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Students' abstraction in re-cognizing, building with and constructing a quadrilateral

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This study aims to implement empirically students' abstraction with socio-cultural background of Indonesia. Abstraction is an activity that involves a vertical reorganization of previously constructed mathematics into a new mathematical structure. The principal components of the model are three dynamic nested epistemic actions: recognizing, building-with, and constructing. This study identified the abstraction profile of a junior high school student in constructing quadrilateral relationship. The student was asked a question related to quadrilateral. The interview was developed based on her answers by using keywords, "what, how, or why". The result of the student's abstraction and attributes were used to recognize the differences and similarities of quadrilateral shapes, building-with attributes by linking the characteristics of every two quadrilaterals, and constructing a network of relationships among quadrilaterals by drawing a figure of such networks.

Key words: Abstraction, recognizing, building with, constructing, quadrilateral.

INTRODUCTION

Abstraction has been a central issue in mathematics and science education for many years (Kidron and Dreyfus, 2008). Abstraction also has become the object of intensive research in philosophy. Plato saw abstraction as a way of achieving eternal truth, but Russell characterized abstraction as one of the highest human achievement (Hershkowitz et al., 2001).

Hershkowitz et al. (2001) suggested that in order to identify an object as an example of an abstraction, then someone must have a little knowledge about abstraction. Abstraction process starts from the beginning of an abstract entity towards a complex structure. In this study, we need a cognitive mechanism of abstraction, which constructs the existing ideas to be more complex.

Abstraction is a vertical reorganization of mathematical concept activities contracted into a new mathematical structure (Kidron and Dreyfus, 2008; Kouropatov and Dreyfus, 2014). For example, in a construct of the relationship that exists between two shapes (square and a rectangle), a grade nine student has them as an abstraction, because he has learned them previously in class. If the rectangle is described as having "sides of the same length" then it tantamount to the characteristic of a square. He could construct that if quadrilateral is a square, then it is also a rectangle. The results of this construction are more complex than the initial concepts for students.

Guler and Arslan (2015), Hershkowitz et al. (2001) and

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Tsamir and Dreyfus (2002) divided abstraction into three epistemic actions: recognizing, building with and constructing. Recognizing is identifying a mathematic structure in a previous knowledge construct. Building with comprises the combination of recognized constructs in order to achieve a localized goal, such as the actualization of a strategy, justification or the solution of a problem. Constructing consists of assembling and integrating previous constructs by vertical mathematization to produce new construct.

Empirical abstraction is a vertical reorganization of mathematical concept that has previously been constructed to become a new mathematical structure based on Indonesian social-cultural background. But according to the non-classical approach, abstraction is an activity which corresponds to mathematical constructions and a process of constructing knowledge. The advantages of these definitions are in the mathematical concept combined, restructured, organized and built up to be more abstract or more formal. The activities used in these definitions are recognizing, building with and construction, and the context of these definitions is the relationship that exists between quadrilateral and the social background of junior high school students.

The rsearcher intends to identify the profile of a student's abstraction namely "*Prakasita"* in showing the relationship that exists between quadrilateral and the background of junior high school students in Indonesia. This student was selected, because according to the research of Berry and Dasen (1974), background has an effect on cognitive organization individually. In Indonesian culture, there is a high tendency for students to perform poorly in communication, both spoken and written forms. This is different in the western culture, where students are able to communicate their thoughts in spoken or written language and are accustomed to answer the "what, why and how"questions.

Abstraction has been become the focus of many research in various fields, including mathematics education (Hershkowitz et al., 2001). In Indonesia, abstraction is chosen because it has no research related to mathematical abstraction. The material of the research is geometry, because it is difficult for students; and quadrilateral material is a foundation for understanding other geometry topics.

The purpose of this study is to identify the profile characteristic of Prakasita's abstraction in recognizing and understanding the characteristic of quadrilateral, building with two quadrilateral characteristics and constructing the relationships that exist between quadrilaterals.

