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Peristaltic transport of micropolar fluid through porous medium in a symmetric channel with heat and mass transfer in the presence of generation and radiation

Nabil T. M. Eldabe[1], Afaf S. Zaghrout[2], Hameda M. Shawky[2] and Amera S. Awad[2]*

[1]Department of Mathematics, Faculty of Education, Ain-Shams University, Cairo, Egypt.
[2]Department of Mathematics, Faculty of Science (Girls), Al-Azhar University, Cairo, Egypt.

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The effect of heat generation and radiation on the peristaltic motion of micropolar fluid with heat and mass transfer through porous medium in a symmetric channel was investigated. The equations of motion for micropolar fluids were introduced as well as the equations of energy and concentration. The system of these equations written in two dimensions and then transformed using the transformations between a laboratory and fixed frames. The yield equations were solved analytically with the appropriate boundary conditions under the approximation of low Reynolds number and long wavelength. The longitudinal velocity, the microrotation velocity, the temperature and the concentration are plotted and shown graphically and discussed for different physical parameters of the problem.

Key words: Peristaltic transport, micropolar fluid, porous medium, heat and mass transfer, heat generation, thermal radiation, thermal diffusion.

INTRODUCTION

Peristalsis is defined as a wave of relaxation contraction (expansion) imparted by the walls of a flexible conduit, thereby pumping the enclosed material, it is a nature's way of moving the content within hollow muscular structures by successive contraction of their muscular fibers (Eytan et al., 2001; Fung and Yih, 1968; Mishra and Rao, 2003; Mekheimer, 2003). Peristalsis is now well-known to the physiologists to be one of the major mechanisms for fluid transport in many biological systems,as it results physiologically from neuron muscular properties of the tubular smooth muscles (Srivastava and Srivastava, 1982; El-Shehawey et al., 2006; Gharsseldien et al., 2010; Gharsseldien, 2003).

The peristaltic transport may be involved in many biological organs, for instance, moving food through the esophagus; movement of chime in the gastrointestinal tract; urine transport from the kidney to the bladder through the ureter; transport of spermatozoa in the ducts efferents of male reproduction tract and in the cervical canal of the female; movement of ova in the fallopian tub; vasomotion of small blood vessels such as venules and capillaries as well as blood flow in arteries, and in many other glandular ducts (El-Shehawey et al., 2006; El-Shehawey and Husseny, 2000; Mekheimer, 2003; Mishra and Rao, 2003; Srivastava and Srivastava, 1982). There are also many industrial applications of the peristaltic transport like, blood pumps in heart lung machine, transport of corrosive fluid, where the contact of the fluid with the machinery parts is prohibited (Mishra and Rao, 2003).

The theory of microfluid introduced by Eringen, deals with a class of fluids which exhibit certain microscopic

*Corresponding author. E-mail: amerasalem5@yahoo.com.

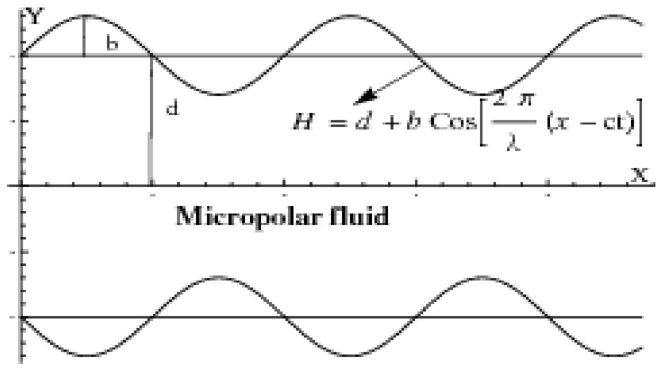

Figure 1. The geometry of the problem.

