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Full Length Research Paper

Heat transfer over an unsteady porous stretching surface embedded in a porous medium with variable heat flux in the presence of heat source or sink

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The unsteady boundary layer flow over a porous stretching surface embedded in a porous medium in the presence of heat source or sink is studied. The unsteadiness in the flow and temperature fields is caused by the time dependence of the stretching velocity and the surface heat flux. The governing partial differential equations are transformed into a system of ordinary differential equations, which is then solved numerically by applying shooting method using Runge-Kutta method. The solution is found to be dependent on the governing parameters including the Prandtl number, porous parameter, heat source /sink parameter, suction or injection parameter and unsteadiness parameter. Comparison of numerical results is made with previously published results under the special cases, and found to be in good agreement. Effects of the Prandtl number, porous parameter, heat source /sink parameter, suction or injection parameter and unsteadiness parameter on the flow and heat transfer are examined.

Key words: Unsteady flow, boundary layer flow, stretching surface, heat transfer, porous medium and source /sink, suction or injection.

INTRODUCTION

The continuous moving surface heat transfer problem has many practical applications in industrial manufacturing processes. Since the pioneering work of Sakiadis (1961a, b) various aspects of the problem have been investigated by many authors. The steady boundary layer flow due to stretching with linear velocity was investigated by Crane (1970). Vleggaar (1977) and Gupta and Gupta (1985) have analyzed the stretching problem with constant surface temperature, while Soundalgekar and Ramana (1980) investigated the constant surface velocity. Grubka and Bobba (1985) have analyzed the stretching problem for a surface moving with a linear velocity and with a variable surface temperature. Ali (1994) has reported flow and heat characteristics on a stretched surface subject to power law velocity and temperature distributions. The flow field of a stretching wall with a power-law velocity variation was discussed by Banks (1983). Ali (1995) and Elbashbeshy (1998) extended Banks's work for a porous stretched surface for different values of the injection parameter. In all the previous investigations, the effects of internal heat source or sink on heat transfer were not studied. When there is an appreciable difference between the surface and the ambient fluid, one need to consider the temperature dependent heat source or sink which may exert strong influence on the heat transfer characteristics. Foraboschi and Federito (2003) assumed the volumetric rate of heat generation as

$$Q = \begin{matrix} Q_0 \quad (T - T_0) \quad if \quad T > T_0 \\ 0 \qquad\qquad if \quad T < 0 \end{matrix}$$

Elbashbeshy and Bazid (2004) studied the heat transfer

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over a stretching surface with internal heat generation. Ali (2007) investigated on the effect of lateral mass flux on the natural convection boundary layer induced by a heated vertical plate embedded in a saturated porous medium with internal heat generation. The unsteady heat transfer problem over a stretching surface, which is stretched with a velocity that depends on time, is considered by Anderson et al. (2000), Elbashbeshy and Bazid (2004) and Ishak et al. (2008). The present work is to study the heat transfer over an unsteady porous stretching surface embedded in a porous medium with variable heat flux in the presence of source /sink.

FORMULATION OF THE PROBLEM

Consider an unsteady two-dimensional laminar boundary layer flow of an incompressible fluid over a continuous moving stretching surface. It is assumed that the surface is stretched with velocity $U_w(x,t) = \dfrac{\alpha x}{1-\gamma t}$ along the x axis keeping the origin fixed, and y axis is normal to it. Also it is assumed that the surface is subjected to a variable heat flux $q_w(x,t) = \dfrac{bx}{1-\gamma t}$. The governing basic boundary layer equations for momentum and energy take the following form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\nu}{K}\, u$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p}(T - T_\infty)$$

Subject to the boundary conditions

$$y = 0: u = U_w \quad, v = v_w, \quad \frac{\partial T}{\partial y} = -\frac{q_w}{k},$$
$$y \to \infty: u = 0, \quad T = T_\infty, \qquad (4)$$

Where x and y represented coordinate axes along the continuous surface in the direction of motion and normal to it, respectively, u and v are the velocity components along the x and y axes, respectively and t is the time. ν is the kinematics viscosity, K is the permeability of the porous medium , T is the temperature inside the boundary layer, c_p is the specific heat at constant pressure, ρ is the density , Q is heat source when $Q > 0$ and Q is heat sink when $Q < 0$, v_w is the velocity of Suction (v_w >0) of the fluid , k is the thermal conductivity, T_∞ is the temperature for away from the stretching surface and α, b , γ are constants, where $\alpha > 0,$ $b \geq 0$, $\gamma \geq 0$ and $\gamma t < 1$ Both α, γ have dimension (time$)^{-1}$. Positive and negative v_w implies to suction and injection.
The equation of continuity is satisfied if we choose a stream function $\psi(x, y)$
such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