MATERIALS AND METHODS

Terminology

Eddie and Tall (2007) noted the term 'abstract' has its origins in the Latin "ab" which means "from" and "trahere", "to drag". Gramatically, to abstract (verb) is a process, to be abstract (adjective) is a property and an abstract (noun) is a concept. Abstraction has two definitions:abstraction is the process of describing a situation and abstraction is the conceptof processing result. It is the result and process of reorganizing vertical mathematical concept which has been constructed earlier into a new mathematical structure.

Freudenthal (1991) provided what mathematicians have in mind when they think of abstraction. Freudenthal has brought forward some of the most important insights into mathematics education in general, and to mathematical abstraction in particular. These insights constitute a cultural legacy that led his collaborators to the idea of vertical mathematization (Treffers and Goffree, 1985). Vertical mathematization points to a process that typically consists of the reorganization of previous mathematical constructs within mathematics and by mathematical means, by which students construct a new abstract (Dreyfus, 2015). Reorganizing activity is a process of collecting, compiling, organizing, and developing mathematical elements into a new element. Vertical reorganizationis a reorganizing activity which changes an abstract form into a more abstract form or more formal than the original.

According to Bikner-Ahsbahs (2014), Celebioglu and Yazgan (2015), Hershkowitz et al. (2007) and Yilmaz (2014) reorganization of mathematical structures occurs through three epistemic actions: recognizing, building-with, and constructing which can be distinguished in any processes of abstraction. Reorganizing is an activity of identifying the characteristics of a quadrilateral. Building with is an activity of combining the characteristics of two quadrilaterals.

Construction is an activity of reorganizing the characteristic of the quadrilateral into a new structure not owned by students. Re-orgnizing, building with and construction activities do not always have a linear shape, but they can occur at the same time.They have been validated and useful for describing and analysing the processes of abstraction of other contents, in other social settings and other learning environments. They have been established by a considerable number of research studies incuding ours (Bikner-Ahsbahs, 2004; Dreyfus and Kidron, 2006; Ozmantar and Roper, 2004; Ron et al., 2006; Stehlíková, 2003; Tabach and Hershkowitz, 2002; Tabach et al., 2001, 2006; Tsamir and Dreyfus, 2002; Williams, 2002, 2003, 2004, 2005; Wood and McNeal, 2003; Wood et al., 2006).

The profile of abstraction is a natural picture of the vertical reorganization of mathematical concept which is constructed earlier to becoming a new mathematical structure (Ergul, 2013; Halverscheid, 2008). Network of relations between a quadrilateral is a representation of the relationship that exists between two shapes that are gridded, charts, graphs, or schema. The diversity of abstraction lies in their differences or similarities; abstraction can either be a process or a result. The quadrilateral network relationship is parallelogram, rectangle, rhombus, square, kite, trapezoid. The representation of a quadrilateral is the shape of a "framework".

Symposium of the American Educational Research Association (AERA, 2004) postulates that four alternative conceptions of abstraction have emerged in an effort to formulate abstraction in a way that is compatible with a situated cognition perspective:

1. Situated abstraction
2. Abstraction in context
3. Collective abstraction, and
4. Actor-oriented abstraction.

Situated abstraction highlights the central role of mediating tools and in particular symbolic tools (Noss and Hoyles, 2002). The artifacts and symbol system have an important meaning in mathematics. Mathematical knowledge can be tied to the ways in which it is learned and used in a socio-cultural practise, yet simultaneously it can be expressed in ways that exhibit invariant mathematical relationships (Hoyles et al., 2001; Noss and Hoyles, 1996; Noss et al., 2002).

While Noss and Hoyles et al. (2002) developed their notion of situated abstraction in relation to the conceptual resources students already have at their disposal, Hershkowitz et al. (2001) developed the idea of abstraction in context by saying that knowledge is constructed within a social practise. They argued that abstraction is a vertical reorganization activity of mathematical concept which had been constructed earlier into a new mathematical structure. Collective abstraction is an activitywhere members of a community collectively contribute at the beginning or end of activities in real terms. In abstraction, everything that is done before and after successively become the object of reflection (Cobb, 2004).

