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Pre-service teachers' cognitive competencies to use the approaches in mathematics teaching: Discovery learning

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This study aims to present the cognitive competences of the pre-service teacher about discovery learning approach in mathematical education. The study was conducted with 37 mathematics pre-service teachers who study Special Teaching Methods lesson in a state university in Turkey. Throughout the lesson, the approaches used in learning were examined with the pre-service teachers. Afterwards, some open-ended questions related to discovery learning approach were asked for the pre-service teachers to answer and they were expected to prepare an activity in which they would apply the approach and then evaluate it. After analyzing the retrieved data with qualitative research techniques, three main findings were achieved: The pre-service teachers have enough theoretical information on discovery learning approach and are able to meaningfully explain that information; their competences at preparing an activity in which they will apply discovery learning are quite low; most of them did not compare the discovery learning approach to other approaches.

Key words: Discovery learning, mathematics education, pre-service teacher, cognitive competency.

INTRODUCTION

The issue regarding what is learning and how it occurs kept many researchers busy (Bruner, 1961; Ausubel, 1968; Inhelder, Sinclair and Bovet, 1974; Lambert and McCombs, 1998) and initiated a lot of discussions for many years. These discussions continued as various learning approaches emerged and some of them were rather accepted in different subjects during different periods (Mayer, 2004).

Cognitive approach which was firstly established by Piaget and is one of the suggested approaches emphasizes that learning occurs in the mind and thus, points out that the activities in the mind shall also be studied (Davis, 1990; English, 1995; OECD, 2003). As the cognitive psychology renewed its interest in the fields such as the concept formation, problem solving, connection among cognitive structures and behavior, a form of cognitivism is referred as constructivism (Noddings, 1990). This theory, which proposes that the

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new information to be achieved shall be associated with the existing information, is named as constructivist approach due to the fact that it resembles forming a new structure in a person's mind (English, 1995; Ojose, 2008). Almost all of the constructivists believe that information is not achieved by simply learning from the teacher but by actively constructing by the student and accept that knowing is not the discovery of the already existing and objective information (Von Glasersfeld, 1987; Kilpatrick, 1987; Noddings, 1990; Lesh, Doerr, Carmona and Hjalmarson, 2003).

With the students having an inefficient academic level in mathematics dragging mathematical education into new quests, constructivism, which is a different perspective in learning, also has been the center of many experimental and theoretical studies in mathematical education (Simon, 1995; Jaworski, 2006). All of these played a part in the formation of the mathematical reform and brought along the reforms applied in mathematics programme (Lin and Cooney, 2001). In many countries, these studies are dependent on various standards and in Turkey, a new mathematics programme based upon constructivism which will be applied in primary and secondary education classes in 2005 are actualized. By this way, it is aimed to bring students in skills such as reasoning, associating, and problem solving by mentally and physically activating them (MEB, 2010).

Implementing the reform movements, in other words, effectively performing new approaches instead of traditional ones depends on teachers (Battista, 1994; Çakıroğlu and Çakıroğlu, 2003). However, constructivism, despite providing a useful structure for mathematical thinking and leading significant ways affecting the reform in mathematical education, does not suggest many ideas on how to teach mathematics and foresee a specific model (Simon, 1995). Thus, it is of utmost importance that prospective teachers have the required proficiencies for that matter. Prospective teachers must be familiar with useful curriculum materials, learning various general and specific education subject models and researching some approaches evaluating the student comprehension during the preparation process before the period of service in which they accumulate basic information for reformist education. However, the focus must be on comprehending when, where, how and why the specific approaches will be used, instead of variety (Feiman-Nemser, 2001).

Many pre-service teachers are not ready to use these approaches which are required by the educational reform (Herman and Gomez, 2009). Therefore, this study aims to have an idea on the cognitive proficiencies formed of prospective teachers before they put the new approaches in reformist thinking into practice from theory. Thus, out of all the approaches, the discovery learning approach was examined. It was aimed to demonstrate what the pre-service teachers understand from this approach, how they think they will apply it in mathematical education and also how much of the competences required for preparing an appropriate activity they have, by doing so, it was targeted to form an opinion about cognitive competences related to the approach.

RESEARCH FRAMEWORK

Discovery learning

Bruner (1966), who improved the ideas of Piaget and asserted the theory of learning through discovering, emphasized that the structure of the topic should be comprehended in learning and concepts should be indicated with basic forms (English, 1995) and defended that comprehending this structure correctly will take place by the individual's discovering the basic principles of the subject actively (Senemoğlu, 2005). Bruner (1966), stated that knowing is not a product but a process, he also remarked that learning how information is formed simplifies comprehending, remembering and using that information in a new context (Altun, 2010) and discussed that, thus, motivation and success will increase (Castronova, 2002). Therefore, students should be forced to analyze, apply and synthesize the information instead of receiving and assimilating it (Baki, 2006).

In discovery learning, learner is not informed about the target information or concept and this could be achieve by evaluating the available circumstances independently (Alfieri et al., 2011). This process, developed inductively (Baki, 2006), continues the assumptions based on systematic researches and intuition by organizing the examples guiding to generalization or conceptualization from basic to advanced (Jacobsen et al., 1985) and states that the teacher's duty is to guide the learner (Hammer, 1997; Svinicki, 1998).

There are two approaches in learning through discovering: First; it's an invention that is unstructured where the teacher lets the student find out the concepts and principles completely on their own, and the students are expected to find out the related concepts and principles on their own like scientists and they start and direct their studies mostly by themselves. Second; it's an inventions that is structured where the teacher determines the behaviors which will be learned, this provides the examples in which related concepts and principles are used and if necessary benefits from the examples contrasting with the related concepts and principles, also the student is expected to come up with the inference (Senemoğlu, 2005).

However, according to Bernstein et al. (2003) even though guidance shows a great performance in practice, too much guidance may affect the next performances negatively. Besides, it is believed that asking students to discover without being guided is not only less effective but also has negative consequences such as finding out information that includes misconceptions and is unorganized and incomplete (Kirschner et al., 2006). Mayer (2004), emphasizes that unguided discovery learning tasks did not help learners discover problem-solving rules, conservation strategies, or programming concepts. Also, although constructivist-based approaches benefits, he mentions its lack of structure.

Cognitive competency

Cognitive field in learning topics includes the learning where individual's mental aspect preponderates and the objectives within this field are sorted from basic to complex, from concrete to abstract in a manner that they are prerequisite for each other: Knowledge, comprehend-sion, application, analysis, synthesis, and evaluation (Bloom et al., 1956; Simpson, 1972; Krathwohl et al., 1973).

Knowledge: Level consists of the behaviors such as the person's recognizing specificities about any object or event when he/she sees it and telling or repeating by heart when he/she is asked.

Comprehension level: Is expression, assimilation, and interpretation of the objectives that are gained on knowledge level without losing their meaning.

Application level: Is the person's applying the knowledge by solving the problem in a situation, which is new to him/her, based on the learning on knowledge and comprehension levels.

