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Mathematics teachers' criteria of dimension

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The aim of the study is to determine mathematics teachers' decisions about dimensions of the geometric figures, criteria of dimension and consistency of decision-criteria. The research is a qualitative research and the model applied in the study is descriptive method on the basis of general scanning model. 15 mathematics teachers attended the research. Familiar geometric objects were given and asked to decide their dimensions. The question of "how do you decide the dimension of a geometric object" was used to determine their criteria of dimension. Descriptive analysis was conducted for this question. The results revealed that half of the mathematics teachers indicated a criterion. The criteria of area-volume and length-width-height were common used criteria. It was seen that their criteria and decisions weren't consistent due to lack of their knowledge of geometric object and concepts of area and volume.

Key words: Criteria of Dimension, Concept of Dimension

INTRODUCTION

Even though the concept of dimension has an important role amongst mathematical ideas (Manin, 2006), it has not been emphasized enough in course books (Skordoulis et al., 2009). In order to explain the concept of dimension, the properties of geometric terms such as point, line, surface, plane, area, volume, region, width, length, height, thickness, depth besides planar and spatial geometric objects must primarily be known. There is more than one definition of dimension in mathematics. The variations on the concept of dimension begin with the definition of Euclid's boundary notation (Manin, 2006). In Euclid geometry, an object has a characteristic in terms of having a length, width or height and if an object has only length, it is 1-dimensional; if the object has both length and width; it is 2-dimensional; if the object has length, width and height, it is 3-dimensional.

Mathematicians began reflecting on the concept of dimension when Poincare defined topology in 1905 (Alexandroff, 1932, 1961; Pears 1975). Until then, the concept of dimension was discussed by an experimental point of view and it was thought that the dimension of an object was related to its spreading of various directions (Skordoulis et al., 2009). According to this definition, a straight line is 1-dimensional, if an object spreads perpendicularly to the first line in a second direction, it is 2-dimensional and if it spreads perpendicularly to the first two lines in a third direction, it is 3-dimensional (Devlin, 1994). This definition is parallel with Euclid's definition. The first systematic didactic approach to the concept of dimension was made by Freudental. Freudenthal (1983)

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noted that the concept of dimension could be addressed from the perspectives of plane geometry, analytic geometry-analysis and topology. In topology, if each element of a set N in space is the centre of a sphere whose all points are in the set N, this set N is called a neighborhood. For instance, inner region of a cube is a neighborhood. The boundary of an N neighborhood is the set of all points which are not belong to N, but the centre of small spheres containing some of the points of N. For instance, the boundary for the inner of a cube is six of its surface. According to these concepts, the definition of dimension as follows: Topologically, a set S of points of space is at most n-dimensional if each point of S lies in arbitrarily small neighborhoods whose boundaries have at most (n-1) dimensional intersections with S and the set S is n-dimensional if it is at most n-dimensional (Menger, 1943). From this perspective, it is understood that linear shapes are 1-dimensional (line, rectangle, circle, curve, etc.), surfaces are 2-dimensional (sphere, circular region, prism, plane, polygonal regions, etc.) and solid objects are 3-dimensional (ex. spherical region, cylindrical region, prismatic region).

According to Euclid's boundary notation, the ends of a line are points, the boundary of a surface is a line, the boundary of an solid object is surface. Topologically, a surface is the boundary of a solid body, a line is the boundary of a surface and points are the boundaries of lines (Skordoulis et. al., 2009). The bounded region and boundary are different sets, and their dimensions are different. The boundary of an object is one dimension less than the dimension of the object that it bounds: A line is bounded by a point, a region is bounded by a line and a volume is bounded by a surface (Jackendoff, 1991). In this context, for example, for a sphere is the boundary of a spherical region, we can say that a sphere is 2-dimensional and the spherical region is 3-dimensional. If we say that a sphere is 3- dimensional then a spherical region must be 4-dimensional. Similarly, it can be said that prisms, cylinder and cone are 2-dimensional, prismatic, cylindrical and conic regions are 3-dimensional.

