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Results on generalized fuzzy soft topological spaces

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In this manuscript, the concept of a generalized fuzzy soft point is introduced and some of its basic properties were studied. Also, the concepts of a generalized fuzzy soft base (subbase) and a generalized fuzzy soft subspace were introduced and some important theorems were established. Finally, the relationship between fuzzy soft set, intuitionistic fuzzy soft set, generalized fuzzy soft set and generalized intuitionistic fuzzy soft set were investigated.

Key words: Fuzzy soft set, generalized fuzzy soft set, generalized fuzzy soft topology, generalized fuzzy soft base (subbase), generalized fuzzy soft subspace, intuitionistic fuzzy soft set.

INTRODUCTION

Most of our real life problems in engineering, social and medical science, economics, environment, etc., involve imprecise data and their solutions involve the use of mathematical principles based on uncertainty and imprecision. To handle such uncertainties, Zadeh (1965) introduced the concept of fuzzy set (FS) and fuzzy set operations. The analytical part of fuzzy set theory was practically started with the paper of Chang (1968) who introduced the concept of fuzzy topological spaces. However, this theory is associated with an inherent limitation, which is the inadequacy of the parametrization tool associated with this theory as it was mentioned by Molodtsov (1999). Molodtsov (1999) introduced the concept of the soft set (SS) theory which is free from the aforementioned problems and started to develop the basics of the corresponding theory as a new approach for modeling uncertainties. Shabir and Naz (2011) studied the topological structures of soft sets. Intuitionistic fuzzy set theory was introduced by Atanassov (1986). In recent times, the process of fuzzification of soft set theory is rapidly progressed. Maji et al. (2001a, b) combined the theory of SS with the fuzzy and intuitionistic fuzzy set theory and called as fuzzy soft set (FSS) and intuitionistic fuzzy soft set (IFSS). Topological structure of fuzzy soft sets was started by Tanay and Burc Kandemir (2011). The study was pursued by some others researchers (Chakraborty et al., 2014; Gain et al., 2013; Mukherjee et al., 2015). Majumdar and Samanta (2010) introduced generalized fuzzy soft set (GFSS) and successfully applied their notion in a decision making problem. Yang (2011) pointed out that some results of Majumdar and Samanta (2010) are not valid in general. Chakraborty and Mukherjee (2015) introduced generalized fuzzy soft union, generalized fuzzy soft intersection and several

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other properties of generalized fuzzy soft sets. Also, they introduced generalized fuzzy soft topological spaces, generalized fuzzy soft closure, generalized fuzzy soft interior and studied some of their properties. Arora and Garg (2017a, b) solved the MCDM problem and established the IFSWA operator and the IFSWG operator under the IFSS environment. Garg (2017) introduced some series of averaging aggregation operators have been presented under the intuitionistic fuzzy environment. Garg and Arora (2017a, b) introduced distance and similarity measures for dual hesitant fuzzy soft sets in multi-criteria decision making problem, also they presented some generalized and group-based generalized intuitionistic fuzzy soft sets in decision-making.

PRELIMINARIES

Here, the basic definitions and results which will be needed in the sequel were presented.

Definition 1

Let X be a non-empty set. A fuzzy set *A* in *X* is defined by a membership function $\mu_A : X \rightarrow [0,1]$ whose value $\mu_A(x)$ represents the "grade of membership" of *x* in *A* for $x \in X$. The set of all fuzzy sets in a set *X* is denoted by I^X, where I is the closed unit interval [0,1].

Theorem

If $A, B \in I^X$, then, we have:

(i) $A \leq B \Leftrightarrow \mu_A(x) \leq \mu_B(x),\ \forall\, x \in X;$
(ii) $A = B \Leftrightarrow \mu_A(x) = \mu_B(x),\ \forall\, x \in X;$
(iii) $C = A \vee B \Leftrightarrow \mu_C(x) = \max(\mu_A(x), \mu_B(x)),\ \forall\, x \in X;$
(iv) $D = A \wedge B \Leftrightarrow \mu_D(x) = \min(\mu_A(x), \mu_B(x)), \forall\, x \in X;$
(v) $E = A^C \Leftrightarrow \mu_E(x) = 1 - \mu_A(x), \forall\, x \in X.$

Definition 2

Let X be an initial universe and E be a set of parameters (Molodtsov, 1999). Let $P(X)$ denotes the power set of *X* and $A \subseteq E$. A pair (f, A) is called a soft set over *X* if *f* is a mapping from *A* into *P(X)*, that is, $f : A \longrightarrow P(X)$.

In other words, a soft set is a parameterized family of subsets of the set *X*. For $e \in A$, $f(e)$ may be considered as the set of e −approximate elements of the soft set (f, A).

Definition 3

Let *X* be an initial universe and *E* be a set of parameters Let I^X be the set of all fuzzy sets in *X* and $A \subseteq E$. A pair (F, A) is called a fuzzy soft set over *X*, where $F : A \longrightarrow I^X$ is a function, that is, for each $e \in A$, $F(e) = F_e : X \longrightarrow I$ is a fuzzy set in *X*.

Definition 4

Let *X* be a universal set of elements and *E* be a universal set of parameters for *X* (Majumdar and Samanta, 2010). Let $F : E \longrightarrow I^X$ and μ be a fuzzy subset of *E*, that is, $\mu : E \longrightarrow I$. Let F_μ be the mapping $F_\mu : E \longrightarrow I^X \times I$ defined as follows:

$F_\mu(e) = (F(e), \mu(e))$, where $F(e) \in I^X$ and $\mu(e) \in I$. Then F_μ is called a generalised fuzzy soft set (GFSS in short) over (X, E).

Definition 5

Let F_μ and G_δ be two GFSSs over (X, E) (Majumdar and Samanta, 2010). Now F_μ is said to be a GFS subset of G_δ or G_δ is said to be a GFS super set of F_μ if:

(i) μ is a fuzzy subset of δ;
(ii) $F(e)$ is also a fuzzy subset of $G(e), \forall e \in E$.

In this case, we write $F_\mu \,\tilde{\subseteq}\, G_\delta$.

Definition 6

Let F_μ be a GFSS over (X, E). The complement of F_μ, denoted by F_μ^c, is defined by $F_\mu^c = G_\delta$, where $\delta(e) = \mu^c(e)$ and $G(e) = F^c(e)$, $\forall e \in E$ (Majumdar and Samanta, 2010). Obviously

$(F_\mu^c)^c = F_\mu$.

Definition 7

Let F_μ and G_δ be two GFSSs over (X, E) (Chakraborty and Mukherjee, 2015). The union of F_μ and G_δ, denoted by $F_\mu \,\tilde{\cup}\, G_\delta$, is the GFSS H_ν, defined as

$H_\nu : E \longrightarrow I^X \times I$ such that $H_\nu(e) = (H(e), \nu(e))$, where $H(e) = F(e) \vee G(e)$ and $\nu(e) = \mu(e) \vee \delta(e), \forall e \in E$.

Let $\{(F_\mu)_\lambda, \lambda \in \Lambda\}$, where Λ is an index set, be a family of GFSSs. The union of these family, denoted by $\tilde{\bigcup}_{\lambda \in \Lambda}(F_\mu)_\lambda$, is the GFSS H_ν, defined as $H_\nu : E \longrightarrow I^X \times I$ such that $H_\nu(e) = (H(e), \nu(e))$, where $H(e) = \vee_{\lambda \in \Lambda}(F(e))_\lambda$, and $\nu(e) = \vee_{\lambda \in \Lambda}(\mu(e))_\lambda, \forall e \in E$.