Analysis level: Is cognitively decomposing a whole or pattern of knowledge in terms of their items, relations and organizations.

Synthesis level: Is gathering and composing the items according to certain relations and rules in such a way that it consists of features such as innovation, originality, and creativity.

Evaluation level: Is the person's determining whether the end product is convenient in terms of providing competences or not (Seddon, 1978; Bloom, 1994; Krathwohl, 2002).

OECD (2003), uses three levels as indicators in order to describe the students' cognitive skills within the scope of Programme for International Student Assessment (PISA) which are as follows:

1. Reproduction: the level in which the known content, previously used knowledge, standard algorithm and elementary formulas are used and basic processes are applied,
2. Connection: the level in which the less known content is interpreted and explained, different systems of repre-sentations are associated, and the required strategies for solving quasi-familiar problems are determined and applied,
3. Reflection: the level in which comprehension is required; reflection, creativity, and the knowledge about how to solve unfamiliar problems are associated; observed results are generalized and justified; and abstraction takes place.

As the knowledge is formed in the mind, the similarities separated from many specific examples create awareness, thus, the knowledge is abstracted in the mind as a consequence of generalizations (Ohlsson and Lehtinen, 1997). Hershkowitz et al. (2001), studied this process that takes place cognitively and developed the RBC model by describing the epistemic actions as recognizing (R), building with (B) and constructing (C). According to this model which allows every action to be observed and helps the process to be understood better, these epistemic actions are discussed as follows:

Recognizing is the process, in which the person deals with activities he/she is familiar with from his previous activities; and associates his/her current activities with his/her old activities.

Building with, is the process during which it is aimed to solve a problem or justify a case; and the elements of the information are combined together.

Construction is the process, in which the person gets his/her knowledge together and so as to create a new structure.

They added the Consolidation process in order to emphasize the independency and elasticity of the abstraction and created the RBC+C model.

In Figure 1, in order to clearly present the cognitive structure that is formed in the mind of a person, the association of the RBC+C Model (Hershkowitz et al., 2001), PISA Competences Clusters (OECD, 2003), and Cognitive Domain Taxonomy (Bloom et al., 1956) which are mentioned above, is demonstrated. The actions enabling the association were formed and it was aimed to achieve a general structure about the cognitive process.

METHODOLOGY

This study is a qualitative research which was done with 37 participants (13 male, 24 female) from mathematics pre-service teachers who are on their 4th year of faculty of education which lasts five years. The participants in the study successfully completed basic education lessons such as Introduction to Educational Science, Educational Psychology, Teaching Principles and Methods on the previous semesters and were taking Special Teaching Methods lesson during the semester in which this study was conducted. Some approaches about education such as Ausubel's (1963) verbal-meaningful learning approach, Freudenthal's (1973) realistic mathematical education approach and Bruner's (1961) discovery learning approach were studied in detail throughout this lesson lasting for four hours in a week and the applications of these approaches in mathematical education were discussed and interpreted.

In this study, discovery learning approach is dealt with in order to reveal how pre-service teachers construct these approaches applied in education in their minds and to determine their cognitive level. Also, the data about the study were collected at the end of the lessons lasting for almost 6 h.

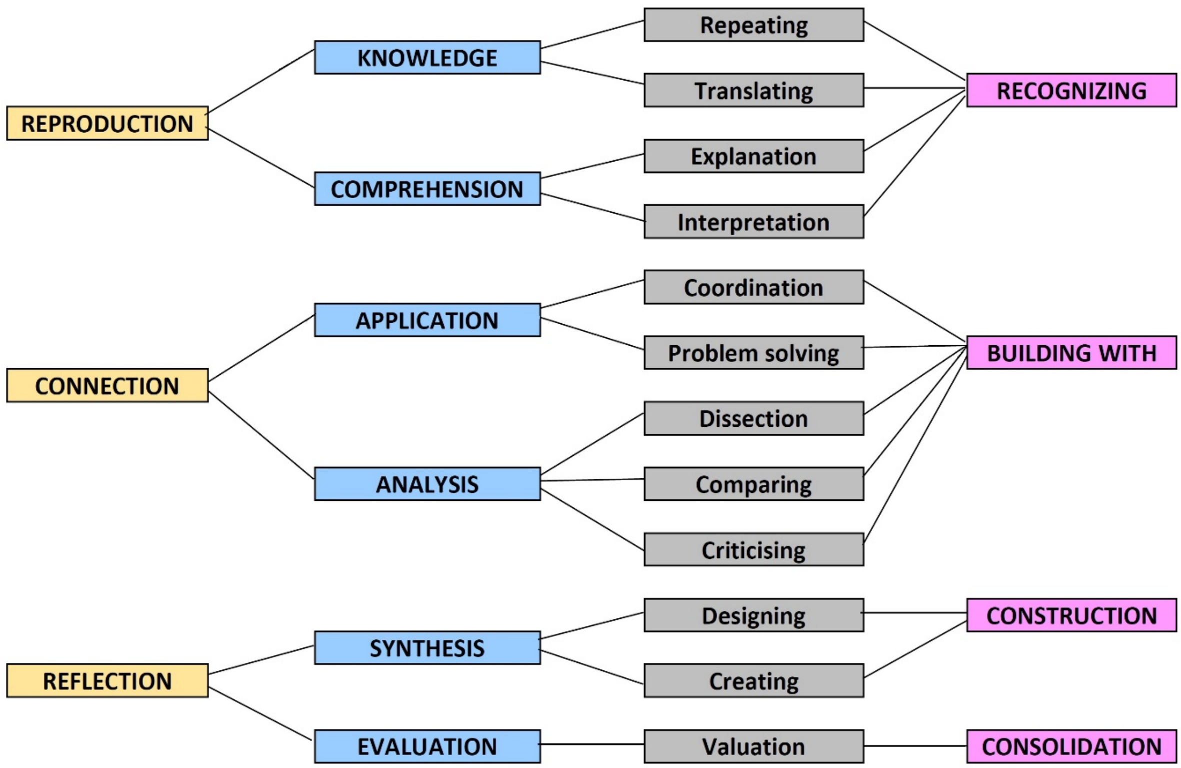

Figure 1. The Relations of Cognitive Competency (Bloom, Engelhart, Furst, Hill and Krathwohl, 1956; Hershkowitz, Schwarz and Dreyfus, 2001; OECD, 2003).

Data collection

First of all, theoretical information regarding discovery learning approach was given to pre-service teachers for 6 h, and later activities on mathematics teaching were applied and discussed in the class. Discussions in the class were mostly about the pre-service teachers' interpretations about the process, teacher's and student's role throughout the teaching and learning process. The activities in the class were about concepts underlying various outcomes that takes place in teaching programme such as Fibonacci Sequence and Golden Ratio, The Sum of Interior Angles of Triangle, π number, Identity of $(x+y)^2=x^2+2xy+y^2$, area of rectangle, and Pythagorean Theorem. In the end of this process, open ended questions were asked to pre-service teachers and furthermore, each of them was asked to prepare an activity to apply discovery learning approach in mathematical education. The pre-service teachers were asked to explain the activities they would prepare and their ideas on the questions, in detail and written format. Thereby, the structure about the approach constructed in the pre-service teachers' minds was studied. The questions asked to the participants are as follows:

1. What is discovery learning?
2. Can you explain the activities applied in the class about the discovery learning approach?
3. Prepare an activity similar to the activities applied in the class about the discovery learning approach. Explain the process in the activity you will prepare step-by-step.
4. Prepare your own activity for mathematical education related to the discovery learning approach. Evaluate the activity you prepared and the whole process.