In terms of differential topology, the dimension of an object is defined as the number of independent parameters used for defining the object (Freudenthal, 1983). In terms of Descartes' coordinate system, a line consists of a movable point, a surface consists of a movable line (not just a straight line) and a space consists of a movable surface (Skordoulis et al., 2009). Descartes' approach to the concept of dimension is in terms of coordinate system and dimension is defined as the number of the independent variables used for defining the object. However, the thought that must be based on here is coordinate system and there are also other coordinate systems expect from rectangular coordinate system. Therefore, if it is approached in terms of all these systems; the dimension of an object can be defined as the minimum number of coordinates needed to specify each point within it. Thus, a surface such as a plane or the sphere is 2-dimensional because two coordinates are needed to specify a point on it (for example, to locate a point on a sphere, its latitude and longitude is needed). The inside of a cube, a cylindrical region or a spherical region is three-dimensional because three coordinates are needed to locate a point within it. Therefore, regardless of its position in the space, we can say that all linear geometric objects (polygons, line segment, curve) are 1-dimensional; all surfaces are 2-dimensional and all solid objects (e.g. spherical region, prismatic region, cylindrical region) are 3-dimensional. This case was also stated by Jackendoff (1991).

Only two researches have been observed on the concept of dimension in education area. A study on candidate teachers' anticipations of the dimensions of some planar objects on Euclidean plane and analytical plane was done by Skordoulis et al. (2009). In this study, it investigated as to what extent the reasons of the difficulties encountered by students in anticipating the dimension of an object are affected either by Cartesian coordinate system or misconceptions. In addition, it was also discussed that whether coordinate system has an epistemological difficulty or just a didactic character.

In the study, it has been identified that;

1. 96.5% of the students identified a line segment as 1-dimensional,
2. They could estimate well the dimension of an object without being affected by whether a curve was drawn freely or in a regular form,
3. Coordinate system did not affect in the case of the students' guessing the dimension of the line segment when it is placed horizontally on coordinate system, but in the case of drawing the figure freely, better results have been obtained than the results when the figure was in coordinate system,
4. Better results have been obtained in students' guessing if the line segment given in the same coordinate system was given parallel to the x-axis than it was given angularly,
5. Students have better estimations if the curve was not given in coordinate system,
6. Consequently, in most cases, it was determined that the coordinate system affected students' ability in identifying the dimension of the given objects .

There was also study about students' perceiving dimension made by Vitsas and Koleza (2000). In this study, the rate of identifying the dimensions of geometric figures by mathematics students and also students' geometrical dimension criteria were researched. That the x-y Cartesian coordinate system has effect in students' determining the dimension of the figure and that students make better guesses about the same figure in the simple Euclidean plane were established.

There are some differences amongst the publications about dimension. For instance, in National Council of Teachers of Mathematics, Principles and Standards for School Mathematics (2000) and in Georgia Mathematics Performance Standards 2009-2010; it is stated that square, rectangle, angle, circle and polygon are two-dimensional; and some surfaces such as cylinder and sphere are three-dimensional. In the Turkish Education Ministry Primary Mathematics Education Curriculum (2004), it is stated that square, rectangle, angle, circle and polygon are one-dimensional and that surfaces are two-dimensional and that some figures such as cylinder and sphere are three-dimensional. Some misusages of geometry terminology or different definitions about dimension criteria may lie behind such different point of views. For instance, the usage or differentiation of different terms such as square and square region without care or not understanding that square and square region are different dimensional can create a complication about the dimensions of these two objects. On the other hand, it is generally thought that the number of variables used for defining the points of an object in rectangular coordinate system shows the dimension of that object. Thus, there can be a thought that square and square region are not different in the dimension aspect since two variables are needed to define both of their points. In the same perception, sphere and spherical region can be seen with the same dimension. Similarly, for example sphere and spherical region can be also seen as the same dimension. However, as the same in square and square region (one has surface while the other not), while sphere does not have a volume, spherical region has a volume and so they have different dimension.

