha_1 y_{n+1} + \alpha_2 y_{n+2} + \alpha_3 y_{n+3} + h^2(\beta_0 y''_n + \beta_1 y''_{n+1} + \beta_2 y''_{n+2} + \beta_3 y''_{n+3} + +\beta_4 y''_{n+4}) \text{.......(2)}$$

is considered in this paper for solution of (1). The parameters $\alpha j's$ and $\beta j's$, j=0(1)4 are determined to ensure that the formula is consistent, convergent and zero stable. Taylor series expansion of variables y_{n+j} j=0(1)4 and their derivatives are used to generate the system of algebraic equations where the values parameters $\alpha j's$ and $\beta j's$, j=0(1)4 are determined. Stability property of the formula is examined using Dahlquist stability model test equation

$$y'' = \lambda y \qquad \text{........(3)}$$

The values of the coefficients are determined from the local truncation error

$$T_{n+k} = y_{n+k} - \sum_{j=0}^{k-1} \alpha_j y_{n+j} + h^2 \sum_{j=0}^{k} \beta_j f_{n+j}\ ,$$

k=4, j=0(1)4 .... (4)

the terms y_{n+j} and f_{n+j} are expressed in Taylor's series with the local truncation error on which accuracy of order six is imposed to obtain the system of equations

$$\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3 = 1$$

$$\alpha_1 + 2\alpha_2 + 3\alpha_3 = 4$$

$$\frac{1}{2}\alpha_1 + \frac{4}{2}\alpha_2 + \frac{9}{2}\alpha_3 + \beta_0 + \beta_1 + \beta_2 + \beta_3 + \beta_4 = \frac{16}{2}$$

$$\frac{1}{6}\alpha_1 + \frac{8}{6}\alpha_2 + \frac{27}{6}\alpha_3 + \beta_1 + 2\beta_2 + 3\beta_3 + 4\beta_4 = \frac{64}{6}$$

$$\frac{1}{24}\alpha_1 + \frac{16}{24}\alpha_2 + \frac{81}{24}\alpha_3 + \frac{1}{2}\beta_1 + \frac{4}{2}\beta_2 + \frac{9}{2}\beta_3 + \frac{16}{2}\beta_4 = \frac{256}{24} \quad \ldots (5)$$

$$\frac{1}{120}\alpha_1 + \frac{32}{120}\alpha_2 + \frac{243}{120}\alpha_3 + \frac{1}{6}\beta_1 + \frac{8}{6}\beta_2 + \frac{27}{6}\beta_3 + \frac{64}{6}\beta_4 = \frac{1024}{120}$$

$$\frac{1}{720}\alpha_1 + \frac{64}{720}\alpha_2 + \frac{729}{720}\alpha_3 + \frac{1}{24}\beta_1 + \frac{16}{24}\beta_2 + \frac{81}{24}\beta_3 + \frac{256}{24}\beta_4 = \frac{4096}{720}$$

$$\frac{1}{5040}\alpha_1 + \frac{128}{5040}\alpha_2 + \frac{2187}{5040}\alpha_3 + \frac{1}{120}\beta_1 + \frac{32}{120}\beta_2 + \frac{243}{120}\beta_3 + \frac{1024}{120}\beta_4 = \frac{16384}{5040}$$

$$\frac{1}{40320}\alpha_1 + \frac{256}{40320}\alpha_2 + \frac{6561}{40320}\alpha_3 + \frac{1}{720}\beta_1 + \frac{64}{720}\beta_2 + \frac{729}{720}\beta_3 + \frac{4096}{720}\beta_4 = \frac{6553}{4032}$$

putting (5) in matrix form and solved with Gaussian process to yield

$$\alpha_0 = -1, \quad \beta_0 = \frac{1}{12}$$

$$\alpha_1 = 4, \quad \beta_1 = \frac{8}{12}$$

$$\alpha_2 = -6, \quad \beta_2 = -\frac{18}{12}$$

$$\alpha_3 = 4, \quad \beta_3 = \frac{8}{12}$$

$$\beta_4 = \frac{1}{12} \quad \ldots\ldots (6)$$

Substituting (6) into (2) yields a 4-step second order scheme

$$y_{n+4} = 4y_{n+3} - 6y_{n+2} + 4y_{n+1} - y_n = \frac{h^2}{12}\left[f_{n+4} + 8f_{n+3} - 18f_{n+2} + 8f_{n+1} - f_n\right] \ldots\ldots\ldots\ldots\ldots (7)$$

The procedure for examining the basic properties of (7) is the same as that of Owolabi et al. (2010). The method is of order $p = 6$. Error constant $C_{P+2} = -\frac{1}{240}$, region of absolute stability $X(\theta) = (-6, \infty)$, it was found to be zero-stable, consistent and convergent. Finally, previous values of y_{n+j}, j=0(1)4 used in the compotation are predicted by Taylor's method (Awoyemi et al., 2003; Owolabi et al., 2010).

NUMERICAL EXAMPLES

Two numerical examples on linear and non-linear were considered to acclaim accuracy and applicability of the new method. The results are compared with that of Owolabi et al. (2010).

Problem 1

$$y'' - x(y')^2 = 0, \; y(0) = 1, \; y'(0) = \frac{1}{2}$$

Exact solution: $y(x) = 1 + \frac{1}{2}\ln(\frac{2+k}{2-x})$

Problem 2

$$y'' + y = 0, \; y(0) = 0, \; y'(0) = 1$$

Table 1. Solution to problem 1.

n	Xn	Owolabi et al. (2010) error	New method error
3	0.75	0.000003815	0.000000000
5	0.125	0.000038981	0.000000834
7	0.175	0.000137210	0.000003695
8	0.200	0.000220060	0.000006437
11	0.275	0.000653028	0.000033855
14	0.350	0.001453042	0.000064492
16	0.400	0.002250195	0.000113964
19	0.475	0.003938079	0.000241637

Table 2. Solution to problem 2.

n	Xn	Owolabi et al. (2010) error	New method error
3	0.75	0.000015609	0.000000015
5	0.125	0.000155993	0.000000246
7	0.175	0.000545457	0.000001371
8	0.200	0.000872254	0.000002652
11	0.275	0.002564847	0.000012845
14	0.350	0.005643636	0.000042707
16	0.400	0.008664817	0.000083184
19	0.475	0.014940143	0.000260181

Exact solution: $y = \sin x$

These problems are solved with fixed step size h=1/40, error = (Exact - Approximate) solution. The solutions to problems 1 and 2 are presented in Tables 1 and 2.

Conclusion

In this paper, a direct four-step formula with step number k=4, has been proposed for direct solution of initial value problems of second order ordinary differential equations as an alternative to Owolabi et al. (2010), to solve problem (1). The two test problems solved by Owolabi et al. (2010) were solved by the new method and the results from the tables indicated that the new method has significant accuracy over Owolabi et al. (2010), on comparison. Analysis and basic properties of the method also revealed that it is consistent, convergent and zero stable.

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