Actor-oriented abstraction is a modification of the two reflective aspects of Piaget (Lobato, 2004). First, effective abstraction is a construct of individual psychology and does not explicitly contribute to the environment, artifacts, curricular tasks and other activities of abstraction. The second is high-level reflective abstraction, which involves decentralization. Abstraction-oriented approach uses the concept of focusing attention on coordinating social and individual levels of abstraction. In abstraction, individuals identify the regularity of mental activity records with a focus and isolate the important properties required, as well as remove properties that are not needed. Social abstraction involves identifying notes, with focus namely on mathematical properties or order emerging as a result of the focus of students when interacting with the environment such as diagrams, strategies and representation.

The main idea of cognitive theory, context is seen as a task or other characteristic of the experimental conditions which is considered to affect the occurrence of mathematical thinking. There are abstraction in condition, abstraction in context, and abstraction-oriented actors in different contexts. Noss and Hoyles (1996) give an idea of the context and symbolic roles and artifacts that are generally used as a means of action and communication. Cobb (2004) takes context into collective class where students participate in and contribute to the collective class. Lobato (2004) shifted from viewing context from the point of view of the researcher as something inherent in the situation where the researcher can manipúlate in considering context from the point of view of the actor.

Davydov (1990) suggested the beginning of abstraction consists of three parts. First, abstraction stems from its early form, the simple, not yet to be developed form; there needs to be consistence both the internal and external. Second, the development of abstraction in the progress of the analysis, from the early stages of abstraction, towards synthesis, and finishing with a final form that is consistent and complicated. Third, abstraction does not run from the concrete to the abstract, but from an abstract form that has not evolved into an abstract shape that develops. The definition of abstraction as an activity is in line with mathematical constructions.

The relationship between quadrilaterals

Definition is an important part of geometry. According to Soedjadi (2000), the definition of a concept is "a phrase that can be used to limit the concept". Quadrilaterals such as parallelogram, rectangle, square, rhombus, trapezoid and kite are examples of concepts, while ‘’a parallelogram is a quadrilateral which has a pair of opposite sides equal" is an example of definition. This definition limits the concept.

Soedjadi (2000) distinguishes definitions into three; they are analytic, genetic and formula definitions. In geometry, formula definition is used. An analytic definition mentions genus Proximum (immediate family) and deferential specifically (special distinction). The definition of parallelogram above is an analytic definition of the genus proximum "quadrilateral" and deferential specifically, "has a pair of opposite sides equal". Genetic definition is a definition that indicates or reveals the occurrence or the formation of the concepts defined. An example of genetic definition is "kite is a quadrilateral shape if two isosceles triangles are congruently combined with pedestal base". There are four elements of the definition: background, genus, defined terms, and attributes. From the example of parallelogram definition above, the background is shaped, the genus is quadrilateral, defined terms are parallelogram, and the attribute is a pair of parallel opposite sides.

Definitions used in the quadrilateral have an impact on the relations between shapes. If the trapezoid is defined as, "a quadrilateral has exactly one pair of parallel sides" or "quadrilateral pair of parallel sides", then both different definitions will have an impact on the relations between the shapes. If the first definition is used, then the set of parallelogram and the set of trapezoidal are disjointed, but if the second definition is used, then the parallelogram set is a subset of the trapezoidal set. Parallelogram can be defined as follows:

1. Parallelogram is a quadrilateral with two pairs of opposite sides parallel
2. Parallelogram is a quadrilateral with two pairs of opposite sides of equal length; and
3. Parallelogram is a quadrilateral with a pair of opposite sides parallel and of equal length.

These definitions are the same. According to Soedjadi (2000), these definitions have an extension (reach) that is equal; and two or more definitions that have equal extension is called definition equivalent. Poespoprojo (1999) said that extension is the whole of an idea that can be applied or an environment (a concept) that may be appointed by the concept. Attributes are used when an object:

1. Has two pairs of sides that are parallel
2. Has two pairs of sides of the same length
3. Has a pair of sides that are parallel and equal in length.