The data analysis

The pre-service teachers' answers to the open ended questions and the activities prepared by them were analyzed by qualitative research techniques in order to determine their cognitive competencies about the approaches. Hence, firstly, the association of taxonomy about cognitive learning domain (Bloom et al., 1956), PISA Competency Clusters (OECD, 2003) and RBC+C model (Hershkowitz et al., 2001), all of which are summarized in Figure 1, was taken as the baseline in creating the categories. Thereafter, there were continuous discussions about creating common themes by dividing these categories into sub-categories. (Creswell, 1998; Patton, 1990). The categories, sub-categories, and common themes were again studied by two experts of the field individually in order to improve their reliability of the study and they reached an agreement on the created categories and themes (Berg, 2001; Yıldırım and Şimşek, 2005).

RESULTS AND DISCUSSION

Answers and activity samples of 37 pre-service teachers who participated in the study were evaluated and common themes were formed according to their cognitive competencies. For this, codifications done for the data analysis, and categories and subcategories formed in finding common themes were summarized in Table 1. And also, frequencies of the findings belonging to the categories and subcategories were demonstrated in this

Table 1. The Frequencies and Percentages of the Categories and Sub-Categories of the Pre-Service Teachers' Competences

CATEGORY			SUBCATEGORY	FREQUENCY	(%)
REPRODUCTION	RECOGNIZING	KNOWLEDGE	Recalling of information by *repeating* or *translating*	35	94,6
			Deficient or inaccurate recalling of information	2	5,4
			No answer	0	0
		COMPREHENSION	*Interpretation* about the knowledge	13	35,1
			Explanation about the knowledge	21	56,8
			Deficient or inaccurate comprehension about the knowledge	3	8,1
			No answer	0	0
CONNECTION	BUILDING WITH CONSTRUCTION	APPLICATION	Posing a familiar problem about the knowledge and *solving* this *problem* as an application	17	46
			Coordination to pose a familiar problem about the knowledge	13	35,1
			Deficient or inaccurate application about the knowledge	1	2,7
			No answer	6	16,2
		ANALYSIS	*Criticizing* about the parts of knowledge by analyzing the familiar problem.	8	21,6
			Dissection or *comparing* the parts of the knowledge by analyzing the familiar problem.	4	10,8
			Deficient or inaccurate analysis of the knowledge	1	2,7
			No answer	24	64,9
		SYNTHESIS	*Creating* an original or an unfamiliar problem about the knowledge	5	13,5
			Designing an original or an unfamiliar problem about the knowledge	4	10,8
			Deficient or inaccurate synthesis about the knowledge	5	13,5
			No answer	23	62,2
REFLECTION	CONSOLIDATION	EVALUATION	Making *valuation* by passing a judgment on the original or unfamiliar problem about the knowledge	1	2,7
			No answer	36	97,3

table. The findings related to the cognitive competencies of the pre-service teachers were explained within each category by using their exact quotations.

Reproduction recognizing

Knowledge

Out of all 37 pre-service teachers participated in the study, 35 of them (94.9%) answered the question of 'what is discovery learning approach?' correctly by repeating or translating the definition they had learned earlier. Examples from the quotations belonging to the answers given by the participants are as follows:

"Enabling the student to achieve a concept or information with the guidance of the teacher, by the help of teacher's preparing the necessary setting with the knowledge he/she had, and by making comparisons and associations…" (12th pre-service teacher-repeating).

"The teacher draws a road map for the students about how he/she will promote them to find a concept or information, and guides them to continue from this route, enables them to achieve the result by themselves, and gives them the opportunity to discover the information on their own…" (37th pre-service teacher- translating).

"The student's achieving the information on his/her own by following an inductive way…" (2nd pre-service teacher- translating).

2 participants (5.4%) defined the discovery learning approach deficiently or inaccurately. There was not any participant who did not answer the question. The quotations belonging to the deficient and incorrect answers given by the pre-service teachers to the question are as follows:

"It is the teaching of a subject with examples more closely to the everyday-life. What is essential in this method is teaching through concretizing and dividing things into phases" *(24th pre-service teacher-inaccurate*)

"As seen from its name, it is teaching students by helping them discover things …" *(23rd pre-service teacher-deficient).*

Referring to Ayer's (1936) definition, and considering the idea which states that even though the correct definition may not contain the definition itself or synonyms, it can still be translated by using equivalents; most of the sentences uttered by the pre-service teachers can be accepted as correct definitions and thus it is possible to say they are on the knowledge level of the reproduction-recognizing processes. Besides, only two pre-service teachers do not have adequate knowledge on the approach according to the idea which states that the ambiguous, complicated or inappropriate forms of definitions cannot be accepted as correct definitions.

Comprehension

In the answers given to the second question in which the pre-service teachers were asked to explain the in-class activities in order to see if they comprehended the discovery learning approach or not, it was observed that 34 (91.9%) of the participants comprehended the discovery learning approach accurately. When the answers were examined, it was seen that 13 (35.1%) of 34 participants interpreted the activities, and 21(56.8%) of 34 participants explained the activities. Also, 3 (8.1%) of 34 participants were insufficient at explaining the activities. There were not any pre-service teachers who did not answer the question. Examples from the exact quotations from pre-service teachers who explain the activities *through interpretation* are as follows. The parts in quotation marks in the quotations draw attention to interpretations:

"Firstly, '*we tried to promote students to gain introductory information through numerical examples in order to enable them to achieve generalizations*' in the activity of finding the area of a rectangle. In other words, we helped them find the area of a general rectangle by starting from finding the areas of rectangles whose edge lengths are different. For this, for instance, we asked them to separate a rectangle whose edge lengths are 3 and 4 units into unit squares. We asked them to find how many unit squares the area of rectangle is formed of. Later, we applied similar condition for a different rectangle. For instance, for a rectangle whose edges are 2 and 5 units. We asked them how many unit squares there were in this rectangle and what the area of rectangle equals to. In the end, we asked them to find how many unit squares there were for an area of rectangle which was a and b units. For this, we demanded them to think about what to do instead of counting them one by one. We expected them to see that there were b unit square and a row in each row in total given that the width is 'a' unit and the height is 'b' unit. Hence, we waited for them to reach the generalization that they were the multiplication of edge lengths. *'Here our aim was to prepare the suitable setting and to make them obtain the information they did not know, with generalizations and step-by-step'" (35th pre-service teacher).*

