African Journal of Mathematics and Computer Science Research Vol. 4(1), pp. 6-17, January 2011
Available online at http://www.academicjournals.org/AJMCSR
ISSN 2006-9731 ©2011 Academic Journals

Full Length Research Paper

A mathematical approach to transportation related data of South Assam for the optimal age of replacement of a vehicle and daily expenditure minimization

Nabendu Sen[1]*, Banashri Sinha[1] and Tanmoy Som[2]

[1]Department of Mathematics, Assam University, Silchar, India-788011, India.
[2]Department of Applied Mathematics, Institute of Technology Banaras Hindu University (BHU-IT), Varanasi, India-221005, India.

Accepted 7 July, 2010

This paper deals with the replacement issue of an Omni bus plying on Silchar–Guwahati route in India under government and private travel agencies and daily expenditure minimization of travel agencies. In this paper, effort has been given to study the age at which a bus should be replaced on the basis of various expenditure on it. It also attempts to calculate how to minimize daily expenditure of a bus service providing agency with some constraints. The post optimality of the L.P models will be studied at the end .We find the literature of it in various standard books and journals (Hartly, 1969; Harvey, 1969) as listed in the reference section. We find literature for formulation of LP models in (Sen and Som, 2008a, b, c; Sen, 2008d). In (Sen and Som, 2008a), models were developed to maximize daily profit of a service unit while in (Sen and Som, 2008b), a model was developed to minimize passenger fare.

Key words: Omnibus, scrap value, running cost, E-class.

INTRODUCTION

The communication of the southern part of North-Eastern Region of India is not good as compared to the other part of the nation. The heart of South Assam is Silchar which connects the neighboring states like Mizoram, Manipur and Meghalaya. Even the gate way of North-Eastern Region of India Guwahati is not properly connected with Silchar by rail. The main dependable source of communication on Silchar-Guwahati route is bus. So, in the quest for comfortable journey, people are always in search of buses in good condition. On the other hand, travel agencies do not care about the good condition of buses for better service to the passenger on this route. So it becomes necessary to make a mathematical study to calculate the age of replacement of buses on the said route and also simultaneously a mathematical study will be carried out to minimize daily expenditure to run the buses under a particular travel agency on Silchar-Guwahati route. Our study will be carried out on the basis of data collected from the concerned source. Before commencing our actual mathematical study, it is necessary to mention the technique to be applied here.

*Corresponding author. E-mail: nsen_16@yahoo.co.in.

METHODOLOGY

Replacement policy and linear programming technique will be applied to carry out the mathematical study.

Replacement policy

Replacement of an item takes place in the following cases:

(a) When an old item has failed and does not work at all or the item

is expected to fail soon.
(b) Then existing item deteriorated and work badly, and also need expensive maintenance.
(c) A better model of item has been developed.
Replacement problem can also broadly be classified into two categories:

(a) When item deteriorate with time and value of money (i) does not change with time (ii) changes with time. (b) When item fails completely.

Replacement of an item that deteriorate with time (t) is discrete and value of money does not change with time

In this section, we discuss the policy which will suggest the optimal age of replacement of items. Here time period is taken fixed and not taken as 1,2,3,4… Then

$$A(n) = \frac{C-S}{n} + \frac{1}{n}\sum_{t=1}^{n} f(t)$$

Now $A(n)$ will be minimum for that value of n for which

$A(n+1) \geq A(n)$ and $A(n-1) \geq A(n)$.

$\Rightarrow A(n-1) - A(n) \geq 0$ and $A(n) - A(n-1) \leq 0$

For the above, we write

$$A(n+1) = \frac{C-S}{n+1} + \frac{1}{n+1}\sum_{t=1}^{n+1} f(t)$$
$$= \frac{1}{n+1}[C - S + \sum_{t=1}^{n} f(t)] + \frac{1}{n+1} f(t+1)$$
$$= \frac{1}{n+1}[A(n) + f(n+1)]$$
$$\therefore A(n+1) - A(n) = \frac{1}{n+1}[f(n+1) - A(n)]$$

Thus, $A(n+1) - A(n) \geq 0$
$\Rightarrow f(n+1) \geq A(n)$

It can also be proved that

$$A(n-1) - A(n) \leq 0$$

$$\Rightarrow f(n) \leq A(n-1)$$

This suggest that if the maintenance cost in (n+1)th year is more than the average total cost in the nth year and nth years maintenance cost is less than the previous year's average total cost, then the item should be replaced at the end of n years.

LP models

The genera l LP model with n variables and m constraints can be variables X, we

Optimize Z=CX

Subjected to

AX ≤ 0 r = 0 r ≥ b.

Together with X ≥ 0.

In general, LP problem (≥,=≤) means that in any specific problem each constraint may take only one of the three possible form namely less than or equal, equal, greater than or equal.