Except for pure mathematics, only those two studies mentioned above have been made about dimension concept in the mathematics education area. In these studies, students' knowledge about straight line and curve was examined in that whether being on the Cartesian plane is related to it. In the current study, mathematics teachers' decisions about the dimension of 0, 1, 2 and 3 dimensional geometrical objects were determined and also, the criteria used while deciding were revealed. It is expected that the results of this study will contribute to the rethinking of the dimension concept and also that this study will be a source for instructors and students in clarifying their idea about the dimension concept and criteria in school mathematics.

Prior to commencement of this study, the math curriculums of different countries, academic or non-academic articles have been examined within the scope of the concept of dimension. As a result, it has been observed that there are different understandings on the concept of dimension; its significance is not known very well and there is not sufficient number of academic studies on neither teachers nor students in this subject. The aim of this study was both contributing to fill the gap in the literature and examining the perspectives of mathematics instructors in terms of the concept of dimension. In addition, there are some recommendations regarding how the concept of dimension should be taken into account during the mathematics education process.

METHOD

In this study, it has been aimed to determine the knowledge of math teachers regarding the concept of dimension in particular. For this purpose, they have been asked that "how many dimensions the well-known geometric figures have" and also "how do you decide on how many dimensions a geometric figure has". In addition, the conflicts between their decisions and criteria are also given. Qualitative research aims to answer the 'what', 'why' questions. Therefore, the current research is a qualitative study and the model applied is descriptive method on the basis of general scanning model. The obtained findings were presented in frequency tables on the basis of the teachers' decisions and criteria of dimension.

Participants and Instrument

The sample of the study consists of 15 mathematics teachers, attended to the study voluntarily, who work at primary and secondary schools in Burdur center. In order to get the teachers' decisions relating to number of dimensions of geometric figures, a test was prepared including eighteen 0, 1, 2 or 3 dimensional geometric figures whose geometric shapes are well-known. In this test, the teachers were asked to mark the dimensions of the figure given. In addition, in order to discover the teachers' dimension criteria, they were also asked to answer the question "how do you decide on how many dimensions a geometric figure has?" Considering that the teachers could not complete the tests during the school time, they have been delivered to the teacher individually and collected back again when they are finished. The teachers have returned their tests in written form.

Data analysis

In the analysis of the teachers' answers in respond to the question of "how do you decide on the dimension of a geometric figure?" descriptive analysis method was used. Each teacher's dimension criteria were recorded, and then their criteria were categorized in terms of making decisions on the dimensions of 0, 1, 2 and 3 dimensional figures. Internal reliability was checked with an academician in the field of mathematics education. He was given a copy of participants' responses and asked to write down what each student' criteria of dimension was. Afterwards, by coming together, these results were compared to each other. The differences were cleared up by reaching a consensus.

RESULTS

In the first question, eighteen geometric figures with various dimensions were given, and the teachers were asked to mark the dimensions of them. The results on this question are given below.

Results on the 0-dimensional figures

In order to determine the teachers' decisions of dimension related to 0-dimensional geometric objects, dimension of"point" and "corner of a prism" were asked. The data regarding these questions is presented in Table 1.

As it can be seen on the Table 1, 14 teachers could identify that the point is 0-dimensional and 12 students

Table 1. The rates of dimension-identification relating to the 0-dimensional figures.

	0-dimensional	1- dimensional	3- dimensional
Point	14	1	0
Corner of prism	12	1	2

Table 2. The rates of dimension-identification relating to the 1-dimensional figures.

	0-D	1- D	2- D	3- D
Line segment		14	1	
Line		14	1	
Angle	1	8	4	
Parabola		8	6	
Circle		9	6	
Triangle		4	11	
Pentagon		4	10	
Rectangle		4	11	
The figure consisting of only the edges of rectangular prism		5	5	5

Table 3. The rates of dimension-identification regarding the 2-dimensional figures.