Definition 8

Let F_μ and G_δ be two GFSSs over (X, E) (Chakraborty and Mukherjee, 2015). The intersection of F_μ and G_δ, denoted by $F_\mu \tilde{\bigcap} G_\delta$, is the GFSS M_σ, defined as $M_\sigma : E \longrightarrow I^X \times I$ such that $M_\sigma(e) = (M(e), \sigma(e))$, where $M(e) = F(e) \wedge G(e)$ and $\sigma(e) = \mu(e) \wedge \delta(e), \forall e \in E$.

Let $\{(F_\mu)_\lambda, \lambda \in \Lambda\}$, where Λ is an index set, be a family of GFSSs. The intersection of these family, denoted by $\tilde{\bigcap}_{\lambda \in \Lambda}(F_\mu)_\lambda$, is the GFSS M_σ, defined as $M_\sigma : E \longrightarrow I^X \times I$ such that $M_\sigma(e) = (M(e), \sigma(e))$, where $M(e) = \wedge_{\lambda \in \Lambda}(F(e))_\lambda$, and $\sigma(e) = \wedge_{\lambda \in \Lambda}(\mu(e))_\lambda$, $\forall e \in E$.

Definition 9

A GFSS is said to be a generalized null fuzzy soft set, denoted by $\tilde{0}_\theta$, if $\tilde{0}_\theta : E \longrightarrow I^X \times I$ such that $\tilde{0}_\theta(e) = (\tilde{0}(e), \theta(e))$ where $\tilde{0}(e) = \bar{0}\ \forall e \in E$ and $\theta(e) = 0\ \forall e \in E$ (Where $\bar{0}(x) = 0, \forall x \in X$) (Majumdar and Samanta, 2010).

Definition 10

A GFSS is said to be a generalized absolute fuzzy soft set, denoted by $\tilde{1}_\Delta$, if $\tilde{1}_\Delta : E \longrightarrow I^X \times I$, where $\tilde{1}_\Delta(e) = (\tilde{1}(e), \Delta(e))$ is defined by $\tilde{1}(e) = \bar{1}, \forall e \in E$ and $\Delta(e) = 1, \forall e \in E$ (Where $\bar{1}(x) = 1, \forall x \in X$) (Majumdar and Samanta, 2010).

Definition 11

Let T be a collection of generalized fuzzy soft sets over (X, E). Then T is said to be a generalized fuzzy soft topology (GFST, in short) over (X, E) if the following conditions are satisfied:

(i) $\tilde{0}_\theta$ and $\tilde{1}_\Delta$ are in T;
(ii) Arbitrary unions of members of T belong to T;
(iii) Finite intersections of members of T belong to T.

The triplet (X, T, E) is called a generalized fuzzy soft topological space (GFST- space, in short) over (X, E).

The members of T are called a GFS open sets in (X, T, E). The complement of a GFS open set is called GFS closed.

Definition 12

Let (X, T, E) be a GFST-space and F_μ be a GFSS over (X, E). Then the generalized fuzzy soft closure of F_μ, denoted by $\overline{F_\mu}$, is the intersection of all GFS closed supper sets of F_μ.

Clearly, $\overline{F_\mu}$ is the smallest GFS closed set over (X, E) which contains F_μ (Chakraborty and Mukherjee, 2015).

Definition 13

Let F_μ be a GFSS over (X, E) (Chakraborty and Mukherjee, 2015. We say that $(x_\alpha, e_\lambda) \in F_\mu$ read as (x_α, e_λ) belongs to the GFSS F_μ if $F(e)(x) = \alpha\ (0 < \alpha \leq 1)$ and $F(e)(y) = 0, \forall y \in X \backslash \{x\}$, $\mu(e) > \lambda$.

Definition 14

A GFSS F_μ in a GFST-space (X, T, E) is called a generalized fuzzy soft neighborhood [GFS-nbd, in short] of the GFSS G_δ if there exists a GFS open set H_ν such that $G_\delta \tilde{\subseteq} H_\nu \tilde{\subseteq} F_\mu$ (Chakraborty and Mukherjee, 2015).

Definition 15

A GFSS F_μ in a GFST-space (X, T, E) is called a generalized fuzzy soft neighborhood of the generalized fuzzy soft point $(x_\alpha, e_\lambda) \tilde{\in} \tilde{1}_\Delta$ if there exists a GFS open set G_δ such that $(x_\alpha, e_\lambda) \tilde{\in} G_\delta \tilde{\subseteq} F_\mu$

(Chakraborty and Mukherjee, 2015).

Definition 16

Difference of two GFSS F_μ and G_δ, denoted by $F_\mu \setminus G_\delta$, is a GFSS $H_\nu = F_\mu \widetilde{\bigcap} G_\delta^c$, defined as $H(e) = F(e) \wedge G^c(e)$ and $\nu(e) = \mu(e) \wedge \delta^c(e)$, $\forall e \in E$ (Mukherjee, 2015).

Definition 17

Let a set E be fixed. An intuitionistic fuzzy set or IFS 'A' in E is anobject having the form $A = \{(x, \mu_A(x), \nu_A(x) : x \in E)\}$ where the functions $\mu_A : E \to I = [0,1]$ & $\nu_A : E \to I = [0,1]$ define the degree of membership and non-membership, respectively, of the element $x \in E$ to the set A and for every $x \in E$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ (Atanassov, 1986).

Definition 18

Let X be an initial universe and E be a set of parameters (Maji et al., 2001a,b). Let IF^X be the set of all intuitionistic fuzzy subsets of X and $A \subseteq E$. Then, the pair (F, A) is called an intuitionistic fuzzy soft set over X, where F is a mapping given by $F: A \to IF^X$.

For any $e \in A$, $F(e)$ is an intuitionistic fuzzy subset of X. Let us denote $\mu_{F(e)}(x)$ and $\nu_{F(e)}(x)$ by the membership degree and non-membership degree, respectively, that object x holds parameter e, where $x \in X$ and $e \in A$. Then, $F(e)$ can be written as an intuitionistic fuzzy set such that $F(e) = \{(x, \mu_{F(e)}(x), \nu_{F(e)}(x)) : x \in X\}$.

Definition 19

Let X be an initial universe and E be a set of parameters. (Dinda et al., 2012). Let IF^X be the set of all intuitionistic fuzzy subsets of X and $A \subseteq E$. Let F be a mapping given by $F: A \to IF^X$ and μ be a mapping given by $\mu : A \to [0,1]$. Let F_μ be a mapping given by $F_\mu : A \to IF^X \times [0,1]$ and defined by $F_\mu(e) = (F(e), \mu(e)) = (\{(x, \mu_{F(e)}(x), \nu_{F(e)}(x)) : x \in X\}, \mu(e))$, where $e \in A$ and $x \in X$. hen, the pair (F_μ, A) is called a generalized intuitionistic fuzzy soft set over (X, E).

GENERALIZED FUZZY SOFT POINTS AND NEIGHBORHOOD SYSTEMS

Here, a generalized fuzzy soft point was introduced and some of its basic properties were studied. Also, we discuss the concept of a neighborhood of a generalized fuzzy soft point in a generalized fuzzy soft topological space.

Definition 1

The generalized fuzzy soft set $F_\mu \in$ GFS(X, E) is called generalized fuzzy soft point (GFS point in short) if there exists the element $e \in E$ and $x \in X$ such that $F(e)(x) = \alpha \ (0 < \alpha \leq 1)$ and $F(e)(y) = 0$ for all $y \in X - \{x\}$ and $\mu(e) = \lambda \ (0 < \lambda \leq 1)$. This generalized fuzzy soft point was denoted $F_\mu = (x_\alpha, e_\lambda)$. (x, e) and (α, λ) are called, respectively the support and the value of (x_α, e_λ).