Minimization problem can be solved by converting it into maximization form by the relation.

Min Z = -Max bn -Z).

The following are the steps to be followed to formulate an LP model:

Identification of decision variables
#Identification of the problem data
#Formulation of constraints
#Formulation of the objective function.

Solution of LP models

When the model involves two decision variables, then it can be solved by graphical methods. But when it involves more than two variables, simplex method is used to solve it. If all the constraints are of less than type, by introducing slack variable it can be solved. But if some of the constraints are greater than or equal or equal type, we use artificial variable technique.

Post optimality or sensitivity analysis

The scope of linear programming does not end at finding the optimal solution to the linear model of real life problem. Sensitivity analysis of linear programming continues with the optimal solution to provide additional practical insight of the model. Since this analysis examines how sensitive the optimal solution is to changes in the coefficients of the LP model, it is called sensitivity analysis. It is also known as post-optimality analysis as it starts after the optimal solution is obtained. Since we live in a dynamic world where changes takes place constantly, this study of the effects on the solution due to changes in the data of the problem is very useful. In general, we are interested in finding the effects of the following changes on the optimal LP solution:

#changes in profit/unit or cost/unit of the objective function.
#changes in the availability of resources or capacity or limit on demand and in other parameters. Post-optimal analysis of model A and model B will be done by using the following:

#When C_K belongs to C_B, then the change ΔC_k so that the solution remain optimum is given by

Table 1. Capital cost Rs 11, 00,000/-.

End of the year	1st year	2nd year	3rd year	4th year	5th year	6th year	7th year
Maintenance cost (Rs)	47,000	95,000	1,25,000	1,65,000	2,40,000	1,50,000	3,90,000

$$\underset{Y_{kj}>0}{Max}\left\{-(Z_j - C_J)/Y_{kj}\right\} \leq \Delta C_k \leq \underset{Y_{kj}<0}{Min}\{-(Z_j - C_J)/Y_{kj}\}$$

The range for discrete change Δb_k for which the optimum basic feasible solution remains is given by

$$\underset{b_{ik}>0}{Max}\{-X_{B_i}/b_{ik}\} < b_k < \underset{b_{ik}<0}{Min}\{-X_{B_i}/b_{ik}\}$$

DATA

A sample data for capital cost and its maintenance cost of an Omni-bus, which was purchased during the year 2001 has been given in Table 1.

We also listed here some other related data for the year 2007 of a travel agency, engaged in providing bus service on Silchar-Guwahati route which will be useful to minimize daily expenditure.

It is found from the collected data of for a particular year that

Number of ticket booking for one old E-class bus is 25.
Number of ticket booking for one new E-class bus is 28.
Number of ticket booking for one old deluxe bus is 30.
Number of ticket booking for one new deluxe bus is 35.
Daily Expenditure on a old E-class bus is Rs. 4331/-
Daily Expenditure on a new E-class bus is Rs. 4181/-
Daily Expenditure on a old deluxe bus is Rs. 4494/-
Daily Expenditure on a new deluxe bus is Rs. 4331/-

The agency used to run 10 to 14 numbers of different types of buses during this period. It is also obtained from the data that

One old E-class bus requires 115 L of fuel per trip
One new E-class bus requires 110 L of fuel per trip
One old deluxe bus requires 120 L of fuel per trip
One new deluxe bus requires 110 L of fuel per trip

There is only one trip every of each type of bus in the route.

The agency can provide daily 1680 L of fuel for running these buses.

Let us also fix the maximum profit as Rs 18,000/

MATHEMATICAL APPROACH:

In this section, first computation will be carried out to calculate the optimal age of replacement of a bus and then an LP model will be formulated to solve daily expenditure minimization.

As per the data we get, Table 2 displays the necessary related calculation.

LP model formulation and solution

Model A

Let us assume x_1 to be the number of old E-class bus.
Let us assume x_2 to be the number of new E-class bus.
Let us assume x_3 to be the number of old deluxe bus.
Let us assume x_4 to be the number of new E-class bus.

So, the daily total transportation cost = $4331x_1 + 4181x_2 + 4494x_3 + 4331x_4$

Then mathematically the problem can be stated as

Minimize $Z = 4331x_1 + 4181x_2 + 4494x_3 + 4331x_4$

Subject to

Total number of passenger in all the buses satisfy the inequality

$25x_1 + 28x_2 + 30x_3 + 35x_4 \leq 462$… (1*)

Total number of buses satisfying the inequalities

$x_1 + x_2 + x_3 + x_4 \geqq 10$… (2*)

$x_1 + x_2 + x_3 + x_4 \leqq 14$… (3*)

Table 2. Computation to calculate the optimal age of replacement of a bus.