"In the activity done for Pythagorean Theorem, firstly, we made them draw a right triangle whose edge lengths were 3, 4, and 5 units on a squared paper, then we made them draw squares having these edge lengths on hypotenuse and both other sides. After that, we separated these squares into unit squares and we asked them to cover the square which was formed on hypotenuse by using the squares which were formed on other sides. Our aim was to make them realize that the squares of 3 and 4 were equaled to the square of 5. Afterwards, we made them do a similar exercise for a right triangle whose legs were 6 and 8 units. We ensured them to see that the same cases occur in different triangles. *'Hence, we helped them to think of the examples from which they would make generalizations'*. In the end, they comprehended the information suggesting that the sum of the squares of the legs equal to the square of hypotenuse *'as if they had found out that information on their own'" (27th pre-service teacher).*

"While teaching the square of a binomial, *'we aimed to make them realize where this identity came from on their own'.* For this, we asked them to calculate the area of a square in the length of x unit. Then, we asked them to calculate its area again by increasing horizontal and vertical edge length by 1 unit, that is, by doing it x+1. '*We led them to associate it with the first square'*. Thus, they saw the identity of $(x+1)^2 = x^2+2x+1$. We made them do a similar thing by extending it two more units, in other words, extending it to x+2 and enabled them to see it was $(x+2)^2 = x^2+4x+4$. After that, we gave clues to enable them to generalize this case. We expected them to realize square of a binomial by this way. *'We tried to* *meaningfully teach them this identity and give them the opportunity to find it on their own*" *(29th pre-service teacher).*

Examples from exact quotations belonging to pre-service teachers who only *explained* the activities are as follows:

"We made them divide a rectangle into unit squares in order to make them find the area of the rectangle. We asked how many unit squares it was formed from. Later, we made them associate it with its edges. We made them do it for other rectangles and find out how to calculate the area of any rectangle" *(16th pre-service teacher).*

"In the activity of finding Pythagorean relation, we first enabled them to see the areas of squares formed by legs is equal to the area of square formed by hypotenuse. For this, we made them to divide the areas into unit squares. We made them do the exercise with different examples such as 3-4-5 or 6-8-10 and they achieved the relation" *(18th pre-service teacher).*

"In the activity of obtaining square of a binomial, we took a square whose edge is in the length of x unit. We asked them what its area was. Next, we asked what its area would be when two consecutive edges were extended by 1 unit. We wanted them to see what is added to the area of the previous square for the area of the new square and expected them to obtain $(x+1)^2=x^2+2x+1$ identity. We asked similar questions when we extended the edges of the square by 2 units and expected them to get $(x+2)^2=x^2+4x+4$ identity. After that, we asked what would happen if the edge was extended by y unit and we expected them to get $(x+y)^2=x^2+2xy+y^2$ identity" *(11th pre-service teacher).*

3 Pre-service teachers, who explained the activities deficiently, were 1st, 23rd, and 24th pre-service teachers. Although, the 1st pre-service teacher defined the discovery learning approach correctly, he was insufficient in explaining on the comprehension level. 23rd and 24th pre-service teachers were seen to be in the category who defined the discovery learning approach deficiently or inaccurately on the knowledge level. Participants' exact quotations from their explanations about activities are as follows:

"We drew 3-4-5 triangle on the board. We made them look like squares and later divided two edges into unit squares and realized that the area of the square, one of whose edge is a hypotenuse, is 25 unit squares. Then, we gave them the formula and showed them examples" *(23rd pre-service teacher).*

"While teaching the area of a rectangle, we firstly explain the area, and then divide the rectangle into unit squares. We see that the sum of each unit square gives the total area of the rectangle. Furthermore, we also say that we can calculate its area when we multiply two edges. Thus, we teach the area of rectangle by starting from unit squares" *(1st pre-service teacher).*

It was observed that the first pre-service teacher comprehend the method inaccurately from his statement in which he mentioned 'firstly explaining the area and then trying to associate it with unit squares for calculating the area of the rectangle' with the application of discovery learning approach. The 23rd pre-service teacher deficiently described the discovery learning on the knowledge level. The wrong expressions he used, such as 'dividing the edges into squares' and 'making them look like squares' and also, the other expressions he used for the approach demonstrate that he did not only comprehend the process deficiently, but also inaccurately. The 24th pre-service teacher's case showed similarities with that of the 23rd pre service teacher. When we consider explaining, which can be referred as giving understanding to somebody else (Brown, 1978), and interpreting, which requires forming a broad understanding (OECD, 2009), as two procedures of unveiling and demystification (Fairclough, 1989); it is possible to say that the reason why most of the pre-service teachers explained or interpreted the approach accurately is because they comprehended the approach in a meaningful manner.

Connection building with

Application

In the answers given to the third question which was asked in order to reveal how much the pre-service teachers applied learning by discovery and in which they were asked to prepare a similar (familiar) activity for the discovery learning, it was observed that 17 participants (46%) developed a problem in preparing a similar (familiar) activity about the discovery learning and solved it; 13 participants (35.1%) tried to develop a similar (familiar) activity and ensure the coordination within the process; 1 participant (2.7%) carried out a deficient or inaccurate application; 6 participants (16.2%) did not answer the question related to the approach. Some examples from exact quotations of the process, in which the participants developed and solved a similar (familiar) problem about the approach, are as follows:

"We draw a square whose edge length is x unit. We ask them the area of that square and write x^2 in it. Then, we ask them to subtract 1 unit from two edge lengths of square and we see that the area of new square which is formed inside is $(x-1)^2$. We demand them to link the area of new square with the area of the first square and we obtain $(x-1)^2=x^2-2x+1$ identity from $x^2=(x-1)^2+(x-1)+(x-1)+1^2$ equation by subtracting the areas of the other parts from the area of the first square. Later, we make them

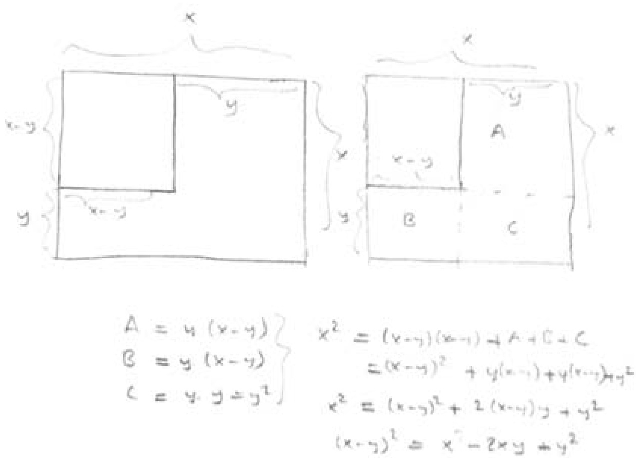

Figure-2. 25th Pre-service Teacher's Description.

think by asking what will happens when we decrease it by 2 units and we obtain $(x-2)^2=x^2-4x+4$ identity from $x^2=(x-2)^2+(x-2).2+(x-2).2+2^2$ equation. Later, we demand them to reach a general idea by asking them what happens when it is decreased by y units and enable them to acquire $(x-y)^2=x^2-2xy+y^2$ identity from $x^2=(x-y)^2+(x-y).y+(x-y).y+y^2$ equation" *(25th pre-service teacher)* (Figure 2).