However, according to Soedjadi (2000), it has a different definition. The definition of parallelogram constructed by the student is said to be accurate if it is equivalent to the definition earlier started. A rectangle can be defined as follows:

1. Rectangle is a quadrilateral that has two pairs of opposite sides equal and a right angle
2. Rectangle is a quadrilateral that has two pairs of opposite sides of equal length and a right angle; and
3. Rectangle is a quadrilateral that has a pair of opposite sides parallel and equal in length as well as a right angle.

Thus, these definitions have equal extension but different intention. Rhombus, square, trapezoid, and kite are defined as follows: rhombus is a quadrilateral that four sides of equal length, square is a quadrilateral that has four sides of equal length and a right angle. Kite is a quadrilateral that two pairs of adjacent sides of equal length with the sides not overlapping; trapezoid is:

1. Quadrilateral that has a pair of opposite sides equal; or
2. A quadrilateral that has exactly one pair of parallel sides.

If the analytical definition is used, then the parallelogram is a rectangle that is a right angle; rhombus is a parallelogram whose four sides are equal or kites whose four sides are equal; and square is a rectangle whose four sides are equal or square is a rhombus with right angles. If the definition of a trapezoid is a

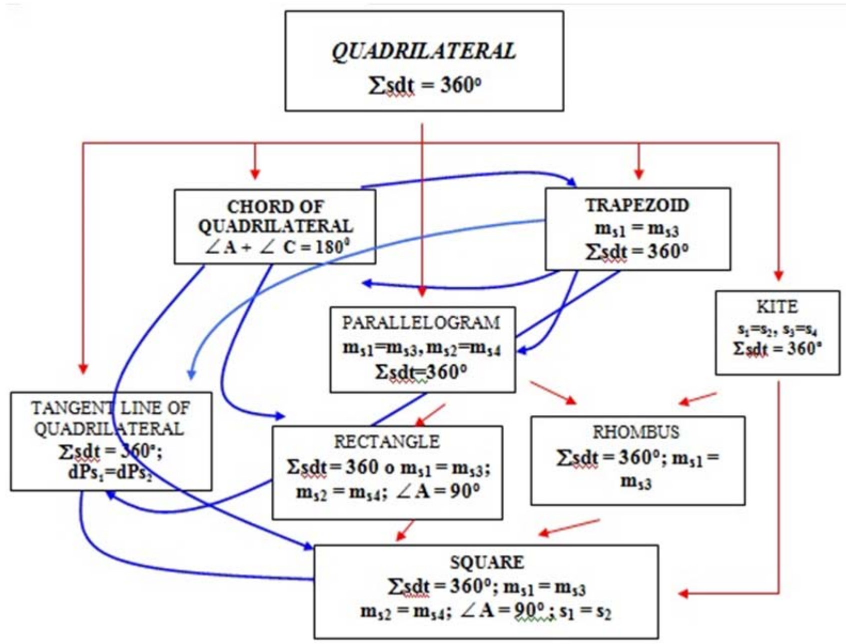

Figure 1. Map concept of quadrilateral (based intension definition).

rectangular with a pair of opposite sides equal, then a parallelogram is a trapezoid with two pairs of parallel sides. The map concept was strongly influenced by the sound definition (semantic) used or preferred relationships. Given quadrilateral ABCD, $\overline{AB} = s_1$, $\overline{BC} = s_2$, $\overline{CD} = s_3$, then $\overline{AD} = s_4$ and with gradient respectively m_{s1}, m_{s2}, m_{s3}, m_{s4}. If P is the center of the circle in quadrilateral ABCD, then dP_{s1} is the distance from P to the side s_1. Budiarto et al (2017) also described quadrilateral relationship in Figure1. Red colour indicates analytical definitions; while blue indicates a result that is related to the red color.