Year of service(n)	Maintenance cost (Rs)	Cumulative maintenance cost	Depreciation /Scrap value	Resale value (C-S)	Total cost	Average total cost
1	47,000	47,000	1,10,000	9,90,000	1037000	1037000
2	95,000	1,42,000	99,000	8,91,000	1033000	516500
3	1,25,000	2,67,000	89,100	8,01,900	1068000	356000
4	1,65,000	4,32,000	80,190	7,21,710	1153710	2884275
5	2,40,000	6,72,000	72,171	649539	1321539	264307.8
6	1,50,000	822000	64,953.9	584585.1	1406585.1	234430.6*
7	3,90,000	1212000	58458.5	526126.6	1738126.6	248303.8

Here we have taken depreciation 10%.

The total daily profit from all these buses has been restricted as

$1400x_1 + 1533x_2 + 833x_3 + 1033x_4 \leq 18,000$…. (4*)

Together with $x_1, x_2, x_3, x_4 \geq 0$

We use two-phase method to solve it

Introducing slack variables x_5, x_7, x_8 and surplus variable x_6 we get

$25x_1 + 28x_2 + 30x_3 + 35x_4 + X_5 = 462$

$x_1 + x_2 + x_3 + x_4 – X_6 = 10$

$x_1 + x_2 + x_3 + x_4 + X_7 = 14$

$1400x_1 + 1533x_2 + 833x_3 + 1033x_4 + X_8 = 18000$

Since there is no initial basic feasible solution, we introduce artificial variable X_9 to the second equation to get an initial basic feasible solution as

$X_5 = 462$, $X_9 = 10$, $X_7 = 14$ and $X_8 = 18,000$

Rewriting the objective function as

Max $Z^* = -4331X_1 - 4181X_2 - 4494X_3 - 4331X_4$

Phase-I

The initial simplex table in phase –I of LP model A is given in Table 3.

Here $Z_1 - C_1$ is most negative. Therefore Y_1 enters the basis.

Here min {462/25, 10/1, 14/1, 18000/1400} =10 which corresponds to X_9.
Therefore X_9 leaves the basis. The pivot element is 1.

The final simplex table in phase-I of LP model A is as shown in Table 4.

Here Max $Z^* = 0$ and no artificial variable appears in the basis. So we proceed to phase-II.

Phase-II

Now if we assign actual cost to the original variables and 0 cost to the other variables we get

Max $Z^* = -4331X_1 - 4181X_2 - 4494X_3 - 4331X_4 + 0X_5 + 0X_6 + 0X_7 + 0X_8$

Initial simplex table for phase-II of LP model A is thus as given in Table 5.

Here $Z_2 - C_2$ is most negative. Therefore Y_2 enters the basis.

Here min {412/3,10/1,14/1,4000/133}=10 which corresponds to X_1.

Therefore X_1 leaves the basis. The pivot element is 1.

The final simplex table in phase-II of LP model A is as shown in Table 6.

Here all $Z_j – C_j \geq 0$ and therefore, the optimal solution $x_2 = 10$ in the first case for which the minimum expenditure

Table 3. The initial simplex table in phase -I of LP model A.

$C_J \rightarrow$	x_B	0	0	0	0	0	0	0	0	-1
c_B		Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9
0	X_5 = 462	25	28	30	35	1	0	0	0	0
-1	X_9 = 10	1*	1	1	1	0	-1	0	0	1
0	X_7 = 14	1	1	1	1	0	0	1	0	0
0	X_8 = 1 8,000	1400	1533	833	1033	0	0	0	1	0
Z_J-$C_J \rightarrow$		-1	-1	-1	-1	0	0	0	0	0

Table 4. Final simplex table in phase-I of LP model A.

$C_J \rightarrow$	x_B	0	0	0	0	0	0	0	0	-1
c_B		Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9
0	X_5 =412	0	3	5	10	1	25	0	0	-25
0	X_1=10	1	1	1	1	0	-1	0	0	1
0	X_7 =4	0	0	0	0	0	1	1	0	-1
0	X_8=4,000	0	133	-567	-367	0	1400	0	1	-1400
Z_J - $C_J \rightarrow$		0	0	0	0	0	0	0	0	1

Table 5. Initial simplex table for phase-II of LP model A.

$C_J \rightarrow$	x_B	-4331	-4181	-4494	-4331	0	0	0	0	0
c_B		Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9
0	X_5 = 412	0	3	5	10	1	25	0	0	-25
-4331	X_1 = 10	1	1*	1	1	0	-1	0	0	1
0	X_7 = 4	0	0	0	0	0	1	1	0	-1
0	X_8 = 4,000	0	133	-567	-367	0	1400	0	1	-1400
Z_J-$C_J \rightarrow$ 0		0	-150	163	0	0	4331	0	0	

Z^* = -41810, that is, Z = 41810.

