African Journal of Mathematics and Computer Science Research Vol. 5(2), pp. 34-38, 15 January, 2012
Available online at http://www.academicjournals.org/AJMCSR
DOI: 10.5897/AJMCSR11.146
ISSN 2006-9731 ©2012 Academic Journals

Full Length Research Paper

A sampling plan with producer's allowable risk (PAR) at maximum allowable proportion defective (MAPD)

T. B. Ramkumar

Department of Statistics, St.Thomas' College, Thrissur, Kerala, India. 680001. E-mail: rktmidhuna@gmail.com.

Accepted October 20, 2011

This paper deals with sampling designs on incoming quality MAPD along with a specified probability of acceptance at this point called producer's allowable risk (PAR). It is defined as the minimum probability of acceptance of the lot with a maximum allowable proportion defective. Tables and graphs are presented, comparing the efficiency of new SSP as to protect AQL. An optimum criterion of sample size and acceptance number is suggested for a fixed ratio of PAR to MAPD within a plausible sample size region.

Key words: Single sampling plan (SSP), maximum allowable proportion defective (MAPD), operating characteristic curve, inflection point, producer's allowable risk (PAR), optimum sampling plan, decisive distance, steepness angle.

INTRODUCTION

Mayer (1956) and Mandelson (1962) suggested a consumer-producer-engineer friendly product quality – MAPD (p^*) below which the proportion of acceptance of the lot was expected to decline stringently. Following poisson distribution by the number of defectives, the ratio c/n is efficient to divide the good and bad lots for the layman and industrialist. Norman (1953), Soundararajan (1975) Ramkumar and Suresh (1996) were derived some basic operating procedure to locate single sampling plan on MAPD and its properties. Ramkumar (2010) had formed a criterion for developing SSP on interval quality design in terms of MAPD in the p axis alike PAR in the Pa(p) axis. Also Ramkumar (2009) had suggested a sampling plan indexed through MAPD and discriminant distance, indicating the efficiency of OC curve on tangential distance and concept of optimum sampling plan under same operating ratio.

From Figure 1, PAR is the minimum probability of acceptance of a lot of quality having a maximum allowable proportion defective. In particular for a product with an incoming quality of MAPD <10%, more than 60% (PAR) lots were accepted saving the interest of producer. Inflection point is the turning point of OC curve with steepest declination tangent indicating the sharpness of OC curve ensuring protection to the consumers. The steepness angle θ is a sensitive measure to define the discrimination of the required OC curve. This angle will be wider for more stringent OC curve. Thus the parameters were capable of protecting both producer and consumer. So this plan is favorable in customer friendly, moderate costly, inspection oriented products like daily using items.

Selection of the plan

For a defined PAR the decisive distance d can be calculated (Equation 2). Inspect the range in which calculated d or PAR falls uniquely and locate c (greater than or equal to d) from Table 1. Hence for a prefixed MAPD, $n=c/p^*$.The steepness angle θ subtended by d on the OC curve with the p axis at $Pa(p^*)$ and given MAPD can also provide d.(Equation 4).

Example 1

A toy quality is fixed at maximum allowable proportion defective 8% and PAR = 0.70. The decisive distance d = 1 - 0.70 = 0.30. From Table 1, $0.2642 \leq d < .0.3233$ for c=2. Sampling plan (n, c) is (25. 2). And the steepness angle θ = 76.10.