Definition 2

The complement of a generalized fuzzy soft point (x_α, e_λ), denoted by $(x_\alpha, e_\lambda)^c$, is defined as follows $(x_\alpha, e_\lambda)^c = (x_{1-\alpha}, e_{1-\lambda})$.

Example 1

Let $X = \{x_1, x_2, x_3, x_4\}$ and $E = \{e_1, e_2, e_3, e_4\}$ the set of parameters. Then $(x_{3(0.1)}, e_{2(0.6)}) = \{(\{\frac{x_1}{0}, \frac{x_2}{0}, \frac{x_3}{0.1}, \frac{x_4}{0}\}, 0.6)\}$ is generalized fuzzy soft point whose complement is $(x_{3(0.1)}, e_{2(0.6)})^c = \{(\{\frac{x_1}{0}, \frac{x_2}{0}, \frac{x_3}{0.9}, \frac{x_4}{0}\}, 0.4)\}$.

The belongingness of a generalized fuzzy soft point to a generalized fuzzy soft set in Definition 13 as follows was redefined:

Definition 3

Let F_μ be a GFSS over (X, E). We say that $(x_\alpha, e_\lambda) \widetilde{\in} F_\mu$ read as (x_α, e_λ) belongs to the GFSS F_μ if for the element $e \in E$, $\alpha \leq F(e)(x)$ and $\lambda \leq \mu(e)$.

Definition 4

Let (x_α, e_λ) and (y_β, e'_δ) be two generalized fuzzy soft points, we say that $(x_\alpha, e_\lambda) \in (y_\beta, e'_\delta) \Leftrightarrow (x, e) = (y, e')$ and $\alpha < \beta$,

$\lambda < \delta$.

Theorem 1

Let F_μ be a GFSS over (X, E), then:

(1) $F_\mu \,\widetilde{\cup}\, F_\mu^c \neq \tilde{1}_\Delta$;
(2) $F_\mu \,\widetilde{\cap}\, F_\mu^c \neq \tilde{0}_\theta$;
(3) if $(x_\alpha, e_\lambda) \,\widetilde{\in}\, F_\mu$ then $(x_\alpha, e_\lambda) \,\widetilde{\notin}\, F_\mu^c$ is not hold;
(4) $(x_\alpha, e_\lambda) \,\widetilde{\in}\, F_\mu \nRightarrow (x_\alpha, e_\lambda)^c \,\widetilde{\in}\, F_\mu^c$.

Example 2

Let $X = \{x_1, x_2\}$ and $E = \{e_1, e_2\}$, consider the GFSS F_μ over (X, E), as:

$$F_\mu = \left\{F_\mu(e_1) = \left(\left\{\frac{x_1}{0.2}, \frac{x_2}{0.7}\right\}, 0.3\right), F_\mu(e_2) = \left(\left\{\frac{x_1}{0.5}, \frac{x_2}{0,4}\right\}, 0.9\right)\right\},$$

$$F_\mu^c = \left\{F_\mu(e_1) = \left(\left\{\frac{x_1}{0.8}, \frac{x_2}{0.3}, \right\}, 0.7\right), F_\mu(e_2) = \left(\left\{\frac{x_1}{0.5}, \frac{x_2}{0,6}, \right\}, 0.1\right)\right\}.$$ Then

$$F_\mu \,\widetilde{\cup}\, F_\mu^c = \left\{F_\mu(e_1) = \left(\left\{\frac{x_1}{0.8}, \frac{x_2}{0.7}, \right\}, 0.7\right), F_\mu(e_2) = \left(\left\{\frac{x_1}{0.5}, \frac{x_2}{0,6}, \right\}, 0.9\right)\right\} \neq \tilde{1}_\Delta,$$

$$F_\mu \,\widetilde{\cap}\, F_\mu^c = \left\{F_\mu(e_1) = \left(\left\{\frac{x_1}{0.2}, \frac{x_2}{0.3}, \right\}, 0.3\right), F_\mu(e_2) = \left(\left\{\frac{x_1}{0.5}, \frac{x_2}{0,4}, \right\}, 0.1\right)\right\} \neq \tilde{0}_\theta.$$

Consider the GFS point $(x_{1(0.1)}, e_{1(0.2)}) = \left\{\left(\left\{\frac{x_1}{0.1}\right\}, 0.2\right)\right\}$, then $(x_{1(0.1)}, e_{1(0.2)}) \in F_\mu$ and $(x_{1(0.1)}, e_{1(0.2)}) \,\widetilde{\in}\, F_\mu^c$.
For Theorem 1, we have:

$(x_{1(0.1)}, e_{1(0.2)}) = \left\{\left(\left\{\frac{x_1}{0.1}\right\}, 0.2\right)\right\} \,\widetilde{\in}\, F_\mu$ but $(x_{1(0.1)}, e_{1(0.2)})^c = \left\{\left(\left\{\frac{x_1}{0.9}\right\}, 0.8\right)\right\} \,\widetilde{\notin}\, F_\mu^c$.

Definition 5

Let (X, T, E) be a GFST-space. The set of all GFS neighborhoods of a generalized fuzzy soft point (x_α, e_λ) is called the GFS neighborhoods system of (x_α, e_λ) and is denoted by $\mathcal{N}_T(x_\alpha, e_\lambda)$.

Theorem 2

Let GFSS (*X*,*E*) be a family of all generalized fuzzy soft sets over soft universe (X, E) and (X, T, E) be GFST-space. Then the following properties are satisfied:

(1) $F_\mu = \widetilde{\bigcup}_{(x_\alpha, e_\lambda) \in F_\mu} (x_\alpha, e_\lambda)$;
(2) $(x_\alpha, e_\lambda) \in \widetilde{\bigcup}\{(F_\mu)_i : i \in J\} \Leftrightarrow \exists i_0 \in J$ such that $(x_\alpha, e_\lambda) \in (F_\mu)_{i_0}$;
(3) $(x_\alpha, e_\lambda) \in \widetilde{\bigcap}\{(F_\mu)_i : i \in J\} \Leftrightarrow \forall i \in J, (x_\alpha, e_\lambda) \in (F_\mu)_i$;
(4) $(x_\alpha, e_\lambda) \in (x_\beta, e_\delta)$ and $(x_\beta, e_\delta) \in \bigwedge_i\{(F_\mu)_i : i \in J\}$. Then $(x_\alpha, e_\lambda) \in \bigwedge_i\{(F_\mu)_i : i \in J\}$;
(5) $F_\mu \in \mathcal{N}_T(x_\alpha, e_\lambda) \Rightarrow \exists\, G_\delta \in \mathcal{N}_T(x_\alpha, e_\lambda)$ such that $F_\mu \in \mathcal{N}_T(x_\beta, e_\delta)$ for each $(x_\beta, e_\delta) \in G_\delta$.

Proof 1

(1) $F_\mu \subseteq \widetilde{\bigcup}_{(x_\alpha, e_\lambda) \in F_\mu} (x_\alpha, e_\lambda)$ straightforward. Let $(x_\beta, e_\delta) \,\widetilde{\in}\, \widetilde{\bigcup}_{(x_\alpha, e_\lambda) \in F_\mu} (x_\alpha, e_\lambda)$, then there exists $(x_{\alpha'}, e_{\lambda'}) \,\widetilde{\in}\, F_\mu$ such that $(x_\beta, e_\delta) \,\widetilde{\in}\, (x_{\alpha'}, e_{\lambda'})$. Therefore, $\beta \leq \alpha'$, $\delta \leq \lambda'$ put $\alpha' \leq F(e)(x)$ and $\lambda' \leq \mu(e)$, then $\beta \leq F(e)(x)$ and $\delta \leq \mu(e)$. This shows that $(x_\beta, e_\delta) \,\widetilde{\in}\, F_\mu$.

