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Full Length Research Paper

A Branch and Bound approach to a state government capital budgeting

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The capital budgeting problem is a multi-constraint Knapsack problem. Hence this paper provides a solution to a capital budgeting problem of Akwa Ibom State, Nigeria using a Branch and Bound approach. The capital projects are summarized in four broad groups: The Economic sector (x_1), The Social Service sector (x_2), The Environmental/Regional Development sector (x_3), and The Administration sector (x_4). Using Tora software (version 11) for the analysis, it was observed that the optimal solution was found in node 60 while the upper bound and the lower bound were 154.46 and 129.76 respectively. The result shows that the second and the third sector will be selected ($x_1, x_4 = 0; x_2, x_3 = 1$).

Key words: Capital budgeting, branch and bound, Knapsack problem, capital projects.

INTRODUCTION

Branch and Bound method is one of the deterministic techniques of solving a Knapsack problem. It is seen as the most efficient tool among other deterministic methods as it is based on a restriction of the search tree growth; (Sedova and Seda, 2008). They also observed that avoiding much enumeration depends on the precise bounds (the lower and the upper bounds, the faster the finding of the solution is). Lawler and Wood (1966) observed Branch and Bound as algorithm for finding optimal solutions to combinatorial problems. The method produces convergent lower and upper bound for the optimal solution using an implicit enumeration scheme. Branch and bound method does not go through iterative partial solution until an optimal solution is found like the case of dynamic programming neither does it enumerate all feasible solutions which may lead to 2^n different solutions and select the one with the highest objective function value; brute force approach. The concept of Branch and Bound is based on an intelligent complete enumeration of the solution space since in many cases, only small subsets of the feasible solutions are enumerated explicitly. It is however guaranteed that the parts of the solution space which were not considered explicitly cannot contain the optimal solution (Kellerer et al., 2004). In this work, we will apply this method in a capital budgeting problem in order to maximize the value of some selected sectors among the other sectors. We are using a state budget; Akwa Ibom State, Nigeria, as our case study for the period of ten years. We are considering the entire Capital budget in terms of their sectors grouping. Each sector has various areas in it. The sectors are, the Economic Sector, the Social Services Sector, the Environment/Regional Development sector and the General Administrative sector. The problem is a multidimensional Knapsack Problem and the model will be stated as we progress.

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Assumptions

(I)All the projects are independent of each other.
(ii) All the sectors have equal importance to the

development of the State.

LITERATURE REVIEW

Branch and Bound approach is one of the exact approaches of solving a Knapsack problem. Researchers have applied it in diverse ways to obtain an optimal solution to the problem of their choice. The first Branch and Bound algorithm for Knapsack problem was presented by Kolesar (1967). Thereafter several variants of the basic framework emerged, which are usually based on the depth-first search to limit the number of open nodes to O(n) at any stage of enumeration (Kellerer et al., 2004). Greenberg and Hegerich (1970) improved on Kolesar (1967) algorithm by reducing the large computer memory and time requirements greatly. Monapo (2008) presented an approach that enhances the performance of the Branch and Bound algorithm for the Knapsack model. He was able to achieve this by generating and adding new objective function and constraint to knapsack model with single constraint. Cotta et al. (1995) used a problem specific Branch and Bound approach for the traveling salesman problem. Volgenant and Jonker (1982) employed a Branch and Bound algorithm for the symmetric traveling problem based on the 1-tree relaxation and made use of an enumeration Algorithm to provide bounds in order to guide the Branch and Bound search. Horowitz and Sahni (1974) derived from the previous scheme a depth-first algorithm in which: (a) selection of the branching variable (x_j) is the same as in Kolesar (1967); (b) the search continues from the node associated with the insertion of item j (condition $(x_j = 1)$), that is, following a greedy strategy. Other algorithms have been derived from the Greenberg-Hegerich approach (Barr and Ross, 1975; Lauriere, 1978) and from different techniques (Lageweg and Lenstra, 1972; Guignard and Spielberg, 1972; Fayard and Plateau 1975; Veliev and Mamedov, 1981).