The mathematical analysis of the problem is simplified by introducing the following dimensionless similarity variables

$$\eta = \sqrt{\frac{\alpha}{\nu(1-\gamma t)}}\,y, \qquad \psi(x, y) = \sqrt{\frac{\alpha \nu x^2}{(1-\gamma t)}}\,f(\eta),$$

$$T = T_\infty + \frac{q_w}{k}\left[\sqrt{\frac{\nu(1-\gamma t)}{\alpha}}\right]\theta(\eta) \qquad (5)$$

Substituting (5) into (2) and (3), we obtain

$$f''' + ff'' - f'^2 - A(f' + \frac{1}{2}\eta f'') - \lambda f' = 0 \qquad (6)$$

$$\theta'' + \Pr[f\theta' - f'\theta - \frac{A}{2}(\theta + \eta\theta') + \delta\theta] = 0 \qquad (7)$$

Where the primes denote differentiation with respect to η , $A = \dfrac{\gamma}{\alpha}$ is a parameter that measure the unsteadiness, $\lambda = \dfrac{\nu(1-\gamma t)}{\alpha K}$ is the porous parameter, $\Pr = \dfrac{\mu c_p}{k}$ is the Prandtl number, (μ is the viscosity) and $\delta = \dfrac{Qk}{\mu c_p}\dfrac{\mathrm{Re}_x}{\mathrm{Re}_k^2}$ is the heat source /sink parameter, $\mathrm{Re}_x = \dfrac{U_w x}{\nu}$ is the local Reynolds number and $\mathrm{Re}_k = \dfrac{U_w\sqrt{k}}{\nu}$.

The boundary conditions (4) now become:

Table 1. Comparison of local Nusselt number for A = 0and λ = $\lambda = f_w = \delta = 0$ at different values of $\Pr$ with previously published data.

Pr Ishak et al. (2008)	Elbashbeshy (1998)	Exact solution (Abramowitz, 1965)	Present results
0.72 0.8086	0.8161	0.8086	0.8086
1.0 1.0	1.0	1.0	1.0
10 3.7202	3.7202	3.7206	3.7204

Table 2. Results of skin friction coefficient and the local Nusselt number at different values of Prandtl number Pr and unsteadiness parameter A at $\lambda = f_w = \delta = 0$.

PR	0.72		1		10	
A	$F''(0)$	$1/\theta(0)$	$F''(0)$	$1/\theta(0)$	$F''(0)$	$1/\theta(0)$
0	1	0.8086	1	1	1	3.7204
0.8	1.3218	0.94	1.3218	1.1386	1.3218	3.9487
1	1.4535	1.0025	1.4535	1.207	1.4535	4.0622
2	1.6828	1.1175	1.6828	1.3345	1.6828	4.2793

$$\eta = 0: \quad f = f_w, \quad f' = 1, \quad \theta' = -1$$
$$\eta \to \infty: \quad f' = 0, \quad \theta = 0 \qquad (8)$$

Where $f_w = -\dfrac{v_w}{\sqrt{\nu U_w}}$ is the suction /injection parameter, f_w is suction if f_w>0 and f_w is injection if f_w<0. The physical quantity of interest in this problem is the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f = \frac{\mu(\frac{\partial u}{\partial y})_{y=0}}{[\rho U_w^2/2]}, \qquad Nu_x = -\frac{x(\frac{\partial T}{\partial y})_{y=0}}{T_w - T_\infty}$$

$$C_f\sqrt{\mathrm{Re}_x} = f''(0), \qquad Nu_x/\sqrt{\mathrm{Re}_x} = -\frac{1}{\theta(0)}$$