(2) Let $(x_\alpha, e_\lambda) \,\widetilde{\in}\, \widetilde{\bigcup}_{i \in J} (F_\mu)_i = \mathrm{H}_\nu$. Then $\alpha \leq H(e)(x) = \bigvee_{i \in J} (F(e)(x))_i$ and $\lambda \leq \nu(e) = \bigvee_{i \in J} (\mu(e))_i \forall e \in E, x \in X$. Then there exists $i_0 \in J$ such that $\alpha \leq (F(e)(x))_{i_0}$ and $\lambda \leq (\mu(e))_{i_0}$, which shows that $(x_\alpha, e_\lambda) \in (F_\mu)_{i_0}$. Conversely, let $(x_\alpha, e_\lambda) \,\widetilde{\in}\, (F_\mu)_{i_0}$ such that $(F_\mu)_{i_0}(e) = \left((F(e))_{i_0}, (\mu(e))_{i_0}\right)$ for some $i_0 \in J$, thus $\alpha \leq (F(e)(x))_{i_0}$ and $\lambda \leq (\mu(e))_{i_0}$ which implies that: $\alpha \leq \bigvee_{i \in J} (F(e)(x))_i$ and $\lambda \leq \bigvee_{i \in J} (\mu(e))_i$. Therefore $\alpha \leq H(e)(x)$ and $\leq \nu(e)$. So $(x_\alpha, e_\lambda) \in H_\nu = \widetilde{\bigcup}_{i \in J} (F_\mu)_i$.

(3) Let $(x_\alpha, e_\lambda) \,\widetilde{\in}\, \widetilde{\bigcap}\{(F_\mu)_i : i \in J\}$, then $(x_\alpha, e_\lambda) \,\widetilde{\in}\, \widetilde{\bigcap}_{i \in J} (F_\mu)_i = M_\sigma$. Therefore:

$\alpha \le M(e)(x) = \wedge_{i\in J}(F(e)(x))_i$ and $\lambda \le \sigma(e) = \wedge_{i\in J}(\mu(e))_i$. Thus $\alpha \le (F(e)(x))_i$, and $\lambda \le (\mu(e))_i\ \forall i \in J$. Therefore $(x_\alpha, e_\lambda) \tilde{\in} (F_\mu)_i\ \forall i \in J$. Conversely, let $(x_\alpha, e_\lambda) \tilde{\in} (F_\mu)_i\ \forall i \in J$, then $\alpha \le (F(e)(x))_i$, and $\lambda \le (\mu(e))_i\ \forall i \in J$. Therefore $\alpha \le \wedge_{i\in J}(F(e)(x))_i$ and $\lambda \le \wedge_{i\in J}(\mu(e))_i$. This implies that $\alpha \le M(e)(x)$ and $\lambda \le \sigma(e)$. Hence $(x_\alpha, e_\lambda) \tilde{\in} M_\sigma = \tilde{\sqcap}_{i\in J}(F_\mu)_i$.

(4) Let $(x_\alpha, e_\lambda) \tilde{\in} (x_\beta, e_\delta) \tilde{\in} (F_\mu)_i$, then $(x_\alpha, e_\lambda) \tilde{\in} (F_\mu)_i$. This implies that $\alpha \le (F(e)(x))_i$ and $\lambda \le (\mu(e))_i$, $\forall i \in J$ which implies that $\alpha \le \min\{(F(e)(x))_i : i \in J\}$ and $\lambda \le \min\{(\mu(e))_i : i \in J\}$. Therefore $\alpha \le \wedge_{i\in J}(F(e)(x))_i$ and $\lambda \le \wedge_{i\in J}(\mu(e))_i$. Then $(x_\alpha, e_\lambda) \tilde{\in} \wedge_i\{(F_\mu)_i : i \in J\}$.

(5) Let $F_\mu \in \mathcal{N}_T(x_\alpha, e_\lambda)$, then there exists a generalized fuzzy soft set $H_\nu \in T$ such that $(x_\alpha, e_\lambda) \tilde{\in} H_\nu \tilde{\subseteq} F_\mu$. Put $G_\delta = H_\nu$. Then for every $(x_\beta, e_\delta) \in G_\delta$, $(x_\beta, e_\delta) \tilde{\in} G_\delta \tilde{\subseteq} H_\nu \tilde{\subseteq} F_\mu$. This implies that $F_\mu \in \mathcal{N}_T(x_\beta, e_\delta)$.

Example 3

We give example GFS neighborhood of GFS set and GFS point which are definitions 14 and 15.

Let $X = \{x, y, z\}$ and $E = \{e, d, h\}$. Consider the following GFSSs over (X, E) defined as:

$$F_\mu = \left\{F_\mu(e) = \left(\left\{\frac{x}{0.6}, \frac{y}{0.5}, \frac{z}{0.4}\right\}, 0.2\right), F_\mu(d) = \left(\left\{\frac{x}{0.2}, \frac{y}{0.3}, \frac{z}{0.8}\right\}, 0.5\right), F_\mu(h) = \left(\left\{\frac{x}{0.7}, \frac{y}{0.4}, \frac{z}{0.3}\right\}, 0.6\right)\right\},$$
$$G_\delta = \left\{G_\delta(e) = \left(\left\{\frac{x}{0.3}, \frac{y}{0.2}, \frac{z}{0.2}\right\}, 0.1\right), G_\delta(d) = \left(\left\{\frac{x}{0.1}, \frac{y}{0}, \frac{z}{0.5}\right\}, 0.3\right), G_\delta(h) = \left(\left\{\frac{x}{0.5}, \frac{y}{0.3}, \frac{z}{0.3}\right\}, 0.5\right)\right\}.$$

Consider $T = \{\tilde{0}_\theta, \tilde{1}_\Delta, F_\mu, G_\delta\}$. Then T forms GFS topology over (X, E). Consider the following GFSS over (X, E),

$$H_\nu = \left\{H_\nu(e) = \left(\left\{\frac{x}{0.7}, \frac{y}{0.6}, \frac{z}{0.5}\right\}, 0.3\right), H_\nu(d) = \left(\left\{\frac{x}{0.3}, \frac{y}{0.4}, \frac{z}{0.8}\right\}, 0.5\right), H_\nu(h) = \left(\left\{\frac{x}{0.9}, \frac{y}{0.4}, \frac{z}{0.7}\right\}, 0.8\right)\right\}$$

If $(x_{(0.4)}, e_{(0.1)}) \in \tilde{1}_\Delta$, then there exists GFS open set F_μ such that $(x_{(0.4)}, e_{(0.1)}) \in F_\mu \tilde{\subseteq} H_\nu$, that is, H_ν is a GFS neighborhood of $(x_{(0.4)}, e_{(0.1)})$. Also, if $M_\sigma \tilde{\subseteq} F_\mu \tilde{\subseteq} H_\nu$, then H_ν is a GFS neighborhood of $M_\sigma \in \mathrm{GFSS}(X, E)$, where GFSS (X, E) the family of all generalized fuzzy soft sets over (X, E).

GENERALIZED FUZZY SOFT BASE AND GENERALIZED FUZZY SOFT SUBBASE

Definition 1

Let (X, T, E) be GFS topological space**.** A collection $\mathfrak{R}$ of generalized fuzzy soft sets over (X, E) is called a generalized fuzzy soft open base or simply a base for generalized fuzzy soft topology on (X, E), if the following conditions hold:

1. $\tilde{0}_\theta \in \mathfrak{R}$

2. $\tilde{\cup}\mathfrak{R} = \tilde{1}_\Delta$ i.e. for each $e \in E$ and $x \in X$, there exists $R_\mu \in \mathfrak{R}$ such that $R(e)(x) = 1$ and $\mu(e) = 1$.