effects arising from the local structure and micro-motions of the fluid elements. A subclass of these fluids is the micropolar fluids, like blood, liquid crystals and polymers, can support couple stresses, body couples and exhibit micro-rotational and micro-inertial effects. Eringen is the first researcher who put out the theory to find a mathematical model of the micropolar fluids (Eringen, 1966). These equations are a generalization of the (Newtonian) Navier-Stokes equations and deals with three fields: velocity vector $\mathbf{V}$, the pressure of the fluid P and microrotation vector $\mathbf{w}$, together with some viscosity parameters and material constants to describe the behavior of the fluid (Eldabe et al., 2008 and El-Sayed et al., 2011). Some researchers attempted studying the peristaltic flow problems concerning the micropolar fluids, (Ali and Hayat 2008; Devi and Devanathan, 1975; Hayat and Ali, 2008; Hayat et al., 2007; Mekheimer and Abd Elmaboud, 2008; Muthu et al., 2003, 2008; Srinivasacharya et al., 2003) and others studied the heat transfer effect with peristalsis or not for different fluids (Abo-Eldahab et al. 2012; Eldabe, 2001; Eldabe et al., 2008; Eldabe and Mohamed, 2002; El-Sayed et al., 2011; Hayat and Hina, 2010; Nadeem and Noreen 2009; Nadeem et al., 2010).

In the current study we investigated the effects of heat generation and radiation with heat and mass transfer on peristaltic transport of micropolar fluid through porous medium in a symmetric channel.

Basic equations

After studying the motion of micropolar fluid through porous medium with heat and mass transfer, the basic equations which describe this motion can be written as:
Continuity equation

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{V}) = 0, \tag{1}$$

Conservation of liner momentum

$$\rho\dot{\mathbf{V}} = (\lambda_v + 2\mu_v + k_v)\nabla(\nabla\cdot\mathbf{V}) - (\mu_v + k_v)\nabla\times\nabla\times\mathbf{V} + k_v\nabla\times\mathbf{w} - \nabla P - \frac{(\mu_v + k_v)}{k}\mathbf{V} \tag{2}$$

Conservation of angular momentum

$$\rho j\dot{\mathbf{w}} = (\alpha_v + \beta_v + \gamma_v)\nabla\nabla\cdot\mathbf{w} - \gamma_v\nabla\times\nabla\times\mathbf{w} + k_v\nabla\times\mathbf{V} - 2k_v\mathbf{w} + \rho\mathbf{l}, \tag{3}$$

Heat equation

$$\rho c_p\frac{dT}{dt} = k_1\nabla^2 T + Q(T - T_0) - \nabla\cdot q_r, \tag{4}$$

Concentration equation

$$\frac{dC}{dt} = D_m\nabla^2 C + D_m k_T\nabla^2 T, \tag{5}$$

where $\mathbf{V}$ is the velocity vector of the fluid, $\mathbf{w} = (0,0,w)$ is the microrotation vector, k is the permeability of porous medium, $\mathbf{l}$ is the body couple, ρ is the fluid density, j is the microinertia parameter, T is temperature, c_p is the specific heat at constant pressure, q_r is the radiation heat flux vector, k_1 is the thermal conductivity, Q is heat generation, C is the concentration, D_m is the coefficient of mass diffusivity, k_T is the thermal diffusion ratio and α_v, β_v, γ_v, λ_v, μ_v and k_v are the material parameters (Eringen, 1966) (different viscosities that characterize the isotropic properties of the fluid), satisfy

$$2\mu_v + k_v \ge 0, \quad k_v \ge 0, \quad 3\alpha_v + \beta_v + \gamma_v \ge 0, \quad \gamma_v \ge |\beta_v|. \tag{6}$$

Mathematical formulation

Consider that the micropolar fluid moves in two-dimensions of cartesian coordinates (X,Y), where X-axis is taken in motion direction while Y-axis is perpendicular on it and (U,V) are the velocity components in X and Y directions respectively as shown in Figure 1. Neglecting body couple, with solenoidal microrotation vector, then the previous equations of become:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \tag{7}$$

$$\rho\left(\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}\right)=(\mu_v+k_v)\nabla^2U+k_v\frac{\partial w}{\partial Y}-\frac{\partial P}{\partial X}-\frac{(\mu_v+k_v)}{k}U, \tag{8}$$

$$\rho\left(\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right)=(\mu_v+k_v)\nabla^2V-k_v\frac{\partial w}{\partial X}-\frac{\partial P}{\partial Y}-\frac{(\mu_v+k_v)}{k}V, \tag{9}$$

$$\rho j\left(\frac{\partial w}{\partial t}+U\frac{\partial w}{\partial X}+V\frac{\partial w}{\partial Y}\right)=\gamma_v\nabla^2w+k_v\left(\frac{\partial V}{\partial X}-\frac{\partial U}{\partial Y}\right)-2k_vw, \tag{10}$$