Aim of the research

Naturally, Government`s responsibility is always to subsidize/provide for its citizen. As such, during this process that government is trying to provide for her citizen, the government is bound to select or choose from all the services needed in different sectors, which among them that he will want to invest on/satisfy. This can be achievable by either minimizing the cost, such that it invests in as much project as possible given the limited resources or by maximizing the values of the selected projects.

METHODOLOGY

This is a technique used only to solve optimization problems. It is an improvement over exhaustive search, because unlike it Branch and Bound constructs candidate solutions one component at a time and evaluates the partly constructed solutions. If no potential value of the remaking components can lead to a solution, the remaining components are not generated at all. This approach makes it possible to solve some large instances of difficulty combinatorial problems, though, in the worst case, it still has an exponential complexity (Hristakeva et al., 2004).

Branch and Bound investigates classes of solutions corresponding to completions on partial solutions in a tree like fashion that gives it the “branch" part of its name. Nodes of this Branch and Bound tree represent partial solutions, with numbers indicating the sequence in which they are investigated. Edges or lines of the tree specify how variables are fixed in partial solutions (Rardin, 1998). Branch and Bound searches terminates, or fathoms a partial solution when they either identify a best completion or prove that none can produce an optimal solution in the over all.

Rardin (1998) further stated that when a partial solution cannot be terminated in a Branch and Bound search of 0-1 discrete optimization model, it is branched by creating two subsidiary partial solutions derived by fixing a previously free binary variable. One of these partial solutions matches the current except that variables chosen is fixed =1, and the other identical except that the variables is fixed = 0. That is to say, this search stops when every partial solution in the tree has been either branched or terminated. Branch and Bound is the most efficient tool among the deterministic techniques as it is based on a restriction of the search tree growth (Sdova and Seda, 2008).

The model

Since the problem is a Capital Budgeting problem, the Capital Budgeting model will be adopted in the work. Capital Budgeting models select a maximum value collection of project, investment and so on, subject to limitations on budgets or other resources consumed (Rardin, 1998) (Table 1).

We aim at selecting at least a sector from each year in such a way that the total cost of executing projects in a certain year does not exceed the total allocation or assigning to capital budget for that year such that the value of that sector is maximized. That is:

$$Max \quad Z = \sum_{j=1}^{n} V_j x_j$$

$$S.t \quad \sum_{j=1}^{n} C_{ij} x_j \le A_i , \quad i = 1,2.,\ldots,m$$

where A_j is the total budget for the n projects in all the sectors for the jth year, C_{ij} is the cost of executing project j in year I, V_j is the value for projects in sector j, x_j is the binary variable .

Therefore we can now present the formulation of capital budgeting of Table 1 as:

$$Max \quad Z = V_1x_1 + V_1x_1 + V_1x_1 + \ldots + V_nx_n$$

$$S.t \qquad C_{11}x_1 + C_{12}x_2 + C_{13}x_3 + \ldots + C_{1n}x_n \le A_1$$

Table 1. Budget requirement.

Years	Sectors							
1	C_{11}	C_{12}	C_{13}	.	.	.	C_{1n}	A_1
2	C_{21}	C_{22}	C_{23}	.	.	.	C_{2n}	A_2
3	C_{31}	C_{32}	C_{33}		.	.	C_{3n}	A_3
.	.							
.	.							
.	.							
M	C_{m1}	C_{m2}	C_{m3}		.	.	C_{mn}	A_m

Table 2. Initial iteration table.