3. If F_μ , $G_\delta \in \mathfrak{R}$ then for each $e \in E$ and $x \in X$, there exists $H_\nu \in \mathfrak{R}$ such that $H_\nu \tilde{\subseteq} F_\mu \tilde{\sqcap} G_\delta$ and $H(e)(x) = \min\{F(e)(x), G(e)(x)\}$ and

$\nu(e) = \min\{\mu(e), \delta(e)\}$.

Example 1

Let $X = \{x_1, x_2, x_3, x_4\}$, $E = \{e_1, e_2, e_3, e_4\}$. Let us consider the collection $\mathfrak{R} = \{\tilde{0}_\theta, \tilde{1}_\Delta, G_\delta, H_\nu, K_\gamma, J_\sigma, M_\eta, N_\psi\}$. Where

$G_\delta = \{G_\delta(e_2) = \left(\left\{\frac{x_2}{0.2}\right\}, 0.2\right)\}$,

$H_\nu = \{H_\nu(e_4) = \left(\left\{\frac{x_4}{0.4}\right\}, 0.4\right)\}$,

$K_\gamma = \{ K_\gamma(e_1) = \left(\left\{\frac{x_1}{0.1}\right\}, 0.1\right), K_\gamma(e_3) = \left(\left\{\frac{x_3}{0.3}\right\}, 0.3\right)\}$,
$J_\sigma = \{ J_\sigma(e_2) = \left(\left\{\frac{x_2}{1}\right\}, 0.2\right)$, $J_\sigma(e_4) = \left(\left\{\frac{x_1}{1}, \frac{x_3}{1}, \frac{x_4}{0.4}\right\}, 1\right)\}$,
$M_\eta = \{ M_\eta(e_1) = \left(\left\{\frac{x_1}{1}, \frac{x_2}{1}, \frac{x_4}{1}\right\}, 0.1\right)$, $M_\eta(e_2) = \left(\left\{\frac{x_1}{1}, \frac{x_2}{0.2}, \frac{x_3}{1}, \frac{x_4}{1}\right\}, 1\right), M_\eta(e_3) = \left(\left\{\frac{x_3}{0.3}, \frac{x_4}{1}\right\}, 0.3\right)$ },
$N_\psi = \{ N_\psi(e_1) = \left(\left\{\frac{x_1}{0.1}, \frac{x_3}{1}\right\}, 1\right)$, $N_\psi(e_3) = \left(\frac{x_1}{1}, \frac{x_2}{1}, \frac{x_3}{1}\}, 1\right), N_\psi(e_4) = \left(\{\frac{x_2}{1}, \frac{x_4}{1}\}, 0.4\right)\}$.

We can see that $\mathfrak{R}$ satisfies the conditions 1 to 3 of Definition (4-1). Therefore, $\mathfrak{R}$ forms a GFS base for a topology on (X, E).

Definition 2

Let $\mathfrak{R}$ be a GFS base for a GFS topology on (X, E). Then, the GFS topology generated by GFS base $\mathfrak{R}$, is denoted by $T_{\mathfrak{R}}$ and is defined as follows:

$$T_{\mathfrak{R}} = \{H_\nu : H_\nu = \tilde{\cup}(R_\mu)_\alpha,\ (R_\mu)_\alpha \in \mathfrak{R}\ \forall\ \alpha \in \nabla, \nabla \text{ an index set}\}.$$

Example 2

Let $E = \{e_1, e_2, e_3\}$, $X = \{x_1, x_2, x_3, x_4\}$ and $\mathfrak{R} = \{\tilde{0}_\theta, F_\mu, G_\delta, H_\nu\}$, where

$$F_\mu = \left\{F_\mu(e_1) = \left(\left\{\frac{x_1}{0.1}, \frac{x_2}{0.2}, \frac{x_3}{1}, \frac{x_4}{1}\right\}, 0.1\right), F_\mu(e_2) = \left(\left\{\frac{x_1}{1}, \frac{x_2}{1}, \frac{x_3}{0.2}, \frac{x_4}{0.4}\right\}, 1\right)\right\},$$
$$G_\delta = \left\{G_\delta(e_1) = \left(\left\{\frac{x_1}{1}, \frac{x_2}{1}, \frac{x_3}{0.3}, \frac{x_4}{0.4}\right\}, 1\right), G_\delta(e_2) = \left(\left\{\frac{x_1}{0.1}, \frac{x_2}{0.2}, \frac{x_3}{1}, \frac{x_4}{1}\right\}, 1\right)\right\},$$
$$H_\nu = \left\{H_\nu(e_1) = \left(\left\{\frac{x_1}{0.1}, \frac{x_2}{0.2}, \frac{x_3}{0.3}, \frac{x_4}{0.4}\right\}, 0.1\right), H_\nu(e_2) = \left(\left\{\frac{x_1}{0.1}, \frac{x_2}{0.2}, \frac{x_3}{0.3}, \frac{x_4}{0.4}\right\}, 0.1\right), H_\nu(e_3) = \left(\left\{\frac{x_1}{1}, \frac{x_2}{1}, \frac{x_3}{1}, \frac{x_4}{1}\right\}, 1\right)\right\}.$$

Then obviously, $\mathfrak{R}$ is a GFS base for a GFS topology on (X, E). The GFS topology generated by $\mathfrak{R}$ is $T_{\mathfrak{R}}$, where

$$T_{\mathfrak{R}} = \{ \tilde{0}_\theta, \tilde{1}_\Delta, F_\mu, G_\delta, H_\nu,$$
$$\{F_\mu \tilde{\cup} G_\delta(e_1) = (\left\{\frac{x_1}{1}, \frac{x_2}{1}, \frac{x_3}{1}, \frac{x_4}{1}\right\}, 1), F_\mu \tilde{\cup} G_\delta(e_2) = (\left\{\frac{x_1}{1}, \frac{x_2}{1}, \frac{x_3}{1}, \frac{x_4}{1}\right\}, 1)\},$$
$$\left\{F_\mu \tilde{\cup} H_\nu(e_1) = \left(\left\{\frac{x_1}{0.1}, \frac{x_2}{0.2}, \frac{x_3}{1}, \frac{x_4}{1}\right\}, 0.1\right), F_\mu \tilde{\cup} H_\nu(e_2) = \left(\left\{\frac{x_1}{1}, \frac{x_2}{1}, \frac{x_3}{0.3}, \frac{x_4}{0.4}\right\}, 1\right), F_\mu \tilde{\cup} H_\nu(e_3) = \left(\left\{\frac{x_1}{1}, \frac{x_2}{1}, \frac{x_3}{1}, \frac{x_4}{1}\right\}, 1\right)\right\},$$
$$\left\{G_\delta \tilde{\cup} H_\nu(e_1) = \left(\left\{\frac{x_1}{1}, \frac{x_2}{1}, \frac{x_3}{0.3}, \frac{x_4}{0.4}\right\}, 1\right), G_\delta \tilde{\cup} H_\nu(e_1) = \left(\left\{\frac{x_1}{0.1}, \frac{x_2}{0.2}, \frac{x_3}{1}, \frac{x_4}{1}\right\}, 1\right), G_\delta \tilde{\cup} H_\nu(e_1) = \left(\left\{\frac{x_1}{1}, \frac{x_2}{1}, \frac{x_3}{1}, \frac{x_4}{1}\right\}, 1\right)\right\}\}.$$

Theorem 1

Let (X, T, E) be a GFST-space and $\mathfrak{R}$ be a sub collection of T such that every member of T is union of some members of $\mathfrak{R}$. Then $\mathfrak{R}$ is GFS base for the GFS topology T on (X, E).