Example 2

A textile company producing cotton and polyester yarn

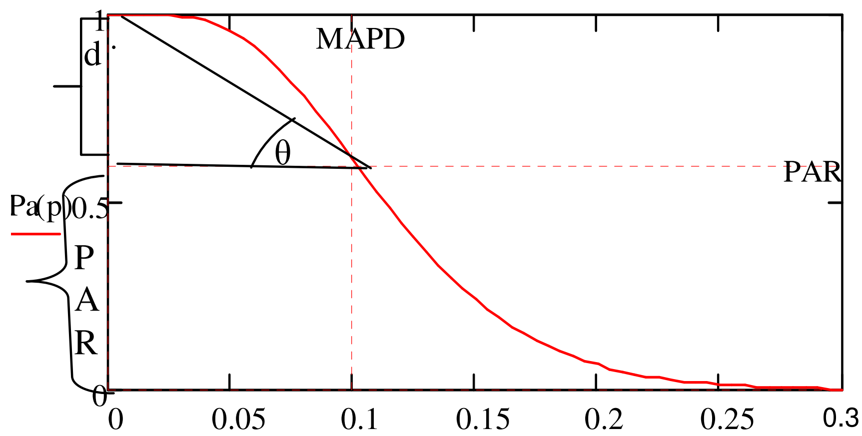

Figure 1. Optimum criterion (OC) curve showing maximum allowable proportion defective (MAPD) and producer's allowable risk (PAR).

Table 1. Values of PAR for c =1 to 20.

c	1	2	3	4	5	6	7	8	9	10
Pa(p^*)	0.7358	0.6767	0.6472	0.6289	0.6159	0.6063	0.5987	0.5926	0.5874	0.5830
d	0.2642	0.3233	0.3528	0.3711	0.3841	0.3937	0.4013	0.4074	0.4126	0.4170
c	11	12	13	14	15	16	17	18	19	20
Pa(p^*)	0.5793	0.5759	0.5731	0.5704	0.5681	0.5659	0.5640	0.5623	0.5606	0.5591
d	0.4207	0.4241	0.4269	0.4296	0.4319	0.4341	0.4360	0.4377	0.4394	0.4409

Table 2. Certain SSP for given PAR (or d) and MAPD.

c	Pa(p*)	d	p*									
			0.01	0.02	0.03	0.04	0.05	0.06	0.08	0.09	0.10	0.11
1	0.7358	0.2642	100	50	33	25	20	17	13	11	10	9
2	0.6767	0.3233	200	100	66	50	40	34	25	22	20	9
3	0.6472	0.3528	300	150	100	75	60	50	38	34	30	18
4	0.6289	0.3711	400	200	133	100	80	67	50	45	40	27
5	0.6159	0.3841	500	250	167	125	100	84	62	55	50	36
6	0.6063	0.3937	600	300	200	150	120	100	75	66	60	45
7	0.5987	0.4013	700	350	234	175	140	116	87	77	70	55
8	0.5926	0.4074	800	400	267	200	160	133	100	88	80	64
9	0.5874	0.4126	900	450	300	225	180	150	112	100	90	73
10	0.5830	0.4170	1000	500	333	250	200	166	125	111	100	82

fixes the MAPD =10% and steepness angle of OC curve as 74 and 68° respectively. Then decisive distances $d_1=p^*.\tan\theta_1 = 0.1 \times 3.487 = 0.3487$ and $d_2=p^*.\tan\theta_2 = 0.1 \text{x} 2.475 = 0.2475$ on OC curve with sampling plans from Table 2 will be (30. 3) and (100.1)

Construction of the plan

Fix p* and Pa (p*) at suitable quality level in an OC curve.

Then

$$\text{Pa}(p^*) = \sum_{r=0}^{c} \frac{e^{-c} \cdot c^{r}}{r\,!} \qquad (1)$$

where the numbers of defectives follow Poisson

Table 3. Certain parametric combinations of PAR, MAPD and Angle θ.