The diagram in Figure1 shows that the position of the quadrilateral chord and the trapezoidal are equal, because both definitions of quadrilateral have two requirements. Likewise, parallelograms and kites are at equal level, because both definitions of quadrilateral have three requirements. Quadrilateral line tangent, rectangle and rhombus are also equal, because the definitions of quadrilateral have four requirements. Square is at the lowest level because its definition has five requirements. Some results of drawing charts that consider position or level are:

1. If one condition of quadrilateral chord is added, then it would be trapezoid.
2. If three conditions of quadrilateral chordare added, then it would be a rectangle.
3. If the four conditions of quadrilateral chord are added, then it would be a square. Likewise, it is a trapezoid if it needs one requirement to be a quadrilateral chord or a parallelogram; if it needs three requirements, then it is a quadrilateral tangent line.

METHODOLOGY

This is a qualitative research which explores students' abstraction in constructing the relationships between quadrilaterals. The subject of the research is Prakasita, a grade 9, Junior high school of Hang Tuah 1 Surabaya, Indonesia. The data of the research were collected by using a task-based interviews (Tsamir and Dreyfus, 2002).

Recorded clinical interviews and audio visual equipment were used as a data collection tecnique. Clinical interviews were used to collect information of a subject's abstraction as a material to draw conclusions. The instrument of the data in this research was the researcher and the supporting instrument was the interview guideline. The interview guidelines were carefully planned and tested on some students from Junior High School Laboratorium of Universitas Negeri Surabaya and from junior high school in Surabaya.

The students were able to recognize quadrilaterals if they could identify their different and similar characteristics and understanding their definitions. They were able to build with the characteristics of two rectangles by combining the characteristics of the two rectangles. For example, by combining the characteristics of a square and a rectangle, they were able to get a square. The students were ableto construct, if they reorganized the characteristics of two rectangular structures to become new structures. For example, the students have builtthe relationship that exists with a rectangle and a square and constructed a network that connects the two shapes.

Attributes are said to be true if the definition of mathematics it is true or have an equivalent with related models of planes. Attribute is said to be non-routine if the attribute was not commonly used in mathematics textbooks to build understanding of the quadrilateral, such as "the diagonal is perpendicular" or "having two axes of symmetry". Attributes are not meaningful if generally attribute does not build understanding of quadrilateral, such as attribute "has an acute angle", "has a hypotenuse", "resembles a rhombus" or "adjacent sides are not equal". The definition of a quadrilateralis accurate, if the attributes used to identify the definition are appropriate. Such as "a rectangle is a quadrilateral with two parallel pairs and a right angle" is an accurate definition. If the results of a series between two shapes and analytically accurate definitions of the subject is more than any other subject, so this subject has better abstraction.

Table 1. Attributes used to distinguish and re-organize the similarity of some models of quadrilateral.

Name	Attributes To recognize	
	Difference	**Similarity**
Parallelogram	The length of side; The size of angle	Two pairs of opposite sides parallel; Two pairs of opposite sides are equal
Rhombus	The length of side	The opposite of angles are equal; The length of two pairs of opposite sides are equal; The length of four sides are equal
Rectangle	Length; Width	Two pairs of opposite sides are parallel; All of angles are right angles
Square	The length of side	Two pairs of opposite sides parallel; The length of two pairs of opposite sides are equal; All of sides are equal; All of angles are right angles
Kite	The length of side	A pair of opposite angles are equal; Two adjacent sides are equal in lengts
Trapezoid	The type of trapezoid	Have 4 of sides; One pair of opposite sides parallel

The analysis process was done after the interview was completed. The analysis began by examining the data, then comparing the data with the transcript of the video recordings. The next step was reduction data, collating data, categorization, coding, examination of data.

RESULTS

The activities of plane are grouped into two phases:

1. Grouping of plane into triangular and quadrilateral groups. Attributes used to classify them were the number sides.
2. Planes were grouped into parallelogram, rectangle, rhombus, square, kite, trapezoid, and irregular quadrilateral. The attribute used to classify them was the name of the shape.

Attributes were used to distinguish some models of quadrilateral and re-organize the similarity of some models of quadrilateral are presented in Table 1. According to Prakasita, the definition of quadrilateral are:

1. Parallelogram was a quadrilateral that had two pairs of opposite sides parallel and equal in length
2. Rectangle it is a quadrilateral with two pairs of opposite sides that are parallel and equal in length, and also had four right angles.
3. Rhombus is a quadrilateral with four sides of equal length
4. Square is a quadrilateral with two pairs of opposite sides that are parallel and equal, hasfour right angles and four sides that are equal.
5. Kite is a quadrilateral with two pairs of adjacent sides that are equal in length; its sides do not overlap, and has a pair of opposite angles that are equal.
6. Trapezoid is a quadrilateral with parallel opposite sides, but are not equal.