$$\rho c_p\left(\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}\right)=k_1\left(\frac{\partial^2T}{\partial X^2}+\frac{\partial^2T}{\partial Y^2}\right)-\left(\frac{\partial q_r}{\partial Y}\right)+Q(T-T_0), \tag{11}$$

$$\left(\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}\right)=D_m\left(\frac{\partial^2C}{\partial X^2}+\frac{\partial^2C}{\partial Y^2}\right)+D_mk_T\left(\frac{\partial^2T}{\partial X^2}+\frac{\partial^2T}{\partial Y^2}\right). \tag{12}$$

Using the Rosselent approximation we have $q_r=-\frac{4}{3}\frac{\sigma_0}{k_0}\frac{\partial T^4}{\partial Y}$ where σ_0 is the Stefan-Boltzmann constant and k_0 is the mean absorption coefficient (Mahmoud and waheed, 2012).

The geometry of the wall surface is defined as

$$Y=H=d+b\cos\frac{2\pi}{\lambda}(X-ct), \tag{13}$$

where d is the half-width of the channel, b is the wave amplitude, λ is the wavelength, c is the velocity propagation, and t is the time. The appropriate boundary conditions are:

$$\psi=0,\quad \frac{\partial^2\psi}{\partial Y^2}=0,\quad w=0,\quad T=T_o,\quad C=C_0\quad at\,Y=0, \tag{14}$$

$$\psi=q,\quad w=\frac{-\alpha}{2}\left(\frac{\partial V}{\partial X}-\frac{\partial U}{\partial Y}\right),\quad \frac{\partial\psi}{\partial Y}=0,\quad T=T_w,\quad C=C_w\quad at\,Y=H, \tag{15}$$

where ψ is the stream function ($U=\frac{\partial\psi}{\partial Y}$ and $V=-\frac{\partial\psi}{\partial X}$), q is the flux of flow, and according to Bayada and Banhaoucha (2008), it is proposed to link the value of the microrotation value with the rotation of the velocity by way of a coefficient α ,

$$\mathbf{w}\times\mathbf{n}=\frac{\alpha}{2}(\nabla\times\mathbf{V})\times\mathbf{n}, \tag{16}$$

where $\mathbf{n}$ is the normal unit vector on the boundary, α characterizes the microrotation on the solid surfaces and its value is evaluated by the relation:

$$0\le\alpha\le\frac{\mu_v+k_v}{k_v}.$$

Introducing a wave frame (x,y) moving with the velocity c away from the laboratory frame (X,Y), by the transformations

$$x=X-ct,\quad y=Y,\quad u=U-c,\quad v=V,\quad p(x)=P(X,t), \tag{17}$$

where u and v are the fluid velocity components and p is pressure in the wave frame of references.
Further, we introduce the following non-dimensional variables:

$$\bar{x}=\frac{x}{\lambda},\quad \bar{y}=\frac{y}{d},\quad \bar{u}=\frac{u}{c},\quad \bar{v}=\frac{v}{c\delta},\quad h=\frac{H}{d},\quad \delta=\frac{d}{\lambda},$$

$$\bar{\psi}=\frac{\psi}{c\,d},\quad \bar{p}=\frac{d^2\,p}{\mu\,\lambda\,c},\quad R_e=\frac{\rho c\,d}{\mu},\quad \bar{j}=\frac{j}{d^2},$$

$$\bar{w}=\frac{wd}{c},\quad k=\frac{d^2}{K^2},\quad \Theta=\frac{T-T_0}{T_w-T_0},\quad \Phi=\frac{C-C_0}{C_w-C_0}. \tag{18}$$

