Full Length Research Paper

Maple (Computer Algebra System) in teaching Pre-Calculus: Example of Absolute Value Function

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Modules in Computer Algebra Systems (CAS) make Mathematics interesting and easy to understand. The present study focused on the implementation of the algebraic, tabular (numerical), and graphical approaches used for the construction of the concept of absolute value function in teaching mathematical content knowledge along with Maple 9. The study group consisted of pre-service teachers attending Department of Primary Education Mathematics teaching at a state university. The pretest (open ended questions) achievement and the posttest (open ended questions) achievement of the group were compared based on their answers. It was concluded that CAS was influential on pre-service teachers' usage of the said learning approaches.

Key words: CAS (Computer Algebra Systems), teaching Pre-Calculus, absolute function.

INTRODUCTION

CAS is a tool aimed at introducing phenomenal, conceptual, procedural knowledge and improving metacognitive knowledge. Traditional approaches emphasize teacher-centered procedural mentality, but alternative approaches encourage more student-centered conceptual mentality and mathematical process (Hiebert; NCTM). Computer Algebra System (Maple) software contribute to in-depth usage of many Mathematics concepts (Algebraic, Numerical and Graphical) (Baki, 2008). Thus, the present study made an attempt to answer the below-mentioned question:

"How does the use of technology (CAS-maple software) in teaching contribute to the Algebraic, Tabular and Graphical representations of absolute value function by

pre-service mathematics teachers?"

When the question "What is the goal of Mathematics course in primary school, middle school and high school?" is asked, the first answer coming to mind is "to prepare for mathematical analysis" through today's constructivist approach, which refers to putting conceptual understanding in the center based on phenomenal knowledge. Mathematical analysis begins with the concepts of equation and function and the search for the ways of combining them with mathematical operations. The concept of function is a dynamic mechanism that represents change and the transformation brought by such change (input-output process). Let's assume that a elementary school teacher or a middle school

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Mathematics teacher who works on the set of numbers says, "$2 + 3 = 5$". Based on the definition, "*Given the sets A, B and C that are different from empty set, but can be equal to one another, each function from a subset of $A \times B$ to C is called a binary operation.*", s/he works on a function that takes two elements in $\square$, and transforms them into 5, a new element, in $\square$ by means of the operator +. We can continue it with multiple representations (example, sequences and series). Textbooks cover the concept of function under different titles, such as absolute value functions. The present study was considered significant in that it attempted to provide an outlook on;

1. The improvement of the analytical thinking of pre-service teachers,
2. How pre-service teachers can assess the analytical thinking skills of their students.

The Maple software is used for evaluating and providing the analytical solutions of many Mathematics problems. Some usage areas are as follows:

1. The application of complex mathematical procedures in mathematical analysis,
2. The evaluation and abbreviation of mathematical expressions, and the usage of Algebraic expressions in different ways,
3. Various operations on functional graphs (example, interventions in the movements of objects, two-dimensional and three-dimensional drawings, animations, etc.).

The review of the related literature shows that difficulties regarding the concept of function are addressed under the titles of:

1. Mathematical thoughts existing in the context of conceptual and procedural knowledge (Vinner, 1983; Hiebert and Levefre, 1986; Harel and Dubinsky, 1992, Baki, 2008), and
2. Multiple representations used for the presentation of the concept (Einsenberg, 1991).

In this regard, the Maple is one of the computer algebra systems that are widely used for symbolic or algebraic use and visualization. In this study, an attempt was made to investigate the effect of Maple 9 on absolute value function and its properties.

The Concept of function and technology

The concept of function is the basis of both General Mathematics and Mathematical Analyses courses (Romberg et al., 1993; Harel and Dubinsky, 1992). It constitutes the framework of other Mathematics concepts. The ways of thinking applied in search of a mathematical relation involve the expression of a concept or rule by use of verbal, graphical, tabular, or algebraic symbols (Choike, 2000; Radford, 2001). Students select among these mathematical representations when solving the problems. They apply and transform such selection (NCTM 2000). According to Breslich (1928), it is impossible to understand and appreciate Mathematics without the functional thinking that focuses on the relationships between the quantities. There are many studies dealing with functions (Bayazıt and Giray, 2004; Baki, 2008, Tuluk and Kaçar, 2007). The concept of function involves many connected concepts. The representations of the function itself and the connected concepts are commutative.