As seen in the exact quotation, the participant developed "$(x-y)^2=x^2-2x+y^2$ identity" as a problem that is similar (familiar) to the activity in which teaching "$(x+y)^2=x^2+2x+y^2$ identity" was taught and solved the problem by explaining it according to the approach. Problem solving, finding what is known (Jonassen, 2000), in other words, can be described as any goal-directed sequence of cognitive operations (Anderson, 1980) from the cognitive point of view. The problems are referred as routine problems in which specific data substitute for general problems with formal solving or a clear example is followed step by step (Polya, 1957) and these are problems whose solutions are known beforehand (Mayer, 1998). In this sense, the process of preparing an activity similar (familiar) to the ones done in the class can be considered to be routine problem solving process. Thus, the fact that almost half of the pre-service teachers tried to solve a similar (familiar) problem can be interpreted as they are successful at the application process and have the cognitive competence in applying the approach. An exact quotation about the process during which the pre-service teachers tried to develop a similar (familiar) activity and obtain coordination for it is as follows:

"We can make the students figure out how to obtain the area of parallelogram. To do so, we draw a parallelogram. They are asked to examine the figure and find out the similarities between the figure and a rectangle. They are expected to figure out the fact that rectangle is a special form of parallelogram whose edges intersect

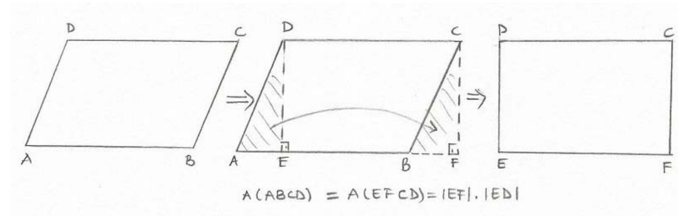

Figure-3. 16th Pre-service Teacher's Description

vertically. We draw heights on the figure and try to see that the triangles formed in the edges are equilateral triangles. According to this, it is observed that the figure, which is obtained by cutting AED triangle and putting it on BFC triangle, is a rectangle. After that, by recognizing that the area of the rectangle is the same as the area of the parallelogram, we find out that we can calculate the area of parallelogram from the area of the rectangle. The area of rectangle can be obtained by separating into unit squares as we did in the activities. Thus, we can teach them to calculate the area of parallelogram by using the area of rectangle which was taught by discovery learning." (16th pre-service teacher) (Figure 3).

The pre-service teacher tried to develop an activity for 'finding the area of parallelogram' that was similar to the activity of 'finding the area of rectangle', which was done in the class. In the activity, they tried to develop a similar problem related to the approach which was examined in the class, however, when the whole activity process is taken into account, the process of applying the approach is rather related to the association of the activity about the area of the rectangle. Therefore, the process was evaluated as enabling the coordination for solving the problem, in a different category. An example for the deficient or inaccurate activity about the approach which was applied by one of the participants is as follows:

"Let's suppose that the student is curious about π. While explaining this, we show them two pictures of two bridges whose rotations and slopes are similar to each other. We can talk about a ratio between these two. In a similar sense, the student can comprehend that the ratio of the circumference to the diameter of the circle is equal to π and it is the same case for all circles by discussing about the relationship among all circles *(1st pre-service teacher).*

It was observed that the first pre-service teacher who performed the application deficiently or inaccurately also fell into the category of those who comprehended the approach deficiently or inaccurately on the comprehension level. Moreover, 2 of the 6 pre-service teachers (23rd and 24th pre-service teachers), who did not give, state their opinion on developing an activity similar to the ones done in the class, failed on knowledge and comprehension level. Although, the others were successful at these processes, they failed at the application processes.

Analysis

In the study, in the third question, in which the pre-service teachers were asked to prepare an activity similar to the ones applied in the class, were demanded to explain these activities which they prepared by themselves step by step.

Thus, it was aimed to examine the applications of the participants about the process, their analysis throughout the process, and the associations the participants created. When the activities and explanations were examined it was observed that 13 participants explained the activity process.

Furthermore, 8 of them (21.6%) developed, solved and then criticized a similar (familiar) problem about the approach, 4 of them (10.8%) only solved the similar (familiar) problem they developed and 1 of them (2.7%) failed at correctly solving the problem. The other 17 participants (46%) who prepared a similar (familiar) activity for the third question did not explain the activities they prepared. 6 participants (16.2%) neither answered to the question nor prepared an activity and 1 participant (2.7%) prepared a inaccurate activity.

An example from the exact quotations of the pre-service teachers who analyzed and *criticized* a similar (familiar) problem related to the approach is as follows:

"Cube of binomial can be taught by discovery learning and the student can be enabled to realize that identity by himself step by step. First of all, we ask students to draw a cube whose edge lengths are x unit, then we see that the volume of the cube is x^3. After that, we ask students to increase the edges of the cube by 1 unit and see that the volume of new figure is $(x+1)^3$. Here, in order to promote them to come up with associations, we ask them what we need to add to the old cube in order to achieve the new cube. Thus, we can mark that the volume of new cube is equal to the sum of the volumes of the four objects in it. Then this will be $x.x.1+x.1.(x+1)+(x+1).(x+1).1+x^3$. Therefore, $(x+1)^3= x^3+3x+3x^2+1$ identity can be obtained from this. We ask them about the edges of the cube when its edges are expanded by 2 units for a similar case and expect them to obtain the identity of $(x+2)^3= x^3+3.2^2.x+3.2.x^2+2^3$. Later, we asked them to make generalization about what the volume of new figure will be if the edges are extended by y unit, and obtain the identity of $(x+y)^3=x^3+3xy^2+3x^2y+y^3$ from $(x+y)^3=x^3+x.y.(x+y)+x.x.y+y.(x+y).(x+y)=x^3+x^2y+xy^2+x^2y+x^2y+xy^2+xy^2+y^3$ equality. (Figure 4).

'Our aim is to help the students realize the information on

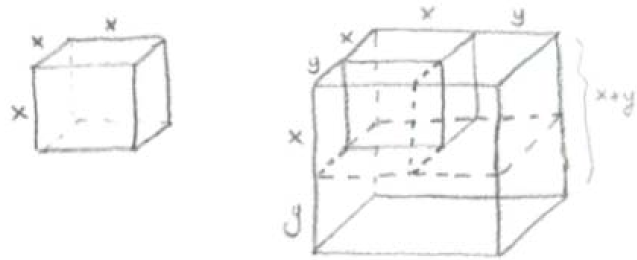

Figure-4. 35 th Pre-service Teacher's Description.

their own by guiding them. We should help them focus on their aim without being distracted by other things and perceive the target information while guiding them. In other words, we should help them see the relation among the given information step by step and obtain a general knowledge. We continue by extending it by 1 unit, 2 units and so on, and obtain the identity by extending it by y units. Since they obtain that information by putting the pieces together by themselves, they can learn this identity more permanently. Instead of giving the whole information and then teach them by dividing it into pieces as done in learning through presentation, we aim to make them put these pieces together by themselves and obtain the whole information inductively' " (35th pre-service teacher).