After using transformation (17), dimensionless variables (18), after dropping bars and under the assumptions of long wave length ($\delta\ll1$) and low Reynolds number (R_e), the Equations 7 to 12 becomes:

$$\frac{\partial p}{\partial x}=\left(\frac{1}{1-N}\right)\frac{\partial^2u}{\partial y^2}+\left(\frac{N}{1-N}\right)\frac{\partial w}{\partial y}-\left(\frac{1}{K(1-N)}\right)(u+1), \tag{19}$$

$$\frac{\partial p}{\partial y}=0, \tag{20}$$

$$\left(\frac{2-N}{\beta^2}\right)\frac{\partial^2w}{\partial y^2}-\frac{\partial u}{\partial y}-2w=0, \tag{21}$$

$$\left(1+\frac{4}{3}\frac{1}{R_n}\right)\frac{\partial^2\Theta}{\partial y^2}+P_rG\Theta=0, \tag{22}$$

$$\frac{1}{S_c}\frac{\partial^2\Phi}{\partial y^2}+S_r\frac{\partial^2\Theta}{\partial y^2}=0 \tag{23}$$

also, the channel wall equation will be :

$$y=h=1+\phi\cos2\pi x, \tag{24}$$

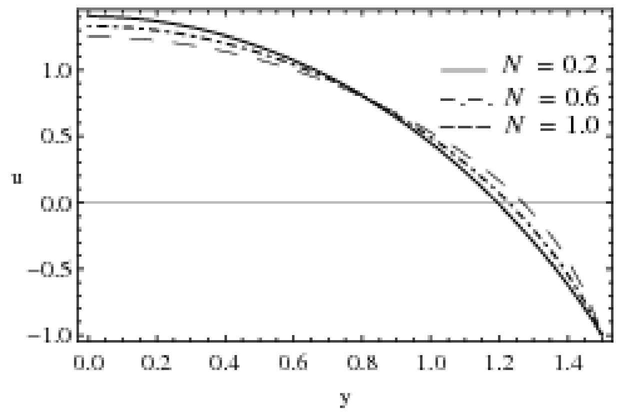

Figure 2. The velocity profile u against y for different values of coupling number N when: $\phi = 0.5$, $x = 1$, $\alpha = 1$, $K = 1.5$, $\beta = 5$, $q = 1$.

where $N = \dfrac{k_v}{\mu_v + k_v}$ is the coupling number $(0 \le N \le 1)$, $\beta^2 = \dfrac{k_v d^2 (2\mu_v + k_v)}{\gamma_v(\mu_v + k_v)}$ is the micropolar parameter, $P_r = \dfrac{\mu_v C_p}{k_1}$ is the Prantdel number, $R_n = \dfrac{k_1 k_0}{4\sigma_0 T_0^3}$ is the radiation parameter, $G = \dfrac{Qd^2}{\mu_v C_p}$ is the generation parameter, $S_r = \dfrac{\rho D_m k_T (T_w - T_0)}{\mu_v (C_w - C_0)}$ is the Soret number, $S_c = \dfrac{\mu_v}{\rho D_m}$ is the Schmidt number and $\phi = \dfrac{b}{d} < 1$ is the amplitude ratio.

Using Equation (20) into (19), the previous Equations (in terms of stream function, will be:

$$\psi_{yyyy} + N w_{yy} - \frac{1}{K}\psi_{yy} = 0, \tag{25}$$

$$\psi_{yy} - \left(\frac{2-N}{\beta^2}\right) w_{yy} + 2w = 0, \tag{26}$$

$$\left(1 + \frac{4}{3}\frac{1}{R_n}\right)\frac{\partial^2 \Theta}{\partial y^2} + P_r G \Theta = 0, \tag{27}$$

$$\frac{1}{S_c}\frac{\partial^2 \Phi}{\partial y^2} + S_r \frac{\partial^2 \Theta}{\partial y^2} = 0 \tag{28}$$

Also, we can write down the non-dimensional boundary conditions in terms of the stream function as follows:

$$\psi = 0, \quad \psi_{yy} = 0, \quad w = 0, \quad \Theta = 0, \quad at\ y = 0, \tag{29}$$

$$\psi = q, \quad w = \frac{-\alpha}{2}\psi_{yy}, \quad \psi_y = -1, \quad \Theta = 1, \quad at\ y = h, \tag{30}$$

Introducing these conditions into the general solutions of the Equations (25-28), the solutions of the stream function and the microrotation velocity, temperature and concentration respectively are given by:

$$\psi = \frac{L_1 y + L_2 \sinh(y\gamma_1) - L_3 \sinh(y\gamma_2)}{L_4} \tag{31}$$

$$w = \frac{L_5 \sinh(y\gamma_1) + L_6 \sinh(y\gamma_2)}{L_7} \tag{32}$$

$$\theta = csch(h\gamma_3)\sinh(y\gamma_3) \tag{33}$$

$$\phi = \left(\frac{S_c S_r + 1}{h}\right) y - S_c S_r csch(h\gamma_3)\sinh(y\gamma_3) \tag{34}$$

where $\gamma_i (i = 1,2,3)$, $L_j (j = 1,...7)$ and $n_k (k = 1,...18)$ are given in the appendix.