The results of building with process presented by Prakasita were:

1. Rectangle should not be called only parallelogram, but is parallelogram with four right angles; so rectangle is parallelogram, but parallelogram is not rectangle.
2. Rhombus is parallelogram with equal four sides; rhombus is parallelogram, but parallelogram is not always rhombus. 3. Square should be called a parallelogram, because a square has four equal sides and four right angles; so square is parallelogram, but parallelogram is not square.
4. Kite is not parallelogram and parallelogram is not always a kite.
5. Parallelogram is not a trapezoid and trapezoid is not always parallelogram.
6. Rhombus is not a rectangle and rectangle is not always a rhombus.
7. Square should be called a rhombus with four right angles; so square is a rhombus but a rhombus is not always square.
8. Rhombus is a kite and kite is not always a rhombus.
9. Rhombus is trapezoid and trapezoid is not always a rhombus.
10. Rhombus is trapezoid and trapezoid is not always a rhombus.
11. Rectangle is not a kite, and kite is not always a rectangle.
12. Rectangle is a trapezoid and a trapezoid is not always a rectangle.
13. Square is a kite and kite is not always a square
14. Square is a trapezoid and a trapezoid is not always a square; and
15. Trapezoid is not always kite and kite is not always a trapezoid.

Prakasita recognized the characteristics and definition of quadrilateral, built with the characteristics of two rectangles, then she constructed inter quadrilateral network of relationships (Figure 2). The arrows from A to B show the characteristics possessed by shape B are included in shape A. The number on the arrow indicated the sequence activities of quadrilateral relationship network creation.

Based on analytical definition, there was a decrease in

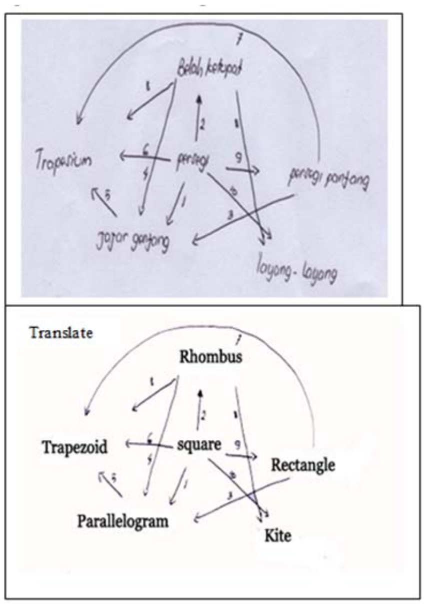

Figure 2. Network relations between the quadrilateral maded by Prakasita.

the relationship from 17 possible relationships into 8 possible relationship. It happened because a genus was used, but it was not Proximum that was used. Prakasita defined trapezoid as a quadrilateral with a pair of parallel sides and a kite is a quadrilateral with two pairs of equal adjacent sides that do not overlap. Therefore, it could be interpreted that Prakasita defined quadrilateral analytically.

1. Rectangle is a parallelogram with a right angle.
2. Rhombus is a kite with four equal sides.
3. Square is a rhombus with a right angle.
4. Rhombus is parallelogram with four equal sides.
5. Square is a rectangle with four equal sides.
6. Parallelogram is a trapezoid with two pairs of parallel sides.

There are 21 possible relationships of quadrilateral between parallelogram, rectangle, rhombus, square, trapezoid and kite. Based on these possibilities, there are only 17 possible relationships. This is caused by the definition of trapezoid as a quadrilateral with a pair of parallel sides. However, Prakasita mentioned 11 out of 17 possible relationships. Six relationships not mentioned by Prakasita are those between quadrilateral with parallelogram, rectangle, rhombus, square, trapezoid and kite.