C_B		V_1	V_2	V_3	V_4	0	0	0	0	0	0	0	0	0	0	
	C_j	x_1	x_2	x_3	x_4	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	Constant
	Basic															
	S_1	C_{11}	C_{12}	C_{13}	C_{14}	1	0	0	0	0	0	0	0	0	0	
	S_2	C_{21}	C_{22}	C_{23}	C_{24}	0	1	0	0	0	0	0	0	0	0	
	S_3	C_{31}	C_{32}	C_{33}	C_{34}	0	0	1	0	0	0	0	0	0	0	
	S_4	C_{41}	C_{42}	C_{43}	C_{44}	0	0	0	1	0	0	0	0	0	0	
	S_5	C_{51}	C_{52}	C_{53}	C_{54}	0	0	0	0	1	0	0	0	0	0	
	S_6	C_{61}	C_{62}	C_{63}	C_{64}	0	0	0	0	0	1	0	0	0	0	
	S_7	C_{71}	C_{72}	C_{73}	C_{74}	0	0	0	0	0	0	1	0	0	0	
	S_8	C_{81}	C_{82}	C_{83}	C_{84}	0	0	0	0	0	0	0	1	0	0	
	S_9	C_{91}	C_{92}	C_{93}	C_{94}	0	0	0	0	0	0	0	0	1	0	
	S_{10}	C_{101}	C_{102}	C_{103}	C_{104}	0	0	0	0	0	0	0	0	0	1	
$\bar{G}$ row																

$$C_{21}x_1 + C_{22}x_2 + C_{23}x_3 + \quad . \quad . \quad . \quad + C_{2n}x_n \leq A_2$$

$$\vdots$$

$$C_{m1}x_1 + C_{m2}x_2 + C_{m3}x_3 + \quad . \quad . \quad . \quad + C_{mn}x_n \leq A_m$$

Optimal solution

Since the problem is a multi-constraint problem, we shall present the model by adopting the simplex approach that was developed by Dantzik (1957). We shall make use of ten constraints representing the ten years considered and four non-basic variables representing the four sectors of the capital projects budgeting. Table 2 shows the initial iteration.

Sector 1 (x_1), is made up of Agriculture, Livestock and Vertinary Service, Forestry, Fisheries, Manufacturing/Craft, Urban Electrification, Commerce and Tourism, Works and Transport.

Sector 2, (x_2) is made up of Education, Science and Technology, Health, Information, Culture, Youth and Sports, Social Development, Water Supply (urban), Rural Development and Utilities.

Sector 3, (x_3) is made up of Sewage, Drainage and Refuge Disposal, Housing, Urban Development.

Sector 4, (x_4) is made up of General Administration, Judiciary-General Administration, House of Assembly-General Administration.

$$\begin{aligned}
\text{Max.} \quad & 62.77x_1 + 64.38x_2 + 65.38x_3 + 65.57x_4 \\
\text{s.t} \quad & 3.31x_1 + 3.17x_2 + 2.96x_3 + 11.67x_4 \leq 16.22 \\
& 20.06x_3 + 45.14x_4 \leq 100.70 \\
& 153.46x_1 + 137.14x_2 + 56.74x_3 + 34.23x_4 \leq 237.03 \\
& 15.68x_1 + 19.54x_2 + 48.18x_3 + 26.48x_4 \leq 70.17 \\
& 52.09x_1 + 53.13x_2 + 31.85x_3 + 41.28x_4 \leq 135.79 \\
& 138.05x_1 + 89.99x_2 + 53.93x_3 + 89.34x_4 \leq 345.41 \\
& 272.29x_1 + 108.84x_2 + 162.42x_3 + 84.61x_4 \leq 558.05 \\
& 260.36x_1 + 181.38x_2 + 275.89x_3 + 191.05x_4 \leq 801.37 \\
& 489.92x_1 + 395.55x_2 + 288.92x_3 + 256.98_4 x \leq 907.60 \\
& 615.65x_1 + 518.47x_2 + 433.15x_3 + 661.05x_4 \leq 1225.98
\end{aligned}$$

where $X_i \in \{0,1\}$ i=1,2,3,4.; and these values are in millions of naira.

RESULTS

Figure 1 below shows the outcome of our optimization effort by branch and bound tree analysis. The upper bound was 154.46 while the lower bound was 129.76 (all figures in millions of naira). Node 31, 33, 43, 51, 53, 55, 61 and 63 recorded an infeasible solution as such they were fathomed; whereas the other nodes provided feasible solutions. However node 60 provided the best bound.