Proof 1

Since $\tilde{0}_\theta \in T$, $\tilde{0}_\theta \in \mathfrak{R}$. Again since $\tilde{1}_\Delta \in T$, $\tilde{1}_\Delta = \cup\mathfrak{R}$. Let $(R_\mu)_1, (R_\mu)_2 \in \mathfrak{R}$. Then $(R_\mu)_1, (R_\mu)_2 \in T$ and so $(R_\mu)_1 \tilde{\cap} (R_\mu)_2 \in T$. Then there exists $(R_\mu)_\alpha \in \mathfrak{R}$, $\alpha \in \nabla$ such that $(R_\mu)_1 \tilde{\cap} (R_\mu)_2 = \tilde{\cup}\{(\tilde{R}_\mu)_\alpha : \alpha \in \nabla\}$. Therefore, $(R_\mu)_1(e) \tilde{\cap} (R_\mu)_2(e) = \tilde{\cup}\{(\tilde{R}_\mu)_\alpha(e) : \in \nabla\}$, for $e \in E$. That is, for each $e \in E$ and $x \in X$, min $\{(R(e)(x))_1, (R((e)(x))_2\}$=max$\{(R(e)(x))_\alpha: \alpha \in \nabla\}$ and min $\{(\mu(e))_1, (\mu(e))_2\}$=max$\{(\mu(e))_\alpha: \alpha \in \nabla\}$. Therefore there exists $\alpha \in \nabla$ such that: min $\{(R(e)(x))_1, (R(e)(x))_2\}= (R(e)(x))_\alpha$ and min$\{(\mu(e))_1, \mu(e))_2\}= (\mu(e))_\alpha$. Thus for $e \in E$ and $x \in X$, we get $(R_\mu)_\alpha \in \mathfrak{R}$ such that $(R_\mu)_\alpha \tilde{\subseteq} (R_\mu)_1 \tilde{\cap} (R_\mu)_2$ and min $\{(R(e)(x))_1, (R(e)(x))_2\}= (R(e)(x))_\alpha$ and min$\{(\mu(e))_1, (\mu(e))_2\}= (\mu(e))_\alpha$. Therefore, $\mathfrak{R}$ is GFS base for the GFS topology T on (X, E).

Definition 3

A collection Ω of some members of GFST-space (X, T, E) is said to be a subbase of T if and only if the collection of all finite intersection of members of Ω is a base for T.

Example 3

Let $X = \{x_1, x_2, x_3, x_4\}, E = \{e_1, e_2, e_3\}$ and $\mathfrak{R} = \{J_\sigma, M_\eta, N_\psi\}$, where

$J_\sigma = \{ J_\sigma(e_2) = \left(\left\{\frac{x_2}{1},\right\}, 0.2\right)$, $J_\sigma(e_4) = \left(\left\{\frac{x_1}{1}, \frac{x_3}{1}, \frac{x_4}{0.4}\right\}, 1\right)\}$,
$M_\eta = \{ M_\eta(e_1) = \left(\left\{\frac{x_1}{1}, \frac{x_2}{1}, \frac{x_4}{1}\right\}, 0.1\right)$, $M_\eta(e_2) = \left(\left\{\frac{x1}{1}, \frac{x2}{0.2}, \frac{x3}{1}, \frac{x4}{1}\right\}, 1\right), M_\eta(e_3) = \left(\left\{\frac{x3}{0.3}, \frac{x4}{1},\right\}, 0.3\right)$ },
$N_\psi = \{ N_\psi(e_1) = \left(\left\{\frac{x1}{0.1}, \frac{x3}{1}\right\}, 1\right)$, $N_\psi(e_3) = \left(\frac{x1}{1}, \frac{x2}{1}, \frac{x3}{1}\right\}, 1), N_\psi(e_4) = \left(\left\{\frac{x2}{1}, \frac{x4}{1}\right\}, 0.4\right)\}$.

The collection of all finite intersection of members of Ω is the base $\mathfrak{R}$ in Example 1. So Ω is a subbase for a GFS topology on (X, E).

Theorem 2

A collection Ω of GFSSs over (X, E) is a subbase for a suitable GFS topology T if and only if:

(1) $\tilde{0}_\theta \in \Omega$ or $\tilde{0}_\theta$ is the intersection of finite number of members of Ω.
(2) $\tilde{1}_\Delta = \tilde{\bigcup}\Omega$.

Proof 2

First let Ω be a subbase for T and $\mathfrak{R}$ be a base generated by Ω. Since $\tilde{0}_\theta \in \mathfrak{R}$, either $\tilde{0}_\theta \in \Omega$ or $\tilde{0}_\theta$ is expressible as an intersection of many finite members of Ω. Now let $x \in X$ and $e \in E$. Since $\tilde{\bigcup}\mathfrak{R} = \tilde{1}_\Delta$, there exists $R_\mu \in \mathfrak{R}$ such that $R(e)(x) = 1 \text{ and } \mu(e) = 1$.

Since $R_\mu \in \mathfrak{R}$ there exists $(K_\gamma)_i \in \Omega, i = 1,2, \dots \dots n$ Such that $R_\mu = \tilde{\bigcap}_{i=1}^n (K_\gamma)_i$.

Therefore $R(e)(x) = \min_{i=1}^n \left(K(e)(x)\right)_i$, $\mu(e) = \min_{i=1}^n \left(\gamma(e)\right)_i$ and so $R(e)(x) = \left(K(e)(x)\right)_i$ for some $i \in \{1,2, \dots, n\}$, $\mu(e) = (\gamma(e))_i$, for some $i \in \{1,2, \dots, n\}$.

Thus $\left(K(e)(x)\right)_i = 1$ and $(\gamma(e))_i = 1$. Hence $\tilde{1}_\Delta = \tilde{\bigcup}\Omega$.

Conversely, let Ω be collection of GFSSs over (X, E) satisfying the conditions 1 and 2. Let $\mathfrak{R}$ be the collection of all finite intersection of members of Ω. Now it enough to show that $\mathfrak{R}$ forms base for suitable GFS topology. Since $\mathfrak{R}$ is the collection of all finite intersection of members of Ω, by assumption (1) we get $\tilde{0}_\theta \in \mathfrak{R}$ and by (2) we get $\tilde{\bigcup}\mathfrak{R} = \tilde{1}_\Delta$. Again let $F_\mu, G_\delta \in \mathfrak{R}$ and $x \in X$, $e \in E$. Since $F_\mu \in \mathfrak{R}$, there exists $(F_\mu)_i \in \Omega$, for $i = 1,2, \dots, n$ such that $F_\mu = \tilde{\bigcap}_{i=1}^n (F_\mu)_i$. Again since $G_\delta \in \mathfrak{R}$, there exists $(G_\delta)_j \in \Omega$, for $j = 1,2, \dots, m$ such that $G_\delta = \tilde{\bigcap}_{j=1}^m (G_\delta)_j$. Therefore, $F_\mu \tilde{\cap} G_\delta = \tilde{\bigcap}_{i=1}^n (F_\mu)_i \tilde{\cap} \left(\tilde{\bigcap}_{j=1}^m (G_\delta)_j\right) \in \mathfrak{R}$.

That is, $F_\mu \tilde{\cap} G_\delta \in \mathfrak{R}$. This completes the proof.