Prakasita could accurately recognize 1 definition, build relationship of two quadrilaterals including 11 relations of shapes, and construct 4 analytic definitions indicated with the blue line. If the accuracy of the definition is not considered, then she could give 6 definitions, build relationship of two quadrilateral including 11 relations of shapes, and construct6 analytic definitions indicated with the dotted lines. Prakasita could recognize a possible definition, the relationship of two rectangle, and give analytic definition indicated with red lines (Figure 3).

DISCUSSION AND CONCLUSION

Based on the aspect of psychology, personality, talent and behavior, Prakasita had an important role in the interview process. Prakasita has tremendous cognitive and communicative abilities in the interview process. In some conditions, Prakasita has a higher thinking level than hers friends. Her mathematical understanding is very good, such as: she explicitly stated that the formula of trapezoid could be used to calculate the area of shapes that have the equal characteristics with a trapezoid having a pair of opposite sides that are parallel. Prakasita showed thatthe formula of area trapezoid can be used to calculate other shapes with equal characteristic with it.

When comparing the equal characteristics, Prakasita explained what to do and why to do it. Specifically, Prakasita could reflect on what she done without the help of the interviewer. She could progress beyond what is expected; like the formula of trapezoid area could be used to find the area of parallelogram, rectangle, square and rhombus, as described below.

In Figure 4, Prakasita could use the analogy of a trapezoid area in determining the other areas of quadrilateral. This is in line with the research work of Black and Solomon (1987), where they found that analogies helped students to learn. They interpreted this finding from a constructivist view. Analogies were helpful because they allowed the students to construct their own knowledge by forcing them to view the new knowledge within the framework of the analogy.

Based on the first didactic aspect, Prakasita still used the model shape in abstraction. Therefore, in the study of geometry, the students still need learning tools, especially students who have the same character with Prakasita. Second, in making a network between two shapes, Prakasita defined trapezoid as a quadrilateral with a pair of parallel sides. In real learning process at school, understanding the definition of trapezoid could be used is quadrilateral that the exactly pair of parallel sides. Therefore, in learning trapezoid, the teacher should explain that both definitions are true.

The subject created relationships between the areas of trapezoid are presented in Figure 5.

Prakasita worked on two levels consistently. She responded to the questions from the interviewer and analyzed them. She tried to find the hidden connections between the areas of a trapezoid with the areas of other

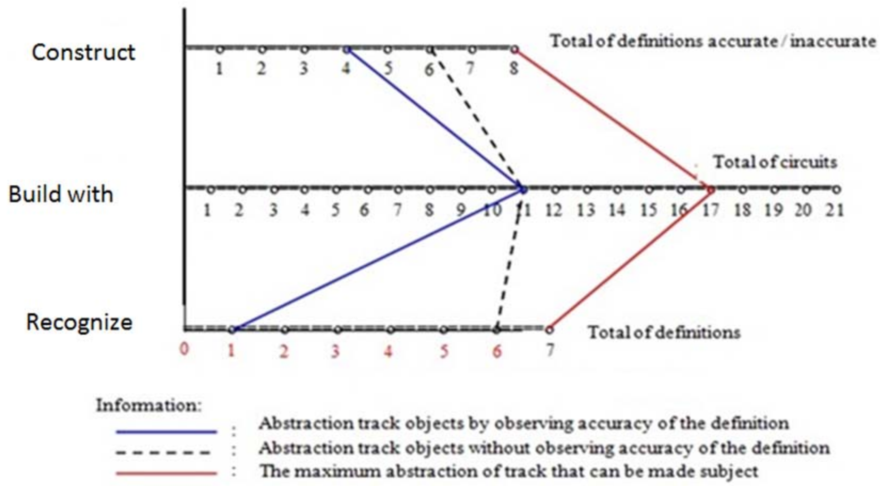

Figure 3. Profile of abstraction Prakasita (Inaccurate definition caused by excessive attributes, both routine and non-routine).