RESULTS AND DISCUSSION

The ordinary differential equations (6) - (8) have been solved numerically by means of shooting method using Runge-Kutta method. In order to check the accuracy of the numerical solution a comparison of heat transfer characteristics at the surface for $A = 0$ (steady-state flow), $\lambda = 0$, $f_w = 0$ and $\delta = 0$ at different values of Prandtl number are made with that of Elbashbeshy (1998), Ishak et al. (2008), and Abramowitz (1965). From Table 1, we note that there is a close agreement with these approaches and thus verifies the accuracy of the method used. From Table 2 - 5, we note that the skin friction coefficient increases with the unsteadiness parameter A and porous parameter λ, while the local Nusselt number increases with the unsteadiness parameter A and decreases porous parameter λ and heat source/sink parameter δ. The local Nusselt number increases with increase of suction parameter and decreases with injection parameter f_w at different values of Prandtl number. Figures 1 and 2, gives the variation of velocity and temperature profiles for various values of suction parameter f_w at Pr=0.72, A=1, $\lambda=0.5, \delta=0.2$ We observe from Figure 1 the velocity decreases with increasing suction (f_w>0) ,fluid velocity is found to decrease , that is suction causes to decrease the velocity of the fluid in the boundary layer region . This effect acts to decrease the skin friction coefficient. Increasing in the suction causes progressive thinning of the boundary layer. From Figure 2 the temperature in the boundary layer also decreases with the increase of suction f_w .The thermal boundary layer thickness decreases with f_w which causes an increase in the rate of heat transfer. Figure 3 presents velocity profiles for various values of po-rous parameter λ, when Pr=0.72, A=1, $f_w = 0.3$, δ =0.2. We observe from Figure 3 that the velocity decreases as the porous parameter λ increases and this implies an accompanying reduction of the

Table 3. Results of skin friction coefficient and the local Nusselt number at different values of Prandtl number Pr and unsteadiness parameter A at $\delta = 0.2$, λ =0.5 and $f_w = 0.3$

PR	0.72		1		10	
A	$F''(0)$	$1/\theta(0)$	$F''(0)$	$1/\theta(0)$	$F''(0)$	$1/\theta(0)$
0	1.3839	0.6236	1.3839	0.9433	1.3839	5.3895
0.8	1.6521	0.9262	1.6521	1.1621	1.6521	5.5352
1	1.7111	0.9678	1.7111	1.2055	1.7111	5.5722
2	1.9735	1.1414	1.9735	1.3927	1.9735	5.7532

Table 4. Results of local Nusselt number for, $A = 0$, $\delta = 0$ and $\lambda = 0$ at different values of $\Pr$ and injection or suction parameter f_w.

f_w PR	-0.6	-0.2	0	0.2	0.3
0.72	0.6341	0.7443	0.8086	0.8796	0.9177
1	0.7440	0.9050	1.0	1.1050	1.1612
10	1.4709	2.7095	3.7204	4.9758	5.6795

Table 5. Results of skin friction coefficient and local Nusselt number for, $A = 1$, $f_w = 0.3$ and $\lambda = 1$ at different values of $\Pr$ and the heat source /sink parameter δ .

PR	0.72		1		10	
δ	$F''(0)$	$1/\theta(0)$	$F''(0)$	$1/\theta(0)$	$F''(0)$	$1/\theta(0)$
-2	1.7111	1.6784	1.7111	2.0155	1.7111	7.6399
-0.1	1.7111	1.1059	1.7111	1.3588	1.7111	5.9096
0	1.7111	1.0631	1.7111	1.3109	1.7111	5.8000
0.2	1.7111	0.9678	1.7111	1.2055	1.7111	5.5722

momentum boundary layer. Figure 4 present temperature profiles for various values of porous parameter λ, when Pr = 0.72, A = 1, $f_w = 0.3$, δ =0.2. We observe from Figure 4 that the temperature increases with the porous parameter λ. Figure 5 present temperature profiles for various values of source/sink parameter at Pr = 0.72, $\lambda = 0.5$, A = 1.We observe from Figure 5.as compared to the case of no source or sink ,one can see , the temperature increases with increasing source parameter (δ >0) .However, the opposite trend is revealed for sink parameter (δ <0) . Figure 6 present the velocity profiles for various values of A when Pr = 0.72, $\lambda = 0.5$, $f_w = 0.3$, and δ =0.2. We observe from Figure 6 that the velocity decreases as the unsteadiness parameter A increases and this implies an accompanying reduction of the momentum boundary layer. Figure 7 present temperature profiles for various values of A when Pr = 0.72, $\lambda = 0.5$, $f_w = 0.3$, and δ = 0.2. We observe from Figure 7 that the temperature decreases as the unsteadiness parameter A increases.