RESLUTS AND DISCUSSION

In the present study, the problem of peristaltic transport of micropolar fluid under the effects of heat generation and radiation with heat and mass transfer through a symmetric channel is modeled mathematically. The governed equations of this problem are formed in two dimensions and transformed from a laboratory frame to a fixed frame, the yield equations written in dimensionless form. The analytical solutions of these equations have been obtained under the conditions of low Reynolds number and long wave length, subject to a set of appropriate boundary conditions using the Mathematica Program. The expressions of velocity profile, microrotation velocity, temperature and concentration distributions have been evaluated for different physical parameters of the problem and have been shown graphically through a set of figures. In our figures we chose y between 0 and 1.5 we observed that in the region $0 \le y \le 0.8$.

Figure 2 explained the effect of coupling number N on the velocity u, it is observed that u decreases with increasing of N and after that, the velocity profile

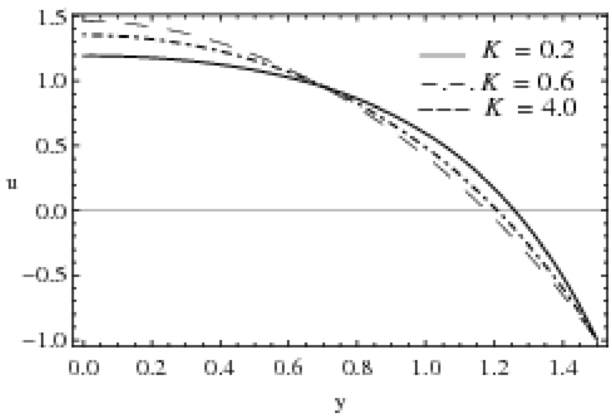

Figure 3. The velocity profile u against y for different values of K when: $\phi = 0.5$, $x = 1$, $\alpha = 1$, $N = 0.1$, $\beta = 5$, $q = 1$.

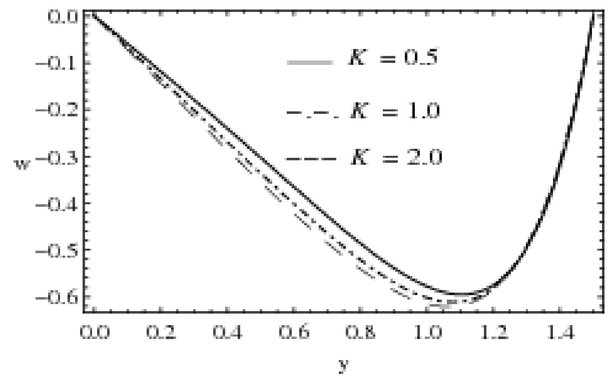

Figure 5. The microrotation w against y for different values of K when: $\phi = 0.5$, $x = 1$, $\alpha = 0$, $\beta = 3$, $N = 0.5$, $q = 0.5$.

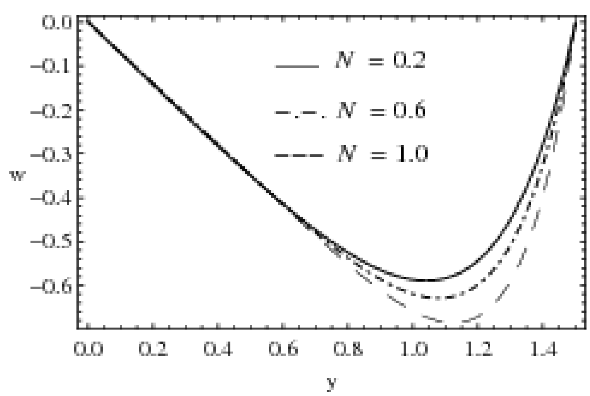

Figure 4. The microrotation w against y for different values of coupling number N when: $\phi = 0.5$, $x = 1$, $\alpha = 0$, $K = 1.5$, $\beta = 3$, $q = 0.5$.