c	d	PAR	p*									
			0.01	0.02	0.05	0.06	0.08	0.1	0.11	0.12	0.15	0.2
1	0.2642	0.7358	θ=87.8	85.67	79.2	77.20	73.15	69.26	67.3	65.57	60.4	52.87
2	0.3233	0.6767	88.22	86.69	81.20	79.48	76.10	72.8	71.2	69.6	65.1	58.25
3	0.3528	0.6472	88.37	86.75	81.93	80.34	77.22	74.1	72.6	71.2	66.96	60.45
4	0.3711	0.6289	88.45	86.55	82.32	80.8	77.8	74.9	73.4	72.08	67.99	61.67
5	0.3841	0.6159	88.50	87	82.58	81.12	78.2	75.4	74	72.6	68.6	62.49
6	0.3937	0.6063	88.52	87.09	82.76	81.3	78.5	75.7	74.3	73.04	69.1	63.06
7	0.4013	0.5987	88.57	87.14	82.89	81.49	78.7	76	74.6	73.35	69.5	63.5
8	0.4074	0.5926	88.59	87.18	83	81.62	78.8	76.2	74.8	73.58	69.78	63.8
9	0.4126	0.5874	88.61	87.2	83.09	81.72	79.02	76.3	75.07	73.78	70.02	64.13
10	0.4170	0.5830	88.62	87.25	83.16	81.8	79.1	76.5	75.22	73.9	70.2	64.23

distribution.

From Figure 1,

$$d = 1 - Pa;\ (p^*) = 1 - \sum_{r=0}^{c} \frac{e^{-c} \cdot c^{r}}{r!} \tag{2}$$

which will be a function of c only, monotonically increasing, so that unique plan holds for each d. Find c matching with the given d (greater than or equal to the nearest d)

$$\text{But } c=np^* \text{. So that } n=c/p^*. \tag{3}$$

Also one can find d from the steepness angle θ opposite to distance d

$$\tan\theta = d/p^* \tag{4}$$

Then d= p* tanθ , can be found for given MAPD and θ.

$$d/np^*=d/c=\tan\theta /n \tag{5}$$

From Pa(p*) or d and θ using (5) tanθ/n is determined and substitute for tanθ ,n can be found or equivalently

$$n= (c/d)^*\tan\theta \tag{6}$$

Construction of table

Table 1 is constructed by substituting c=1,2,.., in(1) and (2). Table 2 represents SSPs for various combinations of p* and d. Find c for each d from Table 1 and then n is determined by (3). Table 3 is the declination angle for various values p* and PAR. Finding d from (2), c is fixed from Table 1 and tanθ is found. It is useful to identify (n,c) for given (p*,θ) Table 4 is the suitable sampling plans at various steepness angles and PA. For PAR and θ, n can be evaluated by the relation d/c=(tanθ)/n from Table 4.

Significance of the sampling plan

Acceptable quality level (AQL) is a quality measure fixing producers risk at 5, 10 and 1% in usual practice. But in an assemblage of components it is not good to prefix only one level of AQL at a fixed producer's risk. Also keeping at a level may exert pressure on the consumers as well as the producer because different items require different levels of acceptance. Thus fixed AQL sampling plans deteriorate the confidence of the vendor and the customer. This is the limitation of all probability based indices with fixed levels as they are inadequate to specify the quality aspiration of the customer. Fixing various level of probability of acceptance at MAPD on the OC curve is called the producers allowable risk (PAR). PAR is probability liberalization to 70, 85 and 93% etc. instead of 95 and 10%. MAPD is an engineer's quality level with steepest declination beyond p*, so that the utmost quality of acceptance is prefixed by MAPD. Thus MAPD and PAR gives a balanced design for both consumer and producer keeping information on steepness of OC. This is the only one point OC plan and may be flexible for various components. The steepness angle produced by d is a direct measure of efficiency of OC curve so the designs are possible in terms of required steepness. Also in the composite production units or in the components, different MAPD and PAR can be fixed. Thus MAPD-PAR sampling plan is a flexible strategy of inspection of different components in a composite product with one point OC curves. Also angleθ and PAR has a strong power of discrimination of lot as good and bad. Considering an example one can show the significance of this sampling plan. Fix the PAR=0.635, (d=0.365) and angle of steepness 80° (1). Then c = 4 and n = 62 with

Table 4. Finding n for given PAR/ (or d) and angle θ.