	0-D	1- D	2- D	3- D
Square region			15	
Triangular region			15	
Circular region	1	11	13	
Sphere			2	13
Rectangular prism			2	12

could identify that the corner of a prism is 0-dimensional. On the other hand, 2 teachers thought that the corner of the prism would be 3-dimensional. These two teachers' criteria of dimension were "width- length-height" and "by looking". In this case, it is seen that the factor of this decision is the thought of that the corner of the prism has width, length and height.

Results on the 1-dimensional figures

In order to determine the teachers' decisions of dimension regarding 1-dimensional geometric objects, the dimension of the 1-dimensional figures were asked, which are given in the Table 2.

As it is apparent on Table 2, there are three groups of figure that differentiate importantly the rates of the teachers identifying the dimension of the 1-dimensional figures. These are straight line and line segment (1^{st} group), angle, parabola and circle (2*nd* group), triangle, pentagon, rectangle and the figure consisting of only the edges of rectangular prism (3^{rd} group). It has been seen that the success rate is 93% for the 1st group, 58% for the 2^{nd} group and 33% for the 3rd group. The average rate is 61%.

Considering the criteria by those who identified the 2nd group figures as 2-dimensional, it has been seen that 3 teachers used the criterion width-length-height and that 2 teachers decided by looking. On the other hand, when examined the criteria by those who identified the 3rd group figures as 2-dimensional, it has been seen that 4 teachers have used the criterion of width-length-height, 3 teachers decided by looking, 1 person stated that the figures that have area are 2-dimensional and 1 person said that 1-dimensional figures formed 2-dimensional figures by coming together.

Results on the 2-dimensional figures

In order to determine the teachers' decisions of dimension about 2-dimensional geometric objects, the dimension of the 2-dimensional figures, given in the Table 3, were asked.

According to Table 3, there are two groups of figures that affect the teachers' identification of the dimension of a 2-dimensional geometric figure. These are square region, triangular region and circular region (1^{st} group), sphere and prism (2^{nd} group). According to the table, the success rate of identification of the dimension of a 2-dimensional figure is 96% in the 1st group and 13% in the 2^{nd} group.

On the other hand, it is attention-drawing that sphere and prism was considered as 3-dimensional with a rate of

Table 4. The identification of 3-dimensional geometric figure.

	0-D	1- D	2- D	3- D
Cylindrical region (Cylinder and inside)				15
Spherical region (Sphere and inside)			1	13

Table 5. The teachers' dimension criteria.

Criteria	F
Width-Length-Height	3
Length-Area-Volume	3
By looking	3
Others	3
No answer	3

83%. Considering the teachers' criteria, it has been understood that the effective factors on this were that these figures had 3 axes on coordinate system, which are width, length and height or a volume.

Results on the 3-dimensional figures

In order to determine the teachers' decisions of dimension about 3-dimensional geometric objects, the dimension of the 3-dimensional figures, given in Table 4, were asked.

As seen from Table 4, 93% of the teachers were able to identify these figures as 3-dimensional. The criteria of the teachers were the number of axis, width-length-height and volume. When examined generally, the most correct decisions of dimension are the identification of cylindrical region as 3-dimensional (100%), of square region and triangular region as 2-dimensional (100%), of the point as 0-dimensional (93%), of the straight line and line segment as 1-dimensional (93%), of the spherical region as 3-dimensional (87%), of the circular region as 2-dimensional (87%), circle, angle and parabola as 1-dimensional (60%) respectively. On the other hand, the falsest answers are the identification of sphere and prism as 3-dimensional instead of 2-dimensional (83%) and the identification of triangle, pentagon and rectangle as 2-dimensional instead of 1-dimensional (67%).

Results on the teachers' criteria of dimension

The answers of the teachers to the question "how do you decide the dimension of a figure?" are given categorically in Table 5.

According to the teachers who stated their criteria as width-length-height, if a figure has width and length, it is 2-dimensional, if a figure has width, length and height, it is 3-dimensional. All of these three teachers have stated that point is dimensionless, straight line and line segment are 1-dimensional, rectangle, triangle, parabola, pentagon, square region and triangular region are 2-dimensional, sphere, rectangular prism, cylindrical region and spherical region are 3-dimensional. It seems that their answers and their criteria stated are coherent. However, the answers and criteria stated for the figures shown in Table 6 are not stable. On the other hand, there is no consistence among these three teachers. The names of the teachers are coded as WLH1, WLH2, WLH3 in the table and their decisions are presented.