GENERALIZED FUZZY SOFT TOPOLOGICAL SUBSPACES

Definition 1

Let (X, T, E) be a GFS topological space. Let Y be an ordinary subset of X and Y_ν be GFSS over (Y, E) such that:

$\forall e \in E$, $Y(e)(x) = \begin{cases} 1 & if\ x \in Y \\ 0 & if\ x \notin Y \end{cases}$, $\nu(e) = 1$ That is $Y(e) = Y, \forall e \in E$. $\nu(e) = 1$

Let $T_Y = \{ Y_\nu \tilde{\cap} G_\delta : G_\delta \in T\}$. We can show that T_Y is a GFS topology on (Y, E) as follows:

(i) Since $\tilde{0}_\theta$, $\tilde{1}_\Delta \in \mathrm{T}$, $(\tilde{1}_\Delta)_Y = Y_\nu \tilde{\cap} \tilde{1}_\Delta$ and $(\tilde{0}_\theta)_Y = Y_\nu \tilde{\cap} \tilde{0}_\theta$, then $(\tilde{0}_\theta)_Y, (\tilde{1}_\Delta)_Y \in T_Y$.
(ii) Suppose that $(F_\mu)_1$, $(F_\mu)_2 \in T_Y$. Then for each $i = 1, 2$, there exist, $(G_\delta)_i \in T$ such that $(F_\mu)_i = Y_\nu \tilde{\cap} (G_\delta)_i$. We have $(F_\mu)_1 \tilde{\cap} (F_\mu)_2 = [Y_\nu \tilde{\cap} (G_\delta)_1] \tilde{\cap} [Y_\nu \tilde{\cap} (G_\delta)_2]$ $= Y_\nu \tilde{\cap} [(G_\delta)_1 \tilde{\cap} (G_\delta)_2]$. Since $(G_\delta)_1 \tilde{\cap} (G_\delta)_2 \in T$, we have $(F_\mu)_1 \tilde{\cap} (F_\mu)_2 \in T_Y$.
(iii) Let $\{(G_\delta)_i \mid i \in J\}$ be a subfamily of T_Y. Then for each $i \in J$, there is a GFSS $(M_\sigma)_i$ of T such that $(G_\delta)_i = Y_\nu \tilde{\cap} (M_\sigma)_i$. We have $\tilde{\bigcup}_{i \in J} (G_\delta)_i = \tilde{\bigcup}_{i \in J} [Y_\nu \tilde{\cap} (M_\sigma)_i] = Y_\nu \tilde{\cap} (\tilde{\bigcup}_{i \in J} (M_\sigma)_i)$.
Since $\tilde{\bigcup}_{i \in J} (M_\sigma)_i \in T$, we have $\tilde{\bigcup}_{i \in J} (G_\delta)_i \in T_Y$.

T_Y is called the GFS subspace topology on (Y, E) and (Y, T_Y, E) is called a GFS subspace of (X, T, E).

Example 1

Let $X = \{x_1, x_2, x_3\}$, $E = \{e_1, e_2, e_3, e_4\}$. Consider F_μ and G_δ as follows:

$$F_\mu = \left\{F_\mu(e_1) = \left(\left\{\frac{x_1}{0.4}, \frac{x_2}{0.1}, \frac{x_3}{0}\right\}, 0.1\right), F_\mu(e_2) = \left(\left\{\frac{x_1}{0.6}, \frac{x_2}{0.5}, \frac{x_3}{0.2}\right\}, 0.2\right), F_\mu(e_3) = \left(\left\{\frac{x_1}{0}, \frac{x_2}{0}, \frac{x_3}{0}\right\}, 0\right), F_\mu(e_4) = \left(\left\{\frac{x_1}{0}, \frac{x_2}{0}, \frac{x_3}{0}\right\}, 0\right)\right\},$$
$$G_\delta = \left\{G_\delta(e_1) = \left(\left\{\frac{x_1}{0.6}, \frac{x_2}{0.1}, \frac{x_3}{0}\right\}, 0.2\right), G_\delta(e_2) = \left(\left\{\frac{x_1}{0.7}, \frac{x_2}{0.9}, \frac{x_3}{0.5}\right\}, 0.3\right), G_\delta(e_3) = \left(\left\{\frac{x_1}{0}, \frac{x_2}{0}, \frac{x_3}{0}\right\}, 0\right), G_\delta(e_4) = \left(\left\{\frac{x_1}{0.5}, \frac{x_2}{0.3}, \frac{x_3}{0.9}\right\}, 0.5\right)\right\}.$$

We consider the GFS topology T on (X, E) as $T = \{\tilde{0}_\theta, \tilde{1}_\Delta, F_\mu, G_\delta\}$

Let $= \{x_1, x_2\} \subseteq X$. $T_Y = \{\tilde{0}_\theta \tilde{\cap} Y_\nu, \tilde{1}_\Delta \tilde{\cap} Y_\nu, F_\mu \tilde{\cap} Y_\nu, G_\delta \tilde{\cap} Y_\nu\}$, where

(i) $\tilde{0}_\theta \tilde{\cap} Y_\nu = (\tilde{0}_\theta)_Y$

(ii) $\tilde{1}_\Delta \tilde{\cap} Y_\nu = (\tilde{1}_\Delta)_Y$

(iii) $F_\mu \tilde{\cap} Y_\nu = J_\sigma = \left\{ J_\sigma(e_1) = \left(\left\{\frac{x_1}{0.4}, \frac{x_2}{0.1}, \frac{x_3}{0}\right\}, 0.1\right), J_\sigma(e_2) = \left(\left\{\frac{x_1}{0.6}, \frac{x_2}{0.5}, \frac{x_3}{0}\right\} 0.2\right),\right.$
$\left. J_\sigma(e_3) = \left(\left\{\frac{x_1}{0}, \frac{x_2}{0}, \frac{x_3}{0}\right\}, 0\right), J_\sigma(e_4) = \left(\left\{\frac{x_1}{0}, \frac{x_2}{0}, \frac{x_3}{0}\right\}, 0\right) \right\}$

(iv) $G_\delta \tilde{\cap} Y_\nu = K_\gamma = \{$
$K_\gamma(e_1) = \left(\left\{\frac{x_1}{0.6}, \frac{x_2}{0.1}, \frac{x_3}{0}\right\}, 0.2\right), K_\gamma(e_2) = \left(\left\{\frac{x_1}{0.7}, \frac{x_2}{0.9}, \frac{x_3}{0}\right\} 0.3\right),$
$K_\gamma(e_3) = \left(\left\{\frac{x_1}{0}, \frac{x_2}{0}, \frac{x_3}{0}\right\}, 0\right), K_\gamma(e_4) = \left(\left\{\frac{x_1}{0.5}, \frac{x_2}{0.3}, \frac{x_3}{0}\right\}, 0.5\right) \}$

Thus, the collection $T_Y = \{(\tilde{0}_\theta)_Y, (\tilde{1}_\Delta)_Y, J_\sigma, K_\gamma\}$ is GFS a topology on (Y, E).

Theorem 1

Let (Y, T_Y, E) be a GFS subspace of (X, T, E) and F_μ a GFSS over (Y, E). Then

(i) F_μ is GFS closed in (Y, E) if and only if $F_\mu = Y_\nu \tilde{\cap} G_\delta$ for some GFS closed set G_δ in (X, E).

(ii) $cl(F_\mu)_Y = \overline{F}_\mu \tilde{\cap} Y_\nu$ where $cl(F_\mu)_Y$ is the closure of F_μ in (Y, E) with respective to T_Y.

Proof 1

(i) If F_μ is GFS closed in (Y, E) then we have $F_\mu = Y_\nu \setminus G_\delta$, for some $G_\delta \in T_Y$.

Now, $G_\delta = Y_\nu \tilde{\cap} M_\sigma$, for some $M_\sigma \in T$.