c	1	2	3	4	5	6	7
d	0.2642	0.3233	0.3528	0.3711	0.3841	0.3937	0.4013
Pa(p*)	0.7358	0.6767	0.6472	0.6289	0.6159	0.6063	0.5987
tanθ/n	0.2642	0.1616	0.1176	0.0927	0.0768	0.0656	0.0573
c/d	3.7850	6.1862	8.5034	10.7787	13.0174	15.2400	17.4433
c	8	9	10	11	12	13	14
d	0.4074	0.4126	0.417	0.4207	0.4241	0.4269	0.4296
Pa(p*)	0.5926	0.5874	0.583	0.5793	0.5759	0.5731	0.5704
tanθ/n	0.0509	0.0458	0.0417	0.0382	0.0353	0.0328	0.0306
c/d	19.6367	21.8128	23.9808	26.1468	28.2952	30.4520	32.5884
c	15	16	17	18	19	20	21
d	0.4319	0.4341	0.436	0.4377	0.4394	0.44	0.4419
Pa(p*)	0.5681	0.5659	0.564	0.5623	0.5606	0.56	0.5581
tanθ/n	0.0287	0.0271	0.0256	0.0243	0.0231	0.022	0.0210
c/d	34.7302	36.8578	38.9908	41.1240	43.2407	45.4545	47.5220

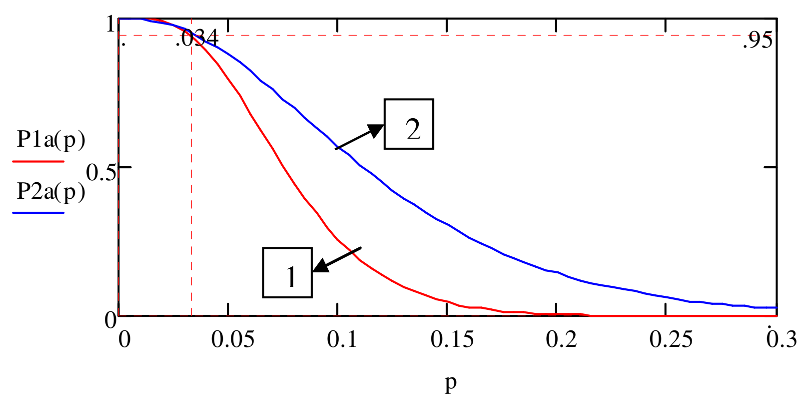

Figure 2. Optimum criterion (OC) curves for fixed acceptable quality level (AQL) defined on producer's allowable risk (PAR) and steepness angle.

MAPD=0.065 and AQL=0.032. For keeping the same AQL, with steepness angle 75°, PAR will be 0.6767, (d=0.3233) (2) and MAPD 0.083 satisfying a sampling plan (24.2) (Figure 2). Thus fixing AQL it is better to adjust PAR and the steepness angle so as to get required quality.

Comparison of OC curves

Figure 2 shows OC curves stringent or moderate at the same quality of AQL which can be finalized on PAR and steepness angle. When PAR is less and angle is more there exists a more discriminating OC curve than higher PAR and lower angle. Also for such parameters MAPD increases on PAR increase. Figure 3 is the feasible acceptance numbers within a sample size 50 to 100 keeping operating ratio d/p* a constant, from which maximum and minimum efficient OC curves can be derived. Figure 4 shows the OC curves satisfying the defined OR=8.0 so that switching rules could be implemented.