According to the teachers, who stated their criteria as length-area-volume, if a figure has only length, it is 1-dimensional; if a figure has area, it is 2-dimensional and if a figure has volume, it is 3-dimensional. All of these three teachers stated that point and the corner of a prism are dimensionless, straight line, line segment, circle, angle, pentagon and parabola are 1-dimensional, square region and triangular region are 2-dimensional, cylindrical region and spherical region are 3-dimensional. Their answers in respond to the questions and their criteria seem consistent. However, the stated answers and criteria for the figures shown in the Table 6 do not seem consistent. On the other hand, there is no consistence among these three teachers. The names of the teachers are coded as LAV1, LAV2, and LAV3 in the table and their answers are presented.

Furthermore, three teachers have not stated a criterion and said that they would decide the dimension of a geometric figure by looking at it (Table 7). The criteria except from the criteria width-length-height and length-area-volume are those:

A teacher's answer in respond to the questions is as follows: "if the figure can be measured based on its length, it gains the dimension concept. Dimensions form 2-dimensional figures by coming together, 2-dimensional figures form 3-dimensional figures by combining." This teacher has called that "straight line, line segment, the corner of the prism and parabola" as 1-dimensional, "rectangle, square region, triangle, circle, circular region, pentagon, traingular region and the figure consisting of only the edges of rectangular prism" as 2-dimensional, "sphere, cylindrical region, rectangular prism and spherical region" as 3-dimensional respectively.

Another teacher has stated that he/she would consider the "edge, surface and depth" of the figure while answering. This teacher has called point and the corner of prism as dimensionless, straight line, line segment, rectangle, triangle, the figure consisting of only the edges of rectangular prism, circle, angle, parabola and pentagon as 1-dimensional, square region, circular region and triangular region as 2-dimensional; and sphere, cylindrical region, rectangular prism and spherical region as 3-dimensional respectively.

DISCUSSION

At the end of the research, it has been seen that while

Table 6. The teachers' inconsistencies of criterion-decision, who stated the criterion of width-length-height dimension.

	WLH1	WLH2	WLH3
The figure consisting of only the edges of rectangular prism	2	2	3
Circle	1	2	0
Circular region	2	2	0
Angle	1	0	2
Corner of a prism	0	0	3

Table 7. The teachers' inconsistencies of criterion-decision, who stated the of length-area-volume.

	LAV1	LAV2	LAV3
Rectangle	1	2	1
Triangle	1	2	1
The figure consisting of only the edges of a prism	3	1	1
Circular region	2	2	1
Sphere	3	3	2
Rectangular prism	3	3	2

deciding the dimension of a geometric figure, 40% of the teachers did not state a criterion, 20% of them have mentioned the "width-length-height" as a criterion and 20% of them mentioned the "length-area-volume" as their criteria to identify the dimensions. Also, it was determined that there are some factors affecting the decisions made on dimensions of a figure negatively. These factors are as follows: the concepts such as "area-volume", "width-length-height, depth" are not understood sufficiently (misconceptions), and the specifications of the geometric figures are not well-known. For instance, a teacher who has determined the length-area-volume criterion correctly can say that rectangle is 2-dimensional, because she thinks that it has an area. Similarly, another one can say that sphere is 3-dimensional, because he thinks that it has a volume.

Considering that whether the teachers, who have mentioned the width-length-height criterion, have made consistent decisions with their criteria; they must have said that "the figure consisting of only the edges of rectangular prism was 3-dimensional and circle, circular region and angle was 2-dimensional. This contradictory behavior may occur because of lack of their knowledge in geometrical figures or in knowing the meaning of having width, length-height of an object.