The study of a participant, who developed, dissected and compared a similar problem, is as follows:

"The identity of the difference between can be taught by discovery learning. First of all, we draw a square whose edge length is x unit. We mark that the area of this square is x^2. Next, we ask them to draw a square whose edges are on the edges of the square and we ask them to link the areas of new squares with the first square. *'Just as we obtained the square of a binomial'*, since the difference between the area of the old square and the area of the new square is the sum of the areas of the other parts, here, $x^2-1^2=(x-1).1+(x-1).x$ equality, that is, $x^2-1^2=(x-1).(x+1)$ is obtained. We ask them to do something similar for a square in the length of 2 units and we obtain $x^2-2^2=(x-2).2+(x-2).x=(x-2).(x+2)$. So, they reach an opinion about both case 1 and 2. Then, we demand them to think about what happens in the square whose edge lengths are y unit in order to make them reach a generalization and expect them to obtain the identity of $x^2-y^2=(x-y).(x+y)$. Thereby, we can obtain $x^2-y^2=(x-y).(x+y)$ identity.

'Our aim is to help the students realize general information by providing them with some cases through which they can reach a generalization. Here, the steps given for generalizations are their realization of what happens for y unit when it continues as 1 unit, 2 units and so on'" (29th pre-service teacher) (Figure 5).

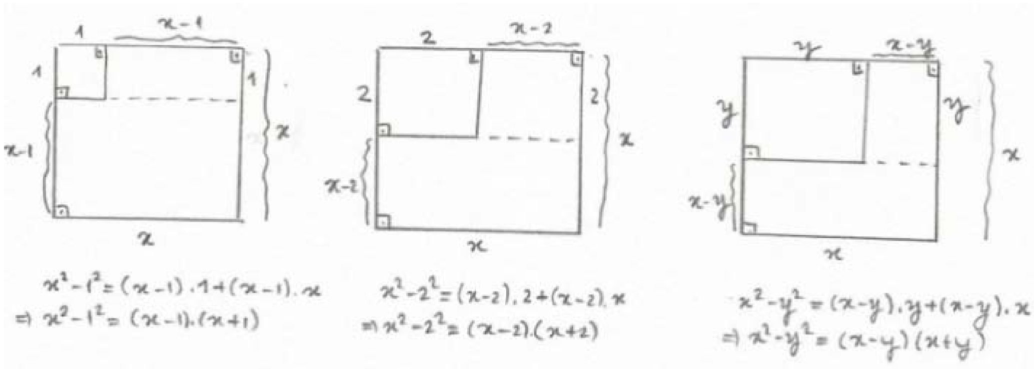

Figure-5. 29th Pre-service Teacher's Description.

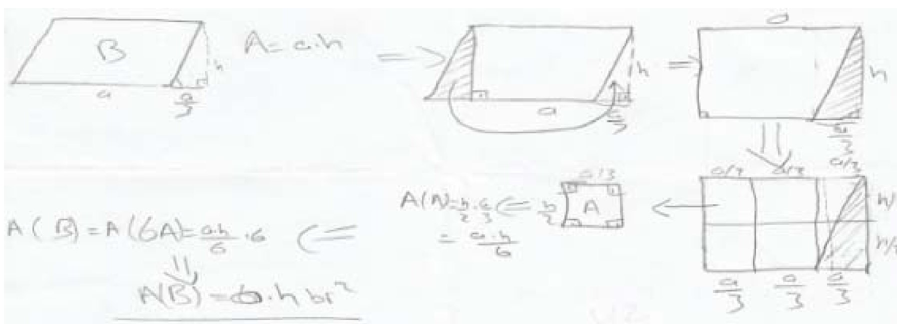

Figure-6. 28th Pre-service Teacher's Description.

The example of the pre-service teacher who performed the application but analyzed it inaccurately is as follows. This example was considered to be similar to the activity example given to enable coordination in the application process. Here the participant tried to develop a similar activity example, and tried to explain the process differently from the other participant but analyzed it falsely. When the example was examined, it was observed that it was not an accurate idea to try to calculate the area of the rectangle again by using the area of the previous rectangle. Moreover, the cases such as trying to obtain unit rectangles by dividing the height as h/2 and the edge of the triangle as a/3 and not being able to conclude a generalization are examples of the inaccurate analysis throughout the process. "While finding the area of a parallelogram, its base is multiplied by its height. Instead of giving this directly as a formula, we can make students find it by themselves. For this, we can consider our figure as a rectangle by having perpendicularities on the bases and then by carrying the triangle which is formed on the other side. Later, we can calculate firstly the area of the rectangle and then the area of the parallelogram by dividing the area of this rectangle into *'unit rectangles'* and by using of the areas of these rectangle. For instance, if we name the base of parallelogram as 'a' and the perpendicular segment as 'h', then the base of triangle is a/3. If the edges of rectangle are divided into pieces as a/3 and h/2, the rectangle is formed from unit rectangles whose areas are $\frac{h}{2}.\frac{a}{3}=\frac{a.h}{6}$ .Therefore, the area of rectangle becomes a.h from $6.\frac{a.h}{6}$ which shows us the area of parallelogram" *(28th pre-service teacher)* (Figure 6).

Reflection

Construction synthesis

Fourth question was asked to pre-service teachers in order to thoroughly see how they would use discovery learning in mathematical education. It was aimed to see how they collect this information in a different manner, and bring about that information as a whole with the help of associations beyond the theoretical and practical knowledge they had. For that reason, they were asked to prepare a different and new activity in which they would use discovery learning. When the activities they prepared were examined, it was observed that 5 participants (13.5%) created an activity that was new or unfamiliar to discovery learning approach, 4 participants (10.8%) designed a new or unfamiliar activity, and 5 participants (13.5%) inaccurately or deficiently designed a new or

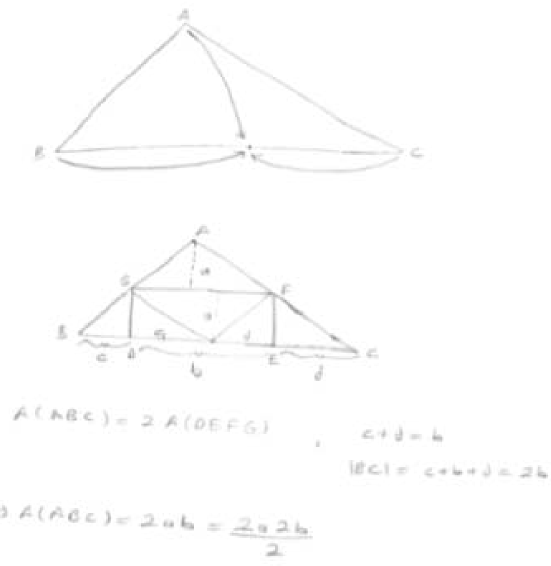

Figure-7. 36th Pre-service Teacher's Description.