Optimum sampling plan

Practically this idea can be used when the producer had little information about the resultant quality, but he is ready to inspect a range of sample size for final decision. Pa(p*) or d is a good measure of efficiency of OC curve, and MAPD is a suitable quality measure so that a ratio of this is fixed as constant and sampling plans were developed on this criteria. There exists a set of plans within a range of sample size and one can start with

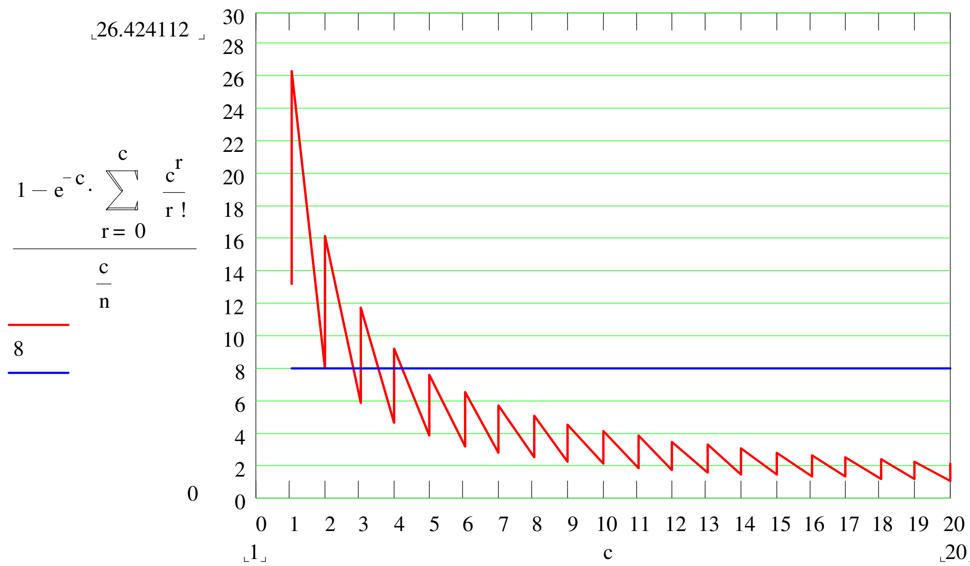

Figure 3. Feasible acceptance numbers for a fixed OR=8.

suitable sampling plan.

Example 3, a yarn industry fix a quality index OR = 8 based on MAPD and decisive distance, provided they are ready to bear an inspection cost of 50 to 100 units. Then what will be the optimum sampling plan?

Since the quality index is fixed at R=8, for a range of sample n = 50 to 100 and c = 1 to 20. Then the various sampling plans obtained were (50.2), (67.3) and (85.4), (Figure 4). Among these sampling plans, optimum is reached at (85.4), and they can start with (50.2) and switch over to (67.3) on successive completion of fixed number of lots. From the OC curves (85. 4), has a optimum probability of acceptance at maximum allowable proportion defective (Figure 4).

Conclusion

The suggestion of this plan is efficient to contain MAPD oriented AQL protection satisfactory for both consumer and producer. Decisive distance and declining angle is a concept to exhibit the quality of OC curve. Switching over of the sampling plans will not highly affect the OC curve even cost reduction is possible.

REFERENCES

Mayer PL (1956). Some Properties of inflection Point of an OC curve for Single Sampling Plan. Proceedings of 2nd Statistical Engineering Symposium, Published in Chemical Corps, Engineering Command.

Mandelson J (1962). The Statistician, Engineer and Sampling Plans. Ind. Qual. Control, 19: 12-15.

Norman B (1953) A method of discrimination for Single and Double sampling OC Curves utilizing of the point of inflection. ENASR Report No.P.R.7 Engineering Agency.

Ramkumar TB, Suresh KK 1996). Selection of sampling plan indexed with Maximum Allowable Average Outgoing Quality. J. Appl. Stat., 23(6): 645-654.

Ramkumar TB (2009). Design of Single Sampling Plan by discriminant at MAPD. Probstat Forum, 2: 104-114.

Ramkumar TB (2010). Designing a Quality Interval Single Sampling Plan. Int. J. Oper. Res. Optim., 1(1): 27-40.

Soundararajan V (1975). Maximum Allowable Percent Defective (MAPD) single sampling inspection by attributes plan. J. Qual. Technol., 7(4): 173-182.