LP Model B

If maximum number of E-class bus (old and new) = 4
If maximum number of deluxe bus (old and new) = 10
Then first problem can be stated as

Minimize $Z = 4331x_1 + 4181x_2 + 4494x_3 + 4331x_4$
Max $Z^* = -4331x_1 - 4181x_2 - 4494x_3 - 4331x_4$
Subjected to

Total number of passenger in all the buses satisfy the inequality

$$25x_1 + 28x_2 + 30x_3 + 35x_4 \leq 462\ldots \quad (5^*)$$

Total number of buses satisfying the inequalities

$$x_1 + x_2 + x_3 + x_4 \geq 10\ldots \quad (6^*)$$

$$x_1 + x_2 + x_3 + x_4 \leq 14\ldots \quad (7^*)$$

$$x_1 + x_2 \leq 4\ldots \quad (8^*)$$

$$x_3 + x_4 \leq 10\ldots \quad (9^*)$$

$$1400x_1 + 1533x_2 + 833x_3 + 1033x_4 \leq 18{,}000\ldots \quad (10^*)$$

Together with $x_1, x_2, x_3, x_4 \geq 0$

Table 6. Final simplex table in phase-II of LP model A.

$C_J \rightarrow$	x_B	-4331	-4181	-4494	-4331	0	0	0	0	0
c_B		Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9
0	$X_5 = 382$	-3	0	2	7	1	28	0	0	-28
-4181	$X_2 = 10$	1	1	1	1	0	-1	0	0	1
0	$X_7 = 4$	0	0	0	0	0	1	1	0	-1
0	$X_8 = 2670$	-133	0	-700	-500	0	1533	0	1	-1533
$Z_J - C_J \rightarrow$		150	0	311	150	0	4181	0	0	

Table 7. Initial simplex table in phase-I of LP model B.

$C_J \rightarrow$	x_B	0	0	0	0	0	0	0	0	0	0	-1
c_B		Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}	Y_{11}
0	$X_5 = 462$	25	28	30	35	1	0	0	0	0	0	0
-1	$X_{11} = 10$	1	1	1	1	0	-1	0	0	0	0	1
0	$X_7 = 14$	1	1	1	1	0	0	1	0	0	0	0
0	$X_8 = 4$	1*	1	0	0	0	0	0	1	0	0	0
0	$X_9 = 10$	0	0	1	1	0	0	0	0	0	1	0
0	$X_{10} = 18000$	1400	1533	833	1033	0	0	0	0	0	1	0
$Z_J - C_J \rightarrow$		-1	-1	-1	-1	0	1	0	0	0	0	0

Here also we use two-phase method to solve it. Introducing slack variables X_5, X_7, X_8, X_9, X_{10} and surplus variable X_6 we have

$$25x_1 + 28x_2 + 30x_3 + 35x_4 + X_5 = 462$$

$$x_1 + x_2 + x_3 + x_4 - x_6 = 10$$

$$x_1 + x_2 + x_3 + x_4 + x_7 = 14$$

$$x_1 + x_2 + x_8 = 4$$

$$x_3 + x_4 + x_9 = 14$$

$$1400x_1 + 1533x_2 + 833x_3 + 1033x_4 + X_{10} = 18000$$

Since there is no initial basic feasible solution, we introduce artificial variable X_{11} to the second equation, and then initial basic feasible solution becomes $X_5 = 462$, $X_{11} = 10$, $X_7 = 14$, $X_8 = 4$, $X_9 = 10$, $X_{10} = 18000$.

Phase-I

The initial simplex table in phase-I of LP model B is shown in Table 7.

Here $Z_1 - C_1$ is most negative .Therefore Y_1 enters the basis.

Here min {462/25, 10/1, 14/1, 18000/1400} =10 which corresponds to X_8 .

Therefore X_8 leaves the basis. The pivot element is 1.

The next iteration in phase-I of LP model B is given in Table 8.

Here Z_3-C_3 as most negative. Therefore Y_3 enters the basis.

Here min {362/30, 10/1, 10/1, 12400/833} = 6 which corresponds to X_{11}.

Therefore X_{11} leaves the basis. The pivot element is 1.
The final iteration table in phase-I of LP model B is given in Table 9.

Here all $Z_J - C_J \geq 0$ and no artificial variable appears in the basis. Therefore we proceed to phase-II.

Table 8. Initial iteration in phase-I of LP model B.