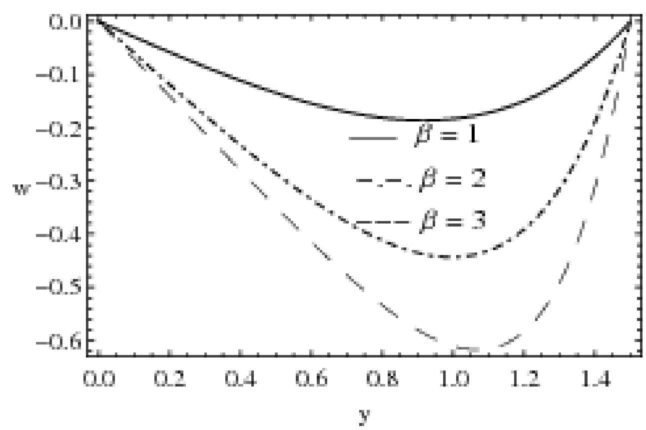

Figure 6. The microrotation w against y for different values of β when: $\phi = 0.5$, $x = 1$, $\alpha = 0$, $N = 0.5$, $K = 1.5$, $\alpha = 0$, $q = 0.5$.

increases with increasing the coupling number N. A similar behavior happens for the micropolar parameter β (when: $\phi = 0.5$, $x = 1$, $\alpha = 1$, $m = 1$, $N = 1$, $K = 0.5$, $q = 1$) and the parameter α (characterizes the microrotation on the solid surfaces, when: $\phi = 0.5$, $x = 1$, $N = 1$, $K = 0.5$, $\beta = 3$, $q = 1$). But the effect of the permeability parameter K on the velocity profile is illustrated in Figure 3. It is clear that velocity increases with increasing of K in the nominated region and then it decreases with increasing of K.

Figure 4 explained the effect of coupling number N on microrotation velocity w, it is observed that in the region $0 \le y \le 0.6$ there is no effect of N on w (w has one behavior) and then w decreases with increasing of N. The effect of the permeability parameter K on the microrotation velocity is illustrated in Figure 5. It is noted that w decreases with increasing of K in the region $0 \le y \le 1.25$ and subsequently this effect disappear and w has one behavior. In Figure 6 it is clear that w decreases with increasing of micropolar parameter β, and its effect of the parameter α on w is reflected as

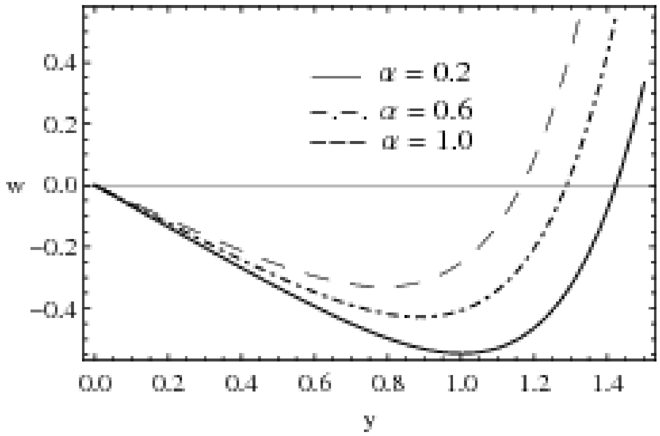

Figure 7. The microrotation w against y for different values of α when: $\phi = 0.5$, $x = 1$, $N = 0.5$, $K = 1.5$, $\beta = 3$, $q = 0.5$.

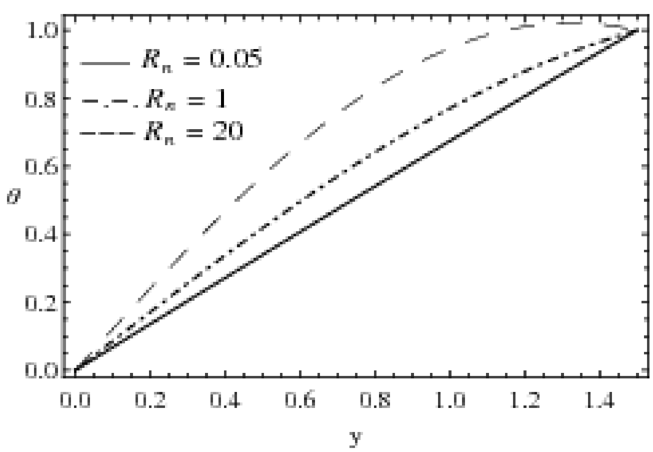

Figure 8. The temperature distribution θ against y for different values of R_n when: $\phi = 0.5$, $x = 1$, $P_r = 5$, $G = 0.3$.