The teachers who had remarked the length-surface-volume criterion made some false decisions even if they have determined a true criterion. These teachers were supposed to state that "rectangle," "triangle" and "the figure consisting of only the edges of rectangular prism" are 1-dimensional and "circular region," "sphere" and "prism" are 2-dimensional, respectively. This case could occur because of the lack of their knowledge in geometrical figures or because of the lack of knowledge in knowing the meaning of having surface or volume. On the other hand, some misusage of terminology in the process of geometry teaching can also affect geometric figure knowledge negatively. For instance, it is said in lectures that "the area of rectangle, square" or "the volume of sphere, cylinder" rather than using "the area of rectangular or square region" and "the volume of spherical region."

A teacher has stated a criterion that "if a figure can be measured based on its length, it gains the dimension concept. 1-dimentional objects create 2-dimensional ones and 2-dimensional objects create 3-dimensional objects by combining them together." When this teacher's answers to the 1st question are examined, it has stated that rectangle, square region, triangle, circle, circular region, pentagon, triangular region and the figure consisting of only the edges of rectangular prism as 2-dimensional; and parabola 1-dimensional. It has been understood from here that this teacher does not know that circle, triangle, rectangle and pentagon can be measured only based on its length. In this case, this teacher's misconceptions about "knowledge of figure", "measurable properties of the figure" seem to be a factor affecting his decision negatively about the dimensions of the figure. On the other hand, the dimension criterion stated by the teacher is insufficient. According to his criterion, square and square region have the same number of dimensions; also sphere and spherical region have the same number of dimensions as well. Another mistake made by him is that the teacher thought the

Table 8. Figure classification used before the dimension concept in the teaching process of geometric figures.

	Planar	Non Planar	Measurable Feature (if it is finite)
1- dimensional (Linear Figures)	Square, triangle, parabola, line, line segment, angle, etc.	the figure consisting of only the edges of rectangular prism (prism framework can be used), a curve in space, etc.	Lenght
2- Dimensional (Surfaces)	Polygonal Regions (sguare region, triangular region, circular region, etc.), plane, any part of a plane, vb.	Sphere, prism, cylinder, pyramid, cone, waved surfaces, etc.	Area
3- Dimensional (Fields)		Cubic, cylindrical, prismatic, conic regions, etc. Also all real objects	Volume

composition of sphere and spherical region as the same. These were identified as 3-dimensional by the teacher in his answers.

Another teacher has stated that she considered the edge, surface and depth while determining the dimension of the figure. When this teacher's answers to the 1st question are examined, it has been seen that she said point and corner of a prism are dimensionless; straight line, line segment, rectangle, triangle, the figure consisting of only the edges of rectangular prism, circle, angle, parabola and pentagon are 1-dimensional; square region, circular region and triangular region are 2-dimensional; sphere, cylindrical region, rectangular prism and spherical region are 3-dimensional, respectively. Considering these answers, it has been understood that this teacher has identified the figures consisting of lines (i.e., the figures that has only length) as 1-dimensional, the areas on plane as 2-dimensional, non planar figures as 3-dimensional.

One of the teachers has not given a certain criterion. Instead, he has given a multiple criteria such as surface-volume, axis number, and width-length-height and plane-space location to be used to determine the number of the dimensions. According to the answers of the teacher in respond to the 1st question, the rectangle, square region, triangle, circular region, angle, parabola, pentagon and triangular region are 2-dimensional. It has been understood from here that according to this teacher, whether the figure has an area or not, it is not a criterion for the 2nd dimension, instead being biaxial is enough to make a decision about the dimensions of a figure. However, there is "area" among his criteria. In addition, the figure consisting of only the edges of rectangular prism is 1-dimensional according to him. Considering the axis criterion, this figure is triaxial. Thus, it is obvious that this teacher has not decided in accordance with a clear criterion. On the other hand, he called sphere, cylindrical region, prism and spherical region as 3-dimensional. For instance, among these figures, spherical region has a volume while sphere has not. In this case, it can be thought that his misconception concerning the concepts of area and volume is another factor affecting his decisions made on the dimension of a figure negatively. If he had considered the number of axis for these figures, he would have needed to call the figure consisting of only the edges of rectangular prism as 3-dimensional.