$C_J \rightarrow$	x_B	0	0	0	0	0	0	0	0	0	0	-1
c_B		Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}	Y_{11}
0	$X_5 = 362$	0	3	30	35	1	0	0	-25	0	0	0
-1	$X_{11} = 6$	0	0	1*	1	0	-1	0	-1	0	0	1
0	$X_7 = 10$	1	1	0	0	0	0	1	-1	0	0	0
0	$X_1 = 4$	1	1	0	0	0	0	0	1	0	0	0
0	$X_9 = 10$	0	0	1	1	0	0	0	0	1	0	0
0	$X_{10} = 12400$	0	133	833	1033	0	0	0	-1400	0	1	0
$Z_J - C_J \rightarrow$		0	0	-1	-1	0	1	0	1	0	0	0

Table 9. Final iteration table in phase-I of LP model B.

$C_J \rightarrow$	x_B	0	0	0	0	0	0	0	0	0	0	-1
c_B		Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}	Y_{11}
0	$X_5 = 182$	0	3	0	5	1	30	0	5	0	0	-30
0	$X_3 = 6$	0	0	1	1	0	-1	0	-1	0	0	1
0	$X_7 = 4$	0	0	0	0	0	1	1	0	0	0	-1
0	$X_1 = 4$	1	1	0	0	0	0	0	1	0	0	0
0	$X_9 = 6$	0	0	0	0	0	1	0	1	1	0	0
0	$X_{10} = 7402$	0	133	0	200	0	833	0	-567	0	1	-833
$Z_J - C_J \rightarrow$		0	0	0	0	0	0	0	0	0	0	1

Table 10. Initial simplex table in phase-II of LP model B.

$C_J \rightarrow$	x_B	-4331	-4181	-4494	-4331	0	0	0	0	0	0	0
c_B		Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}	Y_{11}
0	$X_5 = 182$	0	3	0	5	1	30	0	5	0	0	-30
-4494	$X_3 = 6$	0	0	1	1	0	-1	0	-1	0	0	1
0	$X_7 = 4$	0	0	0	0	0	1	1	0	0	0	-1
-4331	$X_1 = 4$	1	1	0	0	0	0	0	1	0	0	0
0	$X_9 = 6$	0	0	0	0	0	1	0	1	1	0	0
0	$X_{10} = 7402$	0	133	0	200	0	833	0	-567	0	1	-833
$Z_J - C_J \rightarrow$		0	-150	0	-163	0	4494	0	163	0	0	

Phase-II

The initial simplex table in phase-II of LP model B is given in Table 10.

Here Z_4 - C_4 is most negative .Therefore Y_4 enters the basis.
Here min {182/5,6/1,7402/200}=6 which corresponds to X_3

Therefore X_3 leaves the basis. The pivot element is 1.

The next iteration table is Table 11.

Here Z_2 - C_2 is most negative. Therefore Y_2 enters the basis.

Here min {152/3,4/1,6202/133}=4 which corresponds to X_1

Table 11. Initial iteration table for Phase-II of LP model B.

$C_J \rightarrow$	x_B	-4331	-4181	-4494	-4331	0	0	0	0	0	0	0
c_B		Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}	Y_{11}
0	$X_5 = 152$	0	3	-5	0	1	35	0	10	0	0	-35
-4331	$X_4 = 6$	0	0	1	1	0	-1	0	-1	0	0	1
0	$X_7 = 4$	0	0	0	0	0	1	1	0	0	0	-1
-4331	$X_1 = 4$	1	1*	0	0	0	0	0	1	0	0	0
0	$X_9 = 6$	0	0	0	0	0	1	0	1	1	0	0
0	$X_{10} = 6202$	0	133	-200	0	0	1033	0	-367	0	1	-1033
Z_J-$C_J \rightarrow$		0	-150	163	0	0	4331	0	0	0	0	

Table 12. Final iteration table for Phase-II of LP model B.

$C_J \rightarrow$	x_B	-4331	-4181	-4494	-4331	0	0	0	0	0	0	0
c_B		Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}	Y_{11}
0	$X_5 = 140$	-3	0	-5	0	1	35	0	7	0	0	-38
-4331	$X_4 = 6$	0	0	1	1	0	-1	0	-1	0	0	1
0	$X_7 = 4$	0	0	0	0	0	1	1	0	0	0	-1
-4331	$X_2 = 4$	1	1	0	0	0	0	0	1	0	0	0
0	$X_9 = 6$	0	0	0	0	0	1	0	1	1	0	0
0	$X_{10} = 5202$	-133	0	-200	0	0	1033	0	500	0	1	-900
$Z_J - C_J \rightarrow$		150	0	163	0	0	4331	0	150	0	0	

Therefore X_1 leaves the basis. The pivot element is 1.

The next iteration table is Table 12.

Here Z_J-$C_J \geq 0$, therefore the optimal solution is $X_2 = 4$ and $X_4 = 6$ for which min Z = 42710.

Post optimal analysis

We make a post optimal study in C and b for models A and B.

Model A

Now we study the sensitivity analysis which has dealt with minimizing the expenditure of the organization running the transport system.

The optimum simplex table is given in Table 13.