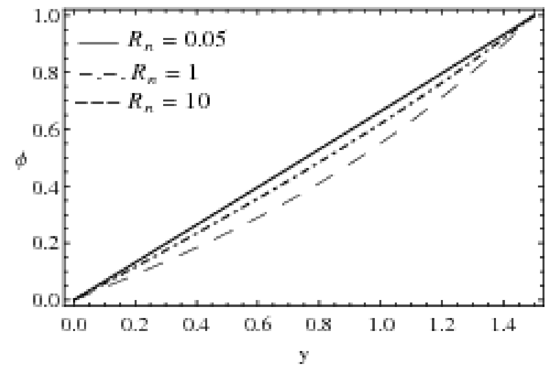

Figure 9. The concentration ϕ against y for different values of R_n when: $\phi = 0.5$, $x = 1$, $P_r = 5$, $G = 0.3$, $S_c = 1.5$, $S_r = 0.3$.

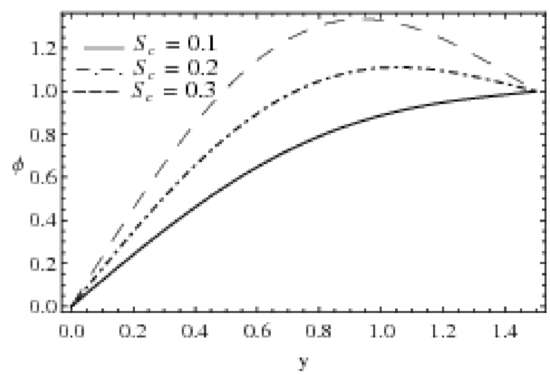

Figure 10. The concentration ϕ against y for different values of S_c when: $\phi = 0.5$, $x = 1$, $R_n = 5$, $P_r = 2o$, $S_r = 0.3$, $G = 0.3$.

shown in Figure 7.

Figures 8 explained the effects of radiation parameter R_n on the temperature distribution θ, it is illustrated that θ increases with increasing of it. The similar behavior occur with Prantdel number P_r (when: $\phi = 0.5$, $x = 1$, $R_n = 5$, $G = 0.3$) and generation parameter G (when: $\phi = 0.5$, $x = 1$, $R_n = 5$, $P_r = 5$). In Figure 9, the effect of R_n on the concentration distribution ϕ is pointed out, it is observed that ϕ decreases with increasing of R_n and the similar behavior happen with P_r (when $\phi = 0.5$, $x = 1$, $R_n = 5$, $G = 0.3$, $S_c = 1.5$, $S_r = 0.3$). The effect of Schmit number S_c on the concentration distribution ϕ is pointed out in Figure 10 and it is shown that ϕ decreases with increasing of S_c. Figure 11 indicated the effect of Soret number S_r on ϕ, it appeared that ϕ decreases with increasing of it and the generation parameter G (when: $\phi = 0.5$, $x = 1$, $R_n = 5$, $P_r = 5$, $S_c = 1.3$, $S_r = 0.3$) has the same effect on ϕ.

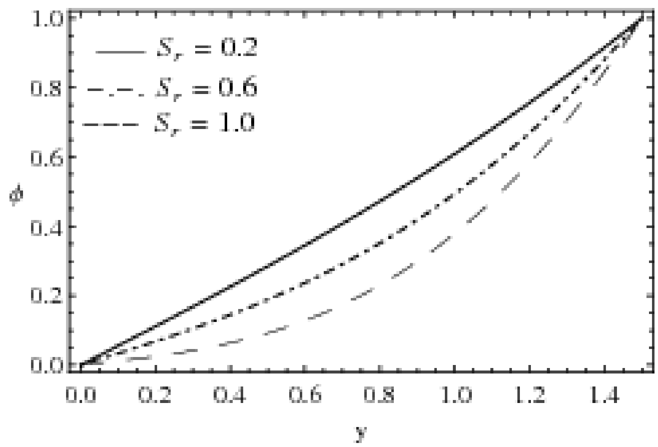

Figure 11. The concentration ϕ against y for different values of S_r when: $\phi = 0.5$, $x = 1$, $P_r = 5$, $G = 0.3$, $R_n = 5$, $S_c = 1.3$.