unfamiliar activity. The other 23 participants (62.2%) neither answered the question nor prepared an activity. An example from the category of participants who *created* a new or unfamiliar activity is as follows:

"We draw a triangle and then cut it. Corners of triangle belonging to any base and the other opposite vertex of triangle are folded by combining them in a common point on the base. By this way, we obtain a rectangle. After that, we realize that the area of the triangle is double the area of the rectangle by asking them to link the whole area of the triangle with this rectangle. Later, we ask them to link it with the edges of the triangle and rectangle. In other words, we see that it is A(ABC)=2.A(DEFG)=2.a.b. Moreover, since c plus d equals to b, we can say that the base is 2b and the height of triangle is 2a by looking at |BC|=b+c+d=2b equality. Thus, as a result of A(ABC)=2.a.b equality which is recognized from the area of rectangle and linking of base (2b) and height (2a), (2.a.b = 2a.2b/2), they can conclude the generalization that the area of a triangle is one-half of multiplication of one of its bases with the height belonging to this base" *(36th pre-service teacher)* (Figure 7).

An example quotation of the pre-service teachers who *designed* a new or unfamiliar activity is as follows:

"We can enable students to achieve a general principle by providing them with different repeating decimals in different forms and asking them to get rid of the repeating parts of these decimals and bring the decimals into the rational shape. For instance, we ask them what we can do to get rid of the repeating part of $2,\bar{3}$. We make the students realize that we can multiply this decimal by 10 and save it from its repeating part by subtracting the decimal from this result since the repeating part of the decimal is made up of a number. In other words, for instance, if our decimal is $x=2,\bar{3}$ then in that case it is $10x=23,\bar{3}$ and as a result $x=\frac{23-2}{9}$ is obtained.

For a different case, we ask them how to get rid of the repeating part of $2,\overline{34}$ and this time, we expect them to realize that it is necessary to multiply the decimal by 100. Thus, if our decimal is $x=2,\overline{34}$ it becomes $100x=234,\overline{34}$ and, we obtain $x=\frac{234-2}{99}$ from here.

We ask them what we can do for $2,1\bar{3}$ again for a different case. Here, we expect them to realize that although the repeating part is again made up of a number, the previous number does not repeat, thus, we must multiply the decimal by both 100 and 10 and subtract it from them in order to get rid of the repeating part. That is to say, if our decimal is $x=2,1\bar{3}$, then it is possible to claim that $100x=213,\bar{3}$ and $10x=21,\bar{3}$ and $x=\frac{213-21}{90}$ is obtained as a result. .

For another sample case, we can examine $2,1\overline{34}$ decimal. From the same point of view, if the decimal is multiplied firstly by 1000, then by 10 and if it is subtracted from them and if we name the decimal as $x=2,1\overline{34}$ then we can say that $1000x=2134,\overline{34}$ and $10x=21,\overline{34}$ and $x=\frac{2134-21}{990}$ is obtained as a result. .

If needed, it is possible to give the students more examples and ask them to achieve a generalization which covers all cases. Thereby, so as to write repeating decimals rationally, considering the repeating part as a single number, the rule of whole number – the part which does not repeat (after comma) 9 in the amount of the number which repeats and 0 in the amount of the number which does not repeat can be obtained" *(34th pre-service teacher).*

Two examples quotations belonging to the participants who designed a deficient or inaccurate activity are as follows:

"Firstly, we ask students about the solutions to various equations with reference to their previous knowledge. For instance, they are asked to solve $x^2+2x+1=0$ equation. From $(x+1)^2=0$ equation, we find x=-1 value. Next, for example, we ask them to solve $x^2-4x-5=0$ equation. We calculate the x=-1 and x=5 values from $(x+1)(x-5)=0$ equation. We give them a few similar examples. Later, we ask them to solve $x^2+1=0$ equation In this example, it is stated that there is not any real number which enables the solution of $x^2=-1$ equation, furthermore, square of any real number is not equal to a negative number and then 'i' number is defined. Thus, the students realize the fact that the solution set to this problem consists of '-i' and 'i' numbers" *(18th pre-service teacher).*

"The students know $\cos(a+b) = \cos a.\cos b - \sin a.\sin b$. Here we expect them to assume $a = b$ and as a result they can calculate $\cos 2a = \cos^2 a - \sin^2 a \ldots$ (1) equation from $\cos(a+a) = \cos a.\cos a - \sin a.\sin a$ equation.

If we continue, we ask them to add and subtract $\sin^2 a$ value to the right side and here, they find $\cos 2a = 1 - 2\sin^2 a \ldots$ (2) equation from $\cos 2a = \cos^2 a - \sin^2 a + \sin^2 a - \sin^2 a = \cos^2 a + \sin^2 a - 2\sin^2 a$ equation. Then, we ask them to add and subtract $\cos^2 a$ equality to the right side in (1) equation and they find $\cos 2a = 2\cos^2 a - 1 \ldots$(3) equation from $\cos 2a = \cos^2 a - \sin^2 a - \cos^2 a + \cos^2 a = 2\cos^2 a - (\sin^2 a + \cos^2 a)$ equation. Thereby, they obtain three different equations in order to calculate the value of cos2a in the end" *(12th pre-service teacher).*

It is seen from the examples given that both of them discussed about defining in the first example and demonstrating in different equations in the second one instead of making the students discover and achieve a new concept (i number) or a new principle (trigonometrical formula). The creative thinking process can be described as bringing ideas or mental images together in order to create an original or appropriate solution to a problem or situation in a manner which was never applied before (Kilgour, 2006). In this process, the person forms a new idea by combining and reorganizing his/her available knowledge structure (Mumford et al., 1997). When we consider the process of designing an original activity by using discovery learning as a creative thinking, it can be said that the pre-service teachers' creative thinking skills are rather weak, judging by the fact that the number of the ones who created such an activity is quite low. We can conclude from this situation that the synthesizing skills of the pre-service teachers about the approach in their minds are very weak. One of the phases of the combination process, which is considered to be the main elements of creative thinking, is generating an idea (Kilgour, 2006). It can be claimed that some pre-service teachers are at the phase of generating an idea given the fact that they can design an activity, even though they cannot create the process of preparing an original activity. Thus, these pre-service teachers can combine the knowledge structure in their minds. Finally, we can conclude from this situation that they have cognitive competence in synthesizing process.