Variation in c_1:

Since c_1 does not belong to C_B , the change in C_1 ΔC_1, so that the solution remains optimum is given by

$\Delta C_1 \leq Z_1 - C_1$

$\rightarrow \Delta C_1 \leq 150$

Therefore the range over which c_1 can vary with maintaining the optimality of the solution is given by

$-\infty \leq C_1 \leq C_1 + \Delta C_1$

$\rightarrow -\infty \leq C_1 \leq -4331+150$

$\rightarrow -\infty \leq C_1 \leq -4181$

For original problem C_1 lies between 4181 and 4331

Variation in C_2:

Since C_2 belongs C_B, the range of ΔC_2 is given by

$$\underset{Y_{2j}>0}{Max}\{-(Z_j - C_J)/Y_{2j}\} \leq \Delta C_2 \leq \underset{Y_{2j}<0}{Min}\{-(Z_j - C_J)/Y_{2j}\}$$

Table 13. Optimum simplex table for model A.

$C_J \rightarrow$	x_B	-4331	-4181	-4494	-4331	0	0	0	0	0
c_B		Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9
0	X_5 = 382	-3	0	2	7	1	28	0	0	-28
-4181	X_2= 10	1	1	1	1	0	-1	0	0	1
0	X_7 = 4	0	0	0	0	0	1	1	0	-1
0	X_8= 2670	-133	0	-700	-500	0	1533	0	1	-1533
Z_J-$C_J \rightarrow$		150	0	3113	150	0	4181	0	0	

$\rightarrow$max {-150/1, 0/1, -3131/1, -150/1} ≤ ΔC_2 ≤ min {-4181/-1}
$\rightarrow 0 \leq \Delta C_2 \leq 4181$

The required range over which C_2 can vary maintaining the condition of optimality is therefore

$-4181 + 0 \leq C_2 \leq -4181 + 4181$
$\rightarrow -4181 \leq C_2 \leq 0$

For actual problem $C_2 \leq 4181$

Variation in C_3:

Since c_3 does not belong to C_B, the change ΔC_3 in C_3, so that the solution remains optimum is given by

$\Delta C_3 \leq Z_3 - C_3$

$\rightarrow \Delta C_3 \leq 313$

Therefore the range over which c_1 can vary with maintaining the optimality of the solution is given by

$-\infty \leq C_3 \leq C_3 + \Delta C_3$

$\rightarrow -\infty \leq C_3 \leq -4494+313$

$\rightarrow -\infty \leq C_3 \leq -4181$

For original problem C_1 lies between 4181 and 4494

Variation in C_4:

Since c_4 does not belong to C_B, the change ΔC_4 in C_4, so that the solution remains optimum is given by

$\Delta C_4 \leq Z_4 - C_4$

$\rightarrow \Delta C_4 \leq 150$

Therefore the range over which c_1 can vary with maintaining the optimality of the solution is given by

$-\infty \leq C_4 \leq C_4 + \Delta C_4$

$\rightarrow -\infty \leq C_4 \leq -4331+150$

$\rightarrow -\infty \leq C_4 \leq -4181$

For original problem C_4 lies between 4181 and 4331

Variation in b:

From the optimum simplex table we have

X_B = [382 10 4 2670]

B = [Y_6 Y_9 Y_7 Y_8]

$$\begin{bmatrix} 1 & -28 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -1533 & 0 & 1 \end{bmatrix}$$

The individual effect of b_i where b = [b_1 b_2 b_3 b_4] such that the optimality of the basic feasible solution is not violated is given by

$$\underset{b_{ik} > 0}{Max}\{-X_{B_i} / b_{ik}\} < b_k < \underset{b_{ik} < 0}{Min}\{-X_{B_i} / b_{ik}\}$$

For b_1 we get

Max {-382/1} ≤ Δb_1

Here $b_1 = 462$,

That is, $462 – 382 \leq b_1$
$80 \leq b_1$

For b_2 we get

Max {-10/1} ≤ Δb_2 ≤ min {-382/-28, -4/-1, -2670/-1533}
$-10 \leq \Delta b_2 \leq 1.7$

Here $b_2 = 10$ therefore $-10+10 \leq b_2 \leq 10+1.7$
$0 \leq b_2 \leq 11.7$

For b_3 we get

Max {-4/1} ≤ Δb_3
$--4 \leq \Delta b_3$

Here $b_3 =14$, therefore $14-4 \leq b_3$
$10 \leq b_3$

For b_4 we get

Max {-2670/1} ≤ Δb_4
$-2670 \leq \Delta b_4$

Here $b_4 = 18,000$, therefore $18000 – 2670 \leq b_4$

That is, $15330 \leq b_4$

Model B

The optimum simplex table for model B is given in Table 14.