Conclusion

The problem of two dimensional peristaltic flow of a miropolar fluid through a symmetric channel with the effects of heat generation and radiation with heat and mass transfer has been investigated. The equations governing the fluid flow, subjected to a set of appropriate boundary conditions, have been solved analytically under the conditions of low Reynolds number and long wave length. The solutions of these equations are obtained as functions of the physical parameters of the problem, the effects of these parameters of the problem on these solutions have been shown graphically.

It appeared that the velocity profile increases with increasing of the parameters N, β and α in the region $0 \le y \le 0.8$. The temperature distribution θ increases with increasing of the parameters R_n, P_r and G. The concentration distribution ϕ decreases with increasing of R_n and P_r and decreases with increasing of the parameters S_c, S_r and G.

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APPENDIX

$$\gamma_1 = \frac{\sqrt{-\dfrac{\sqrt{K^2(N+2)^2\beta^4 - 2K(N-2)^2\beta^2 + (N-2)^2} + K(N+2)\beta^2 - N + 2}{K(N-2)}}}{\sqrt{2}},$$

$$\gamma_2 = 2\sqrt{\frac{\beta^2}{\sqrt{K^2(N+2)^2\beta^4 - 2K(N-2)^2\beta^2 + (N-2)^2} + K(N+2)\beta^2 - N + 2}},$$

$$\gamma_3 = \sqrt{\frac{3GR_nP_r}{-3R_n - 4}},\quad L_1 = n_2 + n_4 + n_8n_9 - n_{11},\ L_2 = n_{10}n_{12},\ L_3 = n_{10}n_{13},\ L_5 = n_{10}n_{14},$$

$$L_4 = n_3 + n_5 + n_6,$$

$$L_6 = n_{10}n_{15},\ L_7 = KN(n_{16} + n_{17} + n_{18}),\ n_1 = K(N\alpha - 2),\qquad n_2 = 2q\gamma_2\sinh(h\gamma_1)\cosh(h\gamma_2),$$

$$n_3 = h\gamma_2(n_1\gamma_1^2 + 2)\sinh(h\gamma_1)\cosh(h\gamma_2),$$

$$n_4 = -q\gamma_1(n_1\gamma_2^2 + 2)\sinh(h\gamma_2)\cosh(h\gamma_1),\qquad n_7 = n_1\gamma_2^2\sinh(h\gamma_2),$$

$$n_5 = n_1\gamma_2^2\sinh(h\gamma_2)(\sinh(h\gamma_1) - h\gamma_1\cosh(h\gamma_1)),$$

$$n_8 = n_1\gamma_1^2\sinh(h\gamma_1),\ n_6 = \gamma_1\sinh(h\gamma_2)(-n_1\gamma_1\sinh(h\gamma_1) - 2h\cosh(h\gamma_1)),\qquad n_9 = q\gamma_2\cosh(h\gamma_2),$$

$$n_{10} = (h+q),\quad n_{11} = n_7\sinh(h\gamma_1) - n_8\sinh(h\gamma_2),\quad n_{12} = 2\sinh(h\gamma_2) + n_7,$$

$$n_{13} = 2\sinh(h\gamma_1) + n_8,\quad n_{17} = n_1\gamma_1^2\sinh(h\gamma_1)(\sinh(h\gamma_2) - h\gamma_2\cosh(h\gamma_2)),$$

$$n_{14} = (K\gamma_1^2 - 1)(n_1\gamma_2^2 + 2)\sinh(h\gamma_2),\ n_{15} = (K\gamma_2^2 - 1)(n_1\gamma_1^2 + 2)(-\sinh(h\gamma_1)),$$

$$n_{16} = h\gamma_1(n_1\gamma_2^2 + 2)\sinh(h\gamma_2)\cosh(h\gamma_1),\ n_{18} = -n_1\gamma_2\sinh(h\gamma_2) - 2h\cosh(h\gamma_2).$$