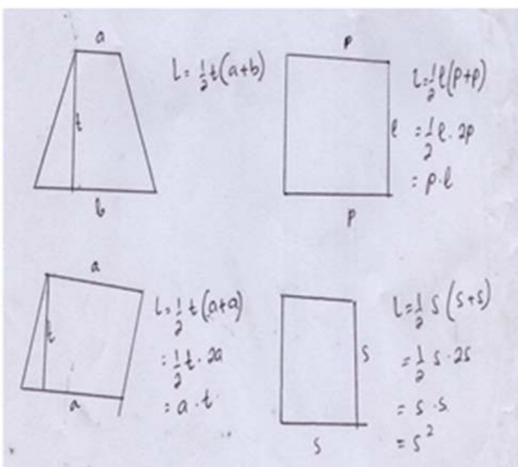

Figure 4. Prakasita determined the formula of area parallelogram, rectangle, square and rhombus.

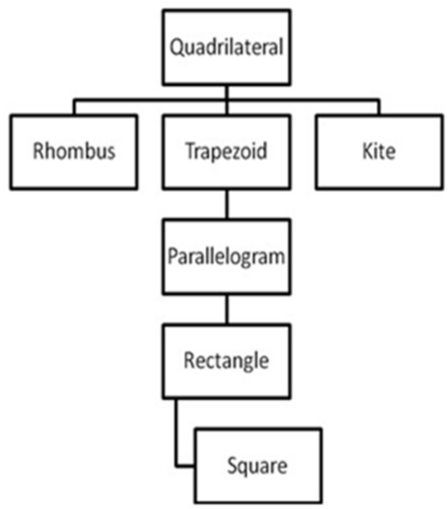

Figure 5. Relationships between shapes with the area of trapezoid made by Prakasita.

shapes which have equal characteristics with trapezoid. She believed that she was directed to a destination, but had no idea about it. When she finally realized that she was directed to make a network of relationships quadrilateral, she identified the related information that is not simple. However, based on the interview result, she not only provided information about the didactic validity of the teaching interview-based, but also showed sufficient detail of the abstraction during the interview.

Based on the theoretical, the analysis of the subject showed that Prakasita's abstraction process was nested. In this process, Prakasita recognized the structures which she constructed and assembled these structures to fulfill what was asked in the interview process. The design of the interview was aimed to create a network of relationships of quadrilateral and offered an opportunity to exit recognized knowledge and construct new structures for Prakasita. As it is known, establishing relationships between variables indicates that mental activities are used. With the fact that abstraction covers the processes that require new structures, constructing new abstract phenomena (Dreyfus and Tsamir, 2004) is taken into consideration. It is observed that abstraction was realized in the study. In this activity, Prakasita recognized, built with and constructed two quadrilaterals relationships which are not nested but more like a series of chain. In other words, constructing, recognizing and building with are linear activities.

Other research results indicated that Prakasita tend to use the rectangle and a parallelogram model, so that rectangle could not be called only a parallelogram. As the characteristic of rectangle is in a parallelogram, Prakasita argued that the rectangle is a parallelogram and parallelogram could not be called a rectangle. She clearly distinguished between the names of shapes and relationships of the equal characteristics of the two shapes. But when determining the relationship between the trapezoid and parallelogram, rectangle, square and rhombus, she suggested that a rectangle could be called a trapezoid and a rectangle is a trapezoid. These results indicated that there is a change in Prakasita's abstraction. Based on the category of Alessi and Trollip (1985), a principle is physical when physical changes are to be observed by the learner, like the case of Prakasita's simulation. All other procedures and principles are non-physical. Generally, Prakasita uses iconic procedure and not symbolic procedure.

This study has not revealed the transitive characteristics of network connections created by Prakasita, such as if the characteristics of shape A are owned by shape B and the characteristics of shape B are owned by shape C, then the characteristics of shape A are owned by shape C. The researcher did not look at the personal background of Prakasita, because of the limited data that could be collected. Teachers' background and the geometry learning process of the student have not been revealed. If those processes are done, the result of the abstraction profile of Prakasita in constructing quadrilateral relationship will be different from the result of this study.

Conflicts of interest

The author has not declared any conflict of interests.

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