Consolidation evaluation

In the study, the fourth question the pre-service teachers were not only asked to prepare a new or different activity in which they would use the discovery learning approach; but also, evaluate the whole process the activities they prepared. Thereby, it was aimed to examine their evaluations about the whole process as well as their configurations about the process; and whether they passed on any judgments or not. It was observed from the pre-service teachers' evaluations that only 1 (2.7%) of them reached an opinion about the approach and the others (97.3%) either did not evaluate at all or did not pass on any judgments from their evaluations. The following quotation belongs to the pre-service teacher who could pass on a judgment from her evaluation:

"Discovery learning is a kind of learning which gives the individual the opportunity to learn the information by constructing it. The individual obtains new information on his/her own by using the information he/she already has. For example, in the activity we prepared, the individual calculates the area of the triangle by using the area of the rectangle. If this type of learning is applied properly, the information obtained can be both more permanent and more meaningful. When compared to other learning approaches, it works in contrast with Ausubel's meaningful verbal learning model. In meaningful verbal learning, flow direction of the information works from the whole to the pieces; and is teacher-centered, however, in discovery learning, it works from the pieces to the whole; and is student-centered. It resembles the individual's getting puzzle pieces together to see the whole picture. In fact, as long as we know the pieces forming the information, we can teach several subjects through discovery learning and design a variety of activities. Moreover, I think if we design the information, which we plan to make the students obtain by discovering, as a problem; and adapt this into daily life, we can actually make a realistic mathematics education happen" (36th pre-service teacher).

Consolidated information enables the use of that type of information in various cases, in a proper and confident manner (Dreyfus and Tsamir, 2004). Evaluation process can be described as the process of consciously passing on judgments based on a clearly defined criterion (Granello, 2001). Therefore, if the information is consolidated, the evaluation of that information will be parallel to its consolidation. The fact that only one of the pre-service teachers passed on judgments about the general structure of the approach and its association with other approaches demonstrates that the pre-service teacher has consolidated information and cognitive competences on the evaluation level.

SUGGESTIONS

While traditional or didactic mathematical education requires creating a routine procedure, modeling a sample problem, and later expects students to apply similar problems, reformist mathematical pedagogy requires designing assignments which expects students to rationalize the quantities, come up with their own strategies, and discuss their opinions (Stigler and Hiebert, 1999; Boaler and Humphreys, 2005). The most fundamental responsibility of the constructivist education knowing your students' mathematical knowledge and harmonizing the teaching methods related to the nature of mathematical knowledge (Steffe and Wiegel, 1992). Hence, the teacher plays an important role in the designing and application of the education. Within this context, it is of utmost importance that pre-service teachers start their service having both contextual and pedagogical skills. In this study, the cognitive skills of pre-service mathematics teachers in using the approaches in mathematical education, within the framework of constructivism, and how they structure the education in their minds were studied. Therefore, out of all approaches, discovery learning and the use of this approach in mathematical education are examined.

In this study, the pre-service teachers were firstly asked questions containing theoretical information about the approach and afterwards, they were asked to evaluate various activities and finally, they were expected to plan an activity which could be used in mathematical education. After the answers, the pre-service teachers had given and the works they had prepared were examined, it was observed that they generally had adequate information about the approach (94.6%), they were able to comprehend and explain this information rationally (91.9%), and they could partially apply an activity related to the approach (46%). These results indicate that the pre-service teachers are successful on 'reproduction-recognizing' level which can be referred as their knowledge and comprehension level. However, on the connection (building with) level, the pre-service teachers' success on application level decreased and on the analysis level, it diminished dramatically. Problems are not the application of mathematical knowledge and procedures but meaningful mathematical activities. Hence, teachers design this activity in order to see the students' level at learning and further improve it by guiding their rationalization (Lampert, 1990; 2001). When an activity, in which discovery learning approach is applied, is accepted as the related activity, it is seen that the pre-service students' success at problem solving (46%) and coordination (35.1%) processes on the application level declines, furthermore their skills at criticizing (21%) and dissection-comparing (10.8%) processes on the analysis level decreases. These results suggest that the pre-service teachers' success on the 'connection-building with' level which can be referred as application and analysis level dramatically declines. These results, which suggest that the level of success related to the comparisons about producing relations of the approach within itself and with other approaches is low, prove that the reflections in the minds of the pre-service teachers do not occur as desired. Hiebert and Carpenter (1992), described understanding as 'correlating ideas, events or procedures' (p.67), furthermore, they mentioned that understanding occurs in two methods which are 'studying differences and similarities; and, specifying the relations in it'. These generated results comply with the mentioned ideas.

In the process of preparing an original activity, it was observed that the pre-service teachers' skills in planning the process greatly decrease (13.5%) and again their skills at designing the process also decline (10.8%). This situation suggests that the success of the pre-service teachers on the 'reflection-construction' level which can be referred as their synthesis level is very low. These results indicate that the pre-service teachers are insufficient at preparing environments suitable for the application of new approaches within the framework of constructivism when we consider Richards' (1991) opinions that the teacher is supposed to come up with assignments and projects which will promote students to ask questions, create problems, and setting objectives and also Bruner's (1961) opinions that students become active learners not by chance but by planning in which they are guided to research and investigate. Besides, unless the teacher has an environment whose objectives, plan, and problem are well developed, the student will not do or learn anything (Brousseau, 1987). The students will learn some other things that might contain close answers to the teacher's questions (Simon, 1995). The results, showing that the pre-service teachers' success at planning an original activity or an activity which they are familiar with was very low and also they carried out the process inaccurately and falsely, suggest that the students will learn incomplete information containing misconceptions and they also support the discussed ideas.

It was observed in the study that the pre-service teachers failed (%2.7) at dealing with the approach by evaluating it with other approaches, and this result demonstrates that they are insufficient on the reflection-consolidation level. There are various discussions on how much guidance should be given to the students in discovery learning. However, Mayer (2004) claims that the most preferred approach in learning is not pure discovery learning or minimalist guidance, but the guidance approach. Since constructivism suggests structures collected in the learners' brains, the teacher's duty is to provide the student with experiences that will guide him/her to start with the old information. Under the guidance of carefully conducted question asking, the student's thinking about structure or problem is planned in a such way that it will head towards the thinking of the teacher or researcher. What is special about these approaches is that the development of the ideas is sequential and linear; also it has steps and orbits planned by the researcher or teacher (Richards, 1991). In this study, the pre-service teachers mentioned that guidance is the basis in the approach while providing the students with the planned information and specifically stressed that in their explanations. The reflections about the approach, which are formed in the minds of the pre-service teachers while they evaluate the in-class activities, comply with the idea of basing education upon on a counseling approach.

It was observed in the study that even though the pre-service teachers might theoretically know how they should apply the approach or evaluate the given activities about the approach, one of the reasons why they are insufficient at planning an activity is that they do not have contextual information about the nature of the knowledge which they are supposed to provide their students with. Thus, the pre-service teachers had a hard time organizing the information they would teach and could not prepare an environment suitable for association.

CONCLUSİON

Considering the results obtained by the study, firstly, the pre-service teachers should have theoretical knowledge about the information which will enable association. In a sense, the pre-service teachers should figure out and accurately structure what the information is about and how it is obtained, in their minds. Moreover, the pre-service teachers should know the information from the pedagogic point of view, select the proper ways to transfer the information to the students, and prepare the learning environment according to the selected approach. That's why the pre-service teachers should be rationally provided with both contextual and pedagogic knowledge and with environments enabling them to associate these two types of knowledge and structure them in their minds.

Conflict of Interests

The author has not declared any conflict of interest.

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