First we go for the sensitivity analysis of C.
Since c_1 does not belong to C_B , the change ΔC_1 in C_1,so that the solution remains optimum is given by

$\Delta C_1 \leq Z_1 - C_1$

$\rightarrow \Delta C_1 \leq 150$

Therefore the range over which c_1 can vary with maintaining the optimality of the solution is given by

$-\infty \leq C_1 \leq C_1 + \Delta C_1$

$\rightarrow -\infty \leq C_1 \leq -4331 + 150$

$\rightarrow -\infty \leq C_1 \leq -4181$

For original problem, C_1 lies between 4181 and 4331

Variation in C_2:

Since C_2 belongs to C_B, the range of ΔC_2 is given by

$$\underset{Y_{2j}>0}{Max}\left\{-(Z_j - C_J)/Y_{2j}\right\} \leq \Delta C_2 \leq \underset{Y_{2j}<0}{Min}\{-(Z_j - C_J)/Y_{2j}\}$$

$\rightarrow$max {-150/1, 0/1, -3131/1, -150/1} ≤ ΔC_2

$\rightarrow 0 \leq \Delta C_2$

The required range over which C_2 can vary maintaining the condition of optimality is therefore

$\rightarrow -4181 \leq C_2$

For actual problem $C_2 \leq 4181$

Variation in C_3:

Since c_3 does not belong to C_B, the change ΔC_3 in C_3, so that the solution remains optimum is given by

$\Delta C_3 \leq Z_3 - C_3$

$\rightarrow \Delta C_3 \leq 163$

Therefore the range over which c_1 can vary maintaining the optimality of the solution is given by

$-\infty \leq C_3 \leq C_3 + \Delta C_3$

$\rightarrow -\infty \leq C_3 \leq -4494 + 163$

$\rightarrow -\infty \leq C_3 \leq -4331$

For original problem C_3 lies between 4331 and 4494
Variation in C_4:
Since C_4 belong to C_B, the range of $\Delta C4$ is given by

$$\underset{Y_{2j}>0}{Max}\left\{-(Z_j - C_J)/Y_{2j}\right\} \leq \Delta C_2 \leq \underset{Y_{2j}<0}{Min}\{-(Z_j - C_J)/Y_{2j}\}$$

$\rightarrow$max {-163/1, 0/1} ≤ ΔC_4 ≤ min {-4331/-1, -150/-1}
$\rightarrow 0 \leq \Delta C_4 \leq 150$

The required range over which C_4 can vary maintaining the condition of optimality is therefore

$\rightarrow -4331 \leq C_4 \leq -4181$

Table 14. Optimum simplex table for model B.

$C_J \rightarrow$	x_B	-4331	-4181	-4494	-4331	0	0	0	0	0	0	0
c_B		Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8	Y_9	Y_{10}	Y_{11}
0	$X_5 = 140$	-3	0	-5	0	1	35	0	7	0	0	-38
-4331	$X_4 = 6$	0	0	1	1	0	-1	0	-1	0	0	1
0	$X_7 = 4$	0	0	0	0	0	1	1	0	0	0	-1
-4331	$X_2 = 4$	1	1	0	0	0	0	0	1	0	0	0
0	$X_9 = 6$	0	0	0	0	0	1	0	1	1	0	0
0	$X_{10} = 5202$	-133	0	-200	0	0	1033	0	500	0	1	-900
$Z_J - C_J \rightarrow$		150	0	163	0	0	4331	0	150	0	0	

For actual problem C_4 lies between 4181 and 4331

Variation in b:

From optimum simplex table we have

X_B = [140 6 4 4 6 5670]

B = [Y_5 Y_{11} Y_7 Y_8 Y_9 Y_{10}]

$$\begin{pmatrix} 1 & -38 & 0 & 7 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & -900 & 0 & 500 & 0 & 1 \\ & & & & & \end{pmatrix}$$

The individual effect effect of b_i where b = [b_1 b_2 b_3 b_4] such that the optimality of the basic feasible solution is not violated is given by

$$Max\{-X_{B_i}/b_{ik}\} < b_k < Min\{-X_{B_i}/b_{ik}\}$$
$$b_{ik} > 0 \qquad\qquad b_{ik} < 0$$

For b_1,
Max {-140/1} ≤ Δb_1
→-140 ≤ Δb_1

Here b_1 = 462, therefore 462 – 140 ≤ b_1
For b_2,

Max {-6/1} ≤ Δb_2 ≤ min {-140/-38, -4/-1, -5670/-900}

-6 ≤ Δb_2 ≤ 3.7

Here b_2 = 10, therefore -6 + 10 ≤ b_2 ≤ 10+3.7
→ 4 ≤ b_2 ≤ 13.7

For b_3,
Max {-4/1} ≤ Δb_3
→-4 ≤ Δb_3

Here b_3 = 14, therefore 14 – 4 ≤ b_3 →10 ≤ b_3
For b_4,

Max {-140/70, -4/1, -6/1} ≤ Δb_4 ≤ min {-6/-1}
→-2 ≤ Δb_2 ≤ 6

Here b_4 = 4, therefore 4 - 2 ≤ b_4 ≤ 6 + 4→2 ≤ b_4 ≤ 10
For b_5,
Max {-6/1} ≤ Δb_5
→-6 ≤ Δb_5