In this research, it was revealed that some teachers' criteria of dimension, decisions for number of dimensions and knowledge about features of geometric objects were not correct. These findings are consistent with the findings of the study conducted by Paksu et al. (2012) and Çetin and Dane (2004) in which it has been revealed that most of the mathematics teachers' concept images about dimension were not correct and they had lack of conceptual knowledge on some geometric objects, and also their criteria of dimension and decisions for number of dimensions were not consistent.

RECOMMENDATIONS

Inadequacy in geometric figure knowledge, the misusage of terminology and the lack of some terms in terminology (or that they are not used commonly), some mistakes made in the process of teaching are thought to be some elements affecting negatively to deal with the dimension concept properly. In this respect, these are considered as significant:

The instructors and students must be informed that prisms, sphere, cone, cylinder have a surface and inside of them is empty and also that a point inside them is not an element of them. Thus, for example it must be said "area of rectangular region, not only rectangle" or "volume of spherical region, not only sphere". These two cases must absolutely be held separated and emphasized. That a closed curve surrounds a specific area does not mean that this curve has an area. Similarly, that a surface surrounds a specific region (volume) does not mean that this surface has a volume.

Similarly, those polygons are linear figure, not a surface and that there is emptiness inside them must be emphasized. Thus, the expression “the length of rectangle” is enough, and there is no need for the expression “the length of the circumference of the rectangle.” This expression is suitable for the “length of the circumference of the rectangular region.”

Use of proper terms for the geometric figure consisting of prisms, sphere, cone, cylinder itself and its inside is significant. “Cylindrical, spherical, conic, prismatic region” terms can be used. In this aspect, some expressions such as “the area of sphere, the volume of the spherical region” are correct.

Figure classification shown below can be used before the dimension concept in the teaching process of geometric figures (Table 8).

Conflict of Interests

The author has not declared any conflict of interests.

REFERENCES

Alexandroff P (1932). Dimensions theorie. Mathematische Annalen, 106:161-238.

Alexandroff P (1961). Elementary concepts of topology. (Translator, Alan E. Farley). New York: Dover Publications.

Çetin ÖF, Dane A (2004). Sınıf öğretmenliği III. sınıf öğrencilerinin geometric bilgilere erişi düzeyleri üzerine. Kastamonu Eğitim Dergisi. 12(2):427-436.

Devlin K (1994). Mathematics: The science of patterns. New York: Scientific American Library.

Freudenthal H (1983). Didactical phenomenology of mathematical structures. Dordrecht: NL Reidel.

Georgia Department of Education (2010). Georgia mathematics performance standards 2009-2010. Atlanta.

Jackendoff R (1991). Parts and boundaries. Cognition 41:9-45.

Manin YI (2006). The notion of dimension in geometry and algebra. Bull. Amer. Math. Soc. (N.S.). 43(2):139-161.

Menger K (1943). What is dimension. Am. Mathematical Monthly. 50(1):2-7.

National Council of Teachers of Mathematics (2000). Principles and standards for school mathematics. Reston, Virginia.

Paksu AD, Musan M, İymen E, Pakmak GS (2012). Preservice elementary teachers’ concept images about dimension. Buca Eğitim Fakültesi Dergisi. 34:53-68.

Pears AR (1975). Dimension theory of general spaces. Cambridge: Cambridge University Press.

Primary Mathematics Education Curriculum (2004). Primary mathematics curriculum (1-5 Classes). Ankara: Republic of Turkey Ministry of National Education.

Skordoulis C, Vitsas T, Dafermos V, Koleza E (2009). The system of coordinates as an obstacle in understanding the concept of dimension. Int. J. Sci. Mathematics Educ. 7:253-272.

Vitsas T, Koleza E (2000). Student ‘s misconceptions on the concept of dimension. Proceedings of the 2nd Mediterranean Conference on Mathematics Education. Nicosia, Cyprus pp.108–119.