Conclusion

Numerical solutions have been obtained to study the flow and heat transfer in a laminar flow of an incompressible fluid past an unsteady porous stretching surface. The effects of unsteadiness parameter A, heat source/sink parameter δ , porous parameter λ suction or injection parameter f_w and Prandtl number on heat transfer

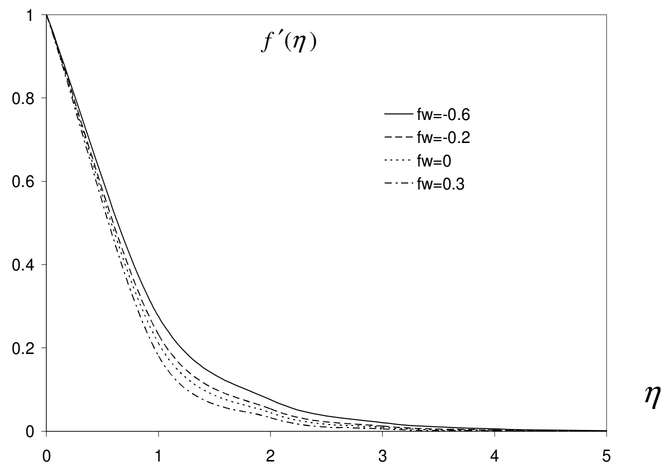

Figure 1. Velocity profiles at Pr=0.72, A=1, $\lambda=0.5, \delta=0.2$ and several values of f_w.

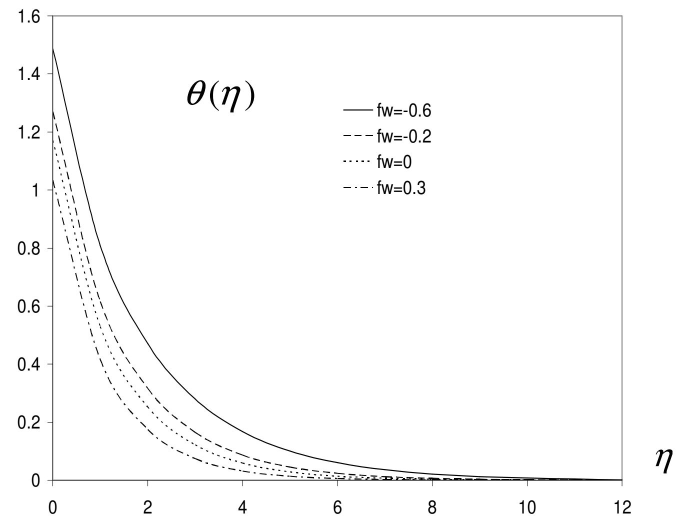

Figure 2. Temperature profiles at Pr=0.72, A = 1, $\lambda=0.5, \delta=0.2$ and several values of f_w.

characteristics were studied. The numerical solution indicated that:

1. The unsteadiness parameter A increases the skin friction coefficient and local Nusselt number.
2. The local Nusselt number decreases with increase of porous parameter λ in the presence of the source/sink parameter and increases with suction.
3. The temperature decreases with an increase in the value of unsteadiness parameter and suction parameter.
4. The unsteadiness parameter A increases the skin

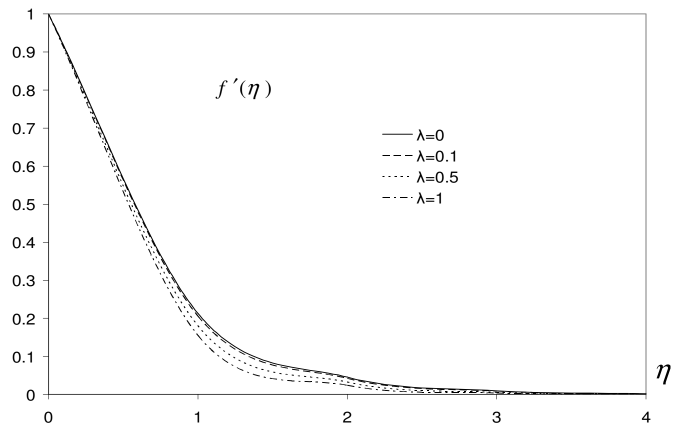

Figure 3. Velocity profiles at Pr=0.72, A=1 , $f_w=0.3$, δ =0.2 and several values of λ.