Conclusion

A Branch and Bound approach was adopted to provide a

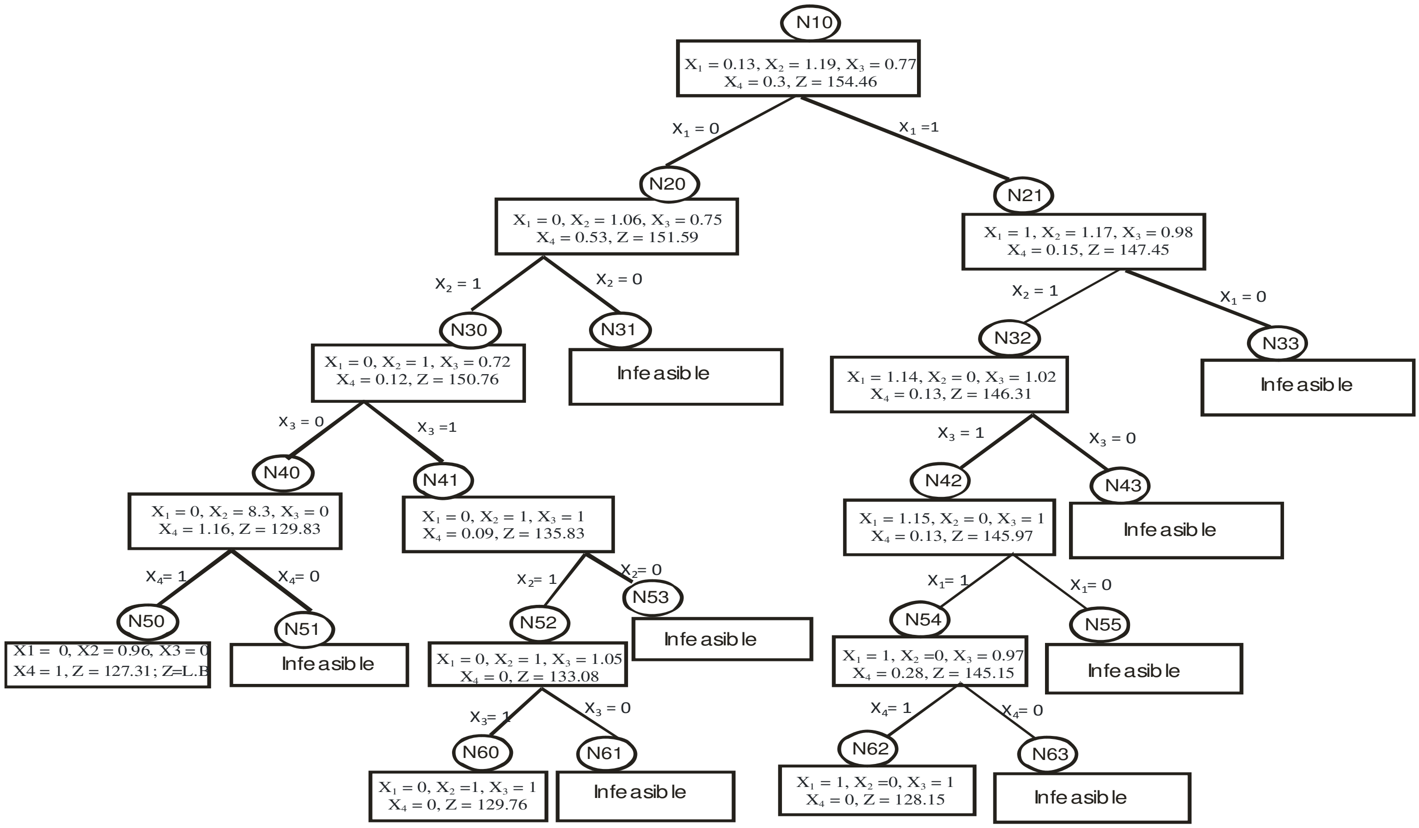

Figure 1. A Branch and Bound tree analysis for the capital budget prob. upper bound = 154.46, lower bound = 129.76.

solution to the problem. It is one of the deterministic methods of providing an optimal solution to a problem. From the analysis it was observed that the upper bound was 154.46 while the lower bound was 129.76 million naira respectively. Although the analysis recorded some infeasible solution, there were good number of feasible solution produced by different nodes, but the best bound was produced by node 60 with ($x_1, x_4 = 0; x_2, x_3 = 1$) and Z =129.76.

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