Here b_5 = 10, therefore 10 – 6 ≤ b_5 →4 ≤ b_5
For b_6,
Max {-5760/1} ≤ Δb_6
→-5760 ≤ Δb_6

Here b_6 = 18000, therefore 18000 – 5760 ≤ b_6 →12330 ≤ b_6.

Conclusion

From the calculation, it may be noted that average total cost at the end of the 6th year is less than the same at the end of the 7th year. This implies that the agency should replace the buses which were purchased in 2001. This will favour the agency on one hand and on the other hand, the introduction of new buses after replacement will give the passengers comfort. Furthermore, similar study can be carried out on the other routes which are connecting Silchar-Agartala and Silchar-Aizwal.

Again from the solution of model A we can say that daily expenditure to run the buses can be minimized if the said agency provide new 10 E-class buses daily when there is no restriction on the number of buses and their types; whereas the solution of model B suggests that daily expenditure can be minimized if the agency provides 6 new deluxe and 4-new E-class. From both results of models A and B, it is clear that a new bus always give comfort to the passengers. Age for replacement of bus can be determined as calculated here.

Post–optimal analysis of model A suggest that C_1 should lie between 4181 and 4331, C_2 must not exceed 4181, C_3 between 4181 and 4494 and C_4 between 4181 and 4331 for the solution to be optimum. Also for the optimality of the solution $b_1 \geq 80$, $b2 \leq 12$, $b3 \geq 10$ and $15330 \leq b_4$.

Post-optimal analysis of model B reveals that C_1 must lie between 4181 and 4331; C_2 should not exceed 4181, C_3 between 4331 and 4494 and C_4 between 4181 and 4331 for the solution to remain optimum. Also optimality of the solution remains unchanged for $b_1 \leq 322$; $4 \leq b_2 \leq 13.7$; $b_3 \geq 10$;$2 \leq b_4 \leq 10$; $b_5 \geq 4$ and $b_6 \geq 12330$.

Further it is suggested these LP model can be solved by using integer programming if our decision variables are restricted to integer values. Moreover the post–optimality study assures us that C_i and b_j are not fixed. Variations in them will give different situations. By considering related data of other parts of the country models can be formulated with different objectives. Our study can be further extended towards transportation related issues in inventory control by using optimization techniques.

Suggestion/practical implication

Our study suggests that agencies providing bus service on different routes should replace their vehicles after considering average total cost and maintenance cost in the future, that is, if the maintenance cost in a particular year is higher than average total cost of the previous year then vehicle should be immediately replaced with alternative arrangement. For the calculation, Table 2 may be referred to.

Again daily expenditure to run these buses on the routes could be minimized in two different ways: a) Firstly, by considering the number of buses plying the route under an agency providing bus service, and number of passengers to whom service can be provided daily and the maximum expectation of daily profit in total. b) Secondly, considering additional two conditions on the number of buses of different types on the basis of new and old category, the daily expenditure could be minimized. Moreover the test post-optimality of models (A and B) suggests that with the variation of data on the number of passengers, daily expenditure per vehicle, total number of buses of different types keeps our solution optimal and we can get the range of variation of different parameters like c_i and b_j. It is also suggested that the present piece of investigation be further applied in other areas as an application of linear programming technique.

ACKNOWLEDGEMENT

Authors are thankful to the travel agency for providing necessary data.

REFERENCES

Hartly RV (1969). "Linear Programming, Some Implifications for Management Accounting", Manage. Accounting, 51: 48 – 51.

Harvey CM (1979). "Operation Research: An Introduction to Linear Optimization and Decision Analysis", Elsevier, North Holand, Inc.

Sen N, Som T (2008a). "Mathematical Modeling of Transportation Related Problem of South Assam and its Optimal Solution", AU.J. Sci., 3(1): 22-27.

Sen N, Som T (2008b). "Mathematical Modeling of Transportation Related Fare Minimization Problem of South Assam and An approach to its Optimal Solution" IJTM., 32(3): 201-208.

Sen N, Som T (2008c)." Mathematical Modeling of Transportation Related Problem of South Assam with an approach to its Optimal Solution", ASR., 22(2): 59-67.

Sen N (2008d) Modeling of Transportation Related Problems of Northeastern Region of India and Their Solutions: A Case Study of Southern Part. PhD Thesis submitted to Assam University, Silchar, India.