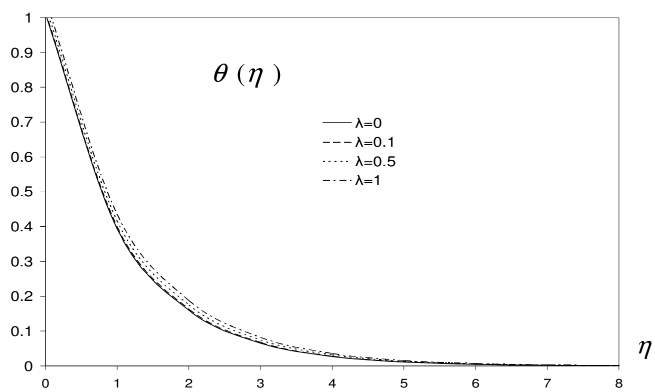

Figure 4. Temperature profiles at Pr=0.72, A = 1, $f_w=0.3$, δ =0.2 and several values of λ.

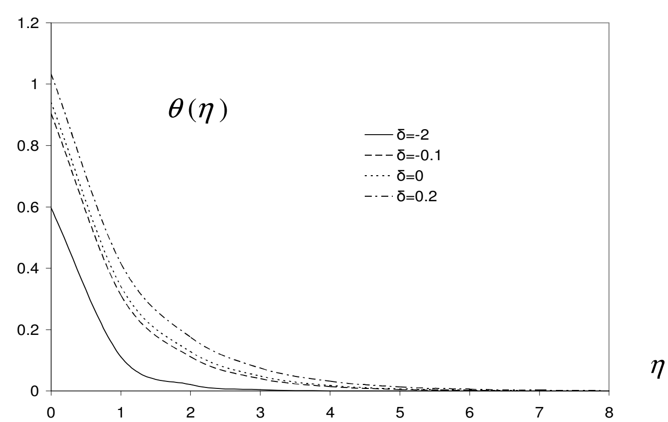

Figure 5. Temperature profiles at Pr=0.72, $\lambda=0.5$, $f_w=0.3$, A=1 and several values of δ **.**

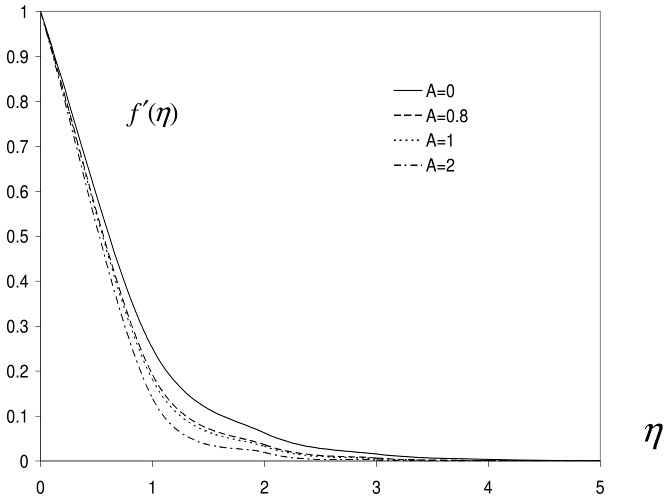

Figure 6. Velocity profiles at Pr=0.72, $\lambda = 0.5$, $f_w = 0.3$, δ =0.2 and several values of A.

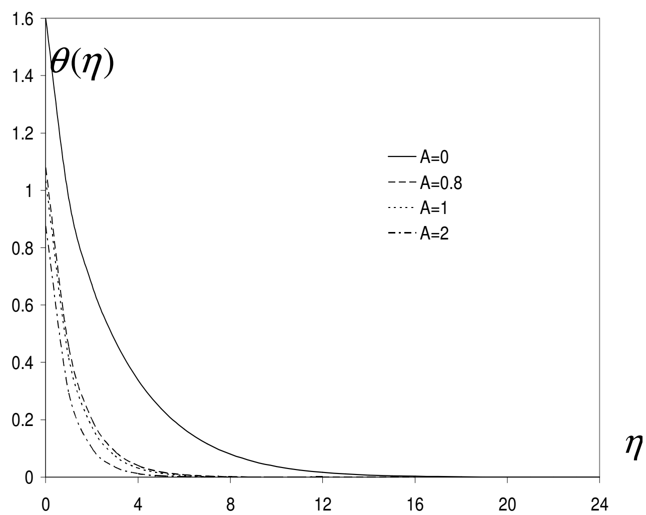

Figure 7. Temperature profiles at Pr=0.72, $\lambda = 0.5$, $f_w = 0.3$, δ =0.2 and several values of A.

friction coefficient and the local Nusselt number at different values of Prandtl number Pr.

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