$F_\mu = Y_\nu \setminus (Y_\nu \tilde{\cap} M_\sigma) = Y_\nu \tilde{\cap} (Y_\nu \tilde{\cap} M_\sigma)^c = Y_\nu \tilde{\cap} (Y_\nu^c \tilde{\cup} M_\sigma^c)$ $= (Y_\nu \tilde{\cap} Y_\nu^c) \tilde{\cup} (Y_\nu \tilde{\cap} M_\sigma^c) = Y_\nu \tilde{\cap} M_\sigma^c$. (since $Y_\nu \tilde{\cap} Y_\nu^c = (\tilde{0}_\theta)_Y$, see Definition 1) where M_σ^c is GFS closed in (X, E) as $M_\sigma \in T$. Conversely, assume that $F_\mu = Y_\nu \tilde{\cap} G_\delta$ for some GFS closed set G_δ in (X, E). This mains that $G_\delta^c \in T$. Now, $Y_\nu \setminus F_\mu = Y_\nu \setminus (Y_\nu \tilde{\cap} G_\delta) = Y_\nu \tilde{\cap} (Y_\nu^c \tilde{\cup} G_\delta^c) = Y_\nu \tilde{\cap} G_\delta^c \in T_Y$ and hence F_μ is GFS closed in (Y, E).

(ii) We have, $\overline{F}_\mu$ is a GFS closed set in (X, E). Then $\overline{F}_\mu \tilde{\cap} Y_\nu$ is a GFS closed set in (Y, E). Now $F_\mu \tilde{\subseteq} \overline{F}_\mu \tilde{\cap} Y_\nu$ and GFS closure of F_μ in (Y, E) is the smallest GFS closed set containing F_μ, so $cl(F_\mu)_Y \tilde{\subseteq} \overline{F}_\mu \tilde{\cap} Y_\nu$.

On other hand $cl(F_\mu)_Y = K_\eta \tilde{\cap} Y_\nu$ where K_η is GFS closed in (X, E).Then K_η is GFS closed set containing F_μ and so $\overline{F}_\mu \tilde{\subseteq} K_\eta$. Therefore, $\overline{F}_\mu \tilde{\cap} Y_\nu \tilde{\subseteq} K_\eta \tilde{\cap} Y_\nu = cl(F_\mu)_Y$. It is useful to investigate the relationship between types of sets that are generalized fuzzy set.

GENERALIZED FUZZY SOFT SET (GFSS) AND INTUITIONISTIC FUZZY SOFT SET(IFSS)

Lemma 1

The relationships between the sets: FS, SS, FSS, IFSS, GFSS and GIFSS that generalized the crisp set (CS) notion are illustrated in Figure 1.

Counter example 1

Let $X = \{x_1, x_2, x_3, x_4\}$, and $E = \{e_1, e_2, e_3\}$.

(1) Let t $(F, E) = \{(e_1 = \{(x_1, 0.8, 0.1), (x_2, 0.9, 0.1), (x_3, 0.8, 0.1), (x_4, 0.7, 0.2)\}),$
$(e_2 = \{(x_1, 0.6, 0.3), (x_2, 0.65, 0.2), (x_3, 0.7, 0.2), (x_4, 0.65, 0.2)\}),$
$(e_3 = \{(x_1, 0.8, 0.2), (x_2, 0.5, 0.3), (x_3, 0.5, 0.4), (x_4, 0.7, 0.2)\})\}$
is an IFSS but not GFSS.

(2) Let t $F_\mu = \{(e_1 = \{(x_1, 0.5), (x_2, 0.7), (x_1, 0.8), (x_1, 0.1)\}, \frac{1}{2}),$
$(e_2 = \{(x_1, 0.4), (x_2, 0.5), (x_1, 0.7), (x_1, 0.2)\}, \frac{1}{3}),$,
$(e_3 = \{(x_1, 0.8), (x_2, 0.3), (x_1, 0.4), (x_1, 0.7)\}, \frac{4}{5})\}$ is GFSS but not IFSS.

(3) Let $(F_\mu, E) = \{(e_1 = \{(x_1, 0.8, 0.1), (x_2, 0.9, 0.1), (x_3, 0.8, 0.1), (x_4, 0.7, 0.2)\}, 0.7),$

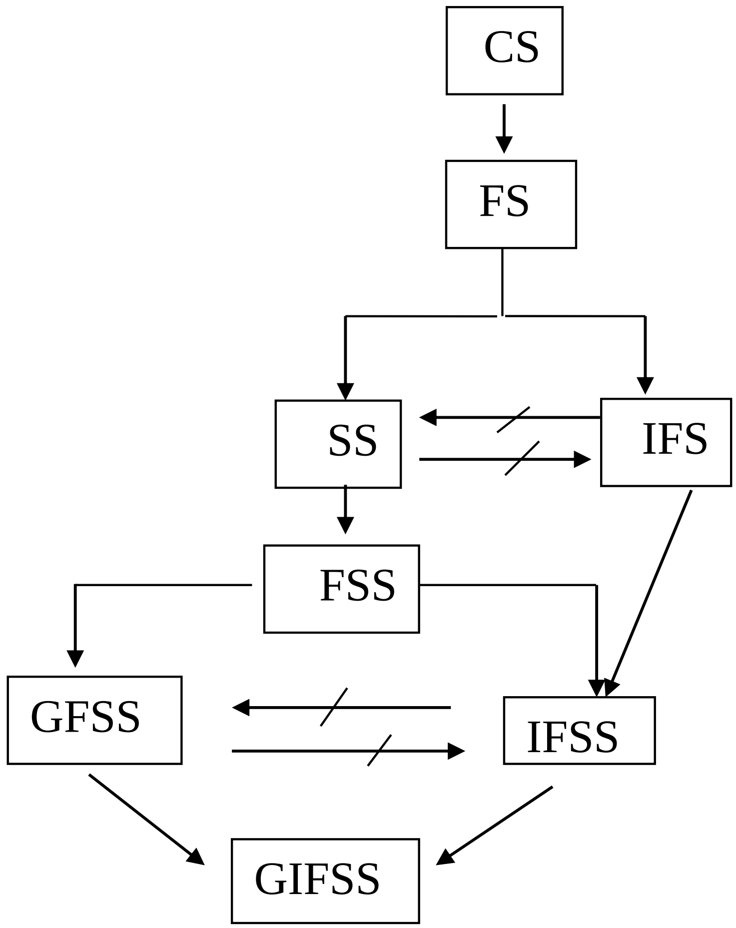

Figure 1. The converse of the arrows of the above diagram need not be true.

$(e_2 = \{(x_1, 0.6,0.3), (x_2, 0.65,0.2), (x_3, 0.7,0.2), (x_4, 0.65,0.2)\}, 0.6)$,, $(e_3 = \{(x_1, 0.8,0.2), (x_2, 0.5,0.3), (x_3, 0.5,0.4), (x_4, 0.7,0.2)\}, 0.5)\}$ is GIFSS but neither IFSS nor GFSS.

From the examples, we see that GFSS and IFSS are independent notions.

Lemma 2

Similarly, one can be deduce similar diagram of the relationship between analogues topologies.

Conclusion

In this paper, we have introduced generalized fuzzy soft point, generalized fuzzy soft open base and subbase. The generalized fuzzy soft topological subspaces is introduced. Finally, we concluded that GFSS and IFSS are independent notions, whereas each of them is GIFSS. So, one can try to introduce some special properties of compactness, some separation axioms, connected nessetc on generalized fuzzy soft topological spaces.

CONFLICT OF INTERESTS

The authors have not declared any conflict of interests.

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