In high school education, the concept of function is constructed based on the principle, "*Given the sets A and B, that are different from empty set, but can be equal to one another, the relation f from A to B that matches each element of A with only one element in B is called function from A to B, and is represented as follows: $f : A \rightarrow B$*" (the definition of Dirichlet-Bourbaki). "$\rightarrow$" shows that these two sets are compared. In addition, it is necessary to know the equation, $y = f(x)$, too. This equation is synonymous with $(x, y) \in f$ or $(x, f(x)) \in f$.

According to that, for each element in A, $f(x)$ image values can be showed with;

1. An arrowed chart.
2. A rule indicating the matching of y with x.
3. Ordered pairs.
4. A table.
5. A graph in an analytical plane (Eisenberg, 1992).

Slavit and Yeidel (1999) states that a teacher needs to establish relationships between functional properties in order to understand the concept of function. For example, 60% of the customers of a store purchased fewer than 5 bags, 30% of them purchased 5 to 10 bags, and 10% of them purchased 10 or more bags. Given that sales are between 100,000 and 200,000, how can you explain this situation algebraically? (Thuy et al., 2008).

```
> a:=piecewise(x<=5,2*x, 5<x and x<=10,10+1.5*(x-5),x>10,17.5+(x-10) );
```

$$a := \begin{cases} 2\,x & x \le 5 \\ 2.5 + 1.5\,x & 5 < x \text{ and } x \le 10 \\ 7.5 + x & 10 < x \end{cases}$$

As in the example provided above, it is necessary "to define different subintervals of the domain as separate equations, and to regard each part defined in this way as

a component of the function". In order to improve the conceptual construction of the absolute value function, the function, $f(x) = ax \pm b$ was taken as basis, and the functions $f(x) = |ax \pm b|$, $f(|x|) = |ax| \pm b$, and $|f(|x|)| = \|ax| \pm b|$ were investigated. The absolute value function is a piecewise function. Thus, the object of the task was to examine $|f(x)|$, $f(|x|)$, and $|f(|x|)|$ separately based on $f(x)$. Therefore, it can be said that the concept of absolute value function can improve one's communication, abstraction, logical thinking, and critical thinking skills. The absolute value was developed by Argand (http://en.wikipedia.org/wiki/Jean-Robert_Argand) in the 19th century in order to construct complex numbers in a complex plane (known as the Argand plane). In school Mathematics, learning goals about the absolute value are as follows:

1- Identifying the absolute value of a real number and showing it with symbols

2- Showing $-|a| \le a \le |a|$ for $\forall a \in \mathbb{R}$

3- Telling and showing the triangle inequality, $-(|a| + |b|) \le a + b \le |a| + |b|$)

4- Showing ($x, y \in \mathbb{R}$, $|x.y| = |x|.|y|$)

5- Showing $\left| \dfrac{x}{y} \right| = \dfrac{|x|}{|y|}$ for $x, y \in \mathbb{R}$, $y = 0$

6- Showing $n \in \mathbb{R}^+ \Rightarrow |x^n| = |x|^n$

In Mathematics, an absolute value or an absolute value function gives the unsigned numerical value of a real number. For instance, $|3|$; $|3| = 3$ and $|-3| = 3$. In computers, the mathematical function which is used for expressing this function is generally abs(...)

> **abs(-3);abs(3);**
3 , 3

Performing operations about absolute value involves;

1- Solving the first-degree equations including only a single term with absolute value in the set of real numbers, and showing solution sets on a numerical axis
2- Solving the first-degree equations including only one unknown and a single term with absolute value in the sets of natural numbers, integers, rational numbers, and reel numbers, and showing solution sets on a numerical axis.

The learning goals regarding the concept of absolute

value function are as follows:

1. Identifying the domain, range, and image set of a function.
2. Identifying the graph of a function.

For mathematical analysis, it is important that pre-service teachers explore the works and discoveries on content knowledge and pedagogical content knowledge concerning the absolute value function as well as the symbols and the roles of such symbols in mathematical interaction. The concept of absolute value is the basis of many mathematical subjects such as series, sequences, convergence, divergence, limit, derivative, etc. (Şandır et al., 2002). After the pre-test was conducted, the concept of function was introduced, and first-degree equations and functions were covered. The difference between linear equations and absolute value equations was highlighted. The lesson on the absolute value started with the question, "place the natural numbers whose sum and difference are 5 in the coordinate plane". Since the pre-service had generally the experiences of hurrying, finding two points in graphic drawings, and drawing the curve in high schools and middle schools, the present study attempted to improve their experiences in thinking about ordered pairs and placing such ordered pairs in the coordinate plane.

```
> restart:with(plots):
a:=plot(0,x=-10..10,y=-10..10):
b:=pointplot({[4,1],[1,4],[2,3],
[3,2],[0,5],[5,0],[6,1],[7,2],
[8,3],[9,4],[10,5]}):display(a,b);
or
>pointplot({[4,1],[1,4],[2,3],
[3,2],[0,5],[5,0],[6,1],[7,2],
[8,3],[9,4],[10,5]},color=red);
```
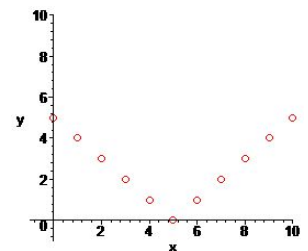

In the analytic plane, the points were discussed and put on the worksheet. They formed a line segment and straight lines. Thus, it became possible to obtain the equations of such straight lines. It was observed that the pre-service teachers were unwilling to place the points given as ordered pairs in the coordinate plane (Knuth, 2000; Leinhardt et al., 1990; Van Dyke and White, 2004), and had difficulty calculating and writing (algebraic – analytical context) the equation of the straight lines based on such points. In the treatment of the equation as a function, the following algebraic process was examined:

```
> with(student):
f:=makeproc([5,0],[10,5]);
```
$f := x \rightarrow x - 5$

```
> with(student):
f:=makeproc([5,0],[0,5]);
```
$f := x \rightarrow -x + 5$

```
> restart:with(plots):f:=x->x-5;
```
$f := x \rightarrow x - 5$
```
> g:=x->-x+5;
```
$g := x \rightarrow -x + 5$
```
> plot(f(x),x=5..10);
```
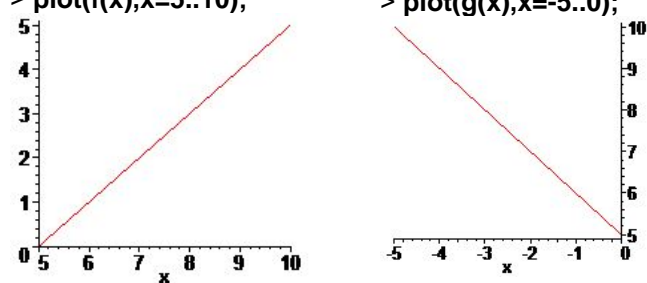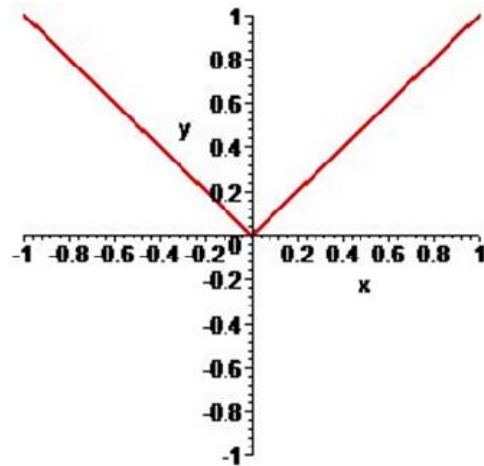```
> plot(g(x),x=-5..0);
```

**Figure 1.** The definition of $|x|$

The determination of the domain and the range failed to attract the attention of the pre-service teachers. In addition, it was difficult to create willingness for changes (example, *x=5..0, x=-5..5*). It was seen that the pre-service teachers watched the computer like a television screen or a presentation screen on which only the lesson would be taught, but no operation would be performed. That may have resulted from the habits of the pre-service teachers regarding representing functions via charts or writing only a couple of ordered pairs and revealing a curve (it can be a straight line, too) in the plane. Inexperience in writing notations via Computer Algebra Systems may have caused that. However, it is thought that awareness should be raised among the pre-service teachers that computer is a writing and calculating tool on whose screen data are entered and operations are performed, and the pre-service teachers should be encouraged for making use of the writing and reading functionality of computer. Following the above-mentioned examinations, the representation of piecewise function was focused on in the algebraic expression of the function.

```
> h:=piecewise(x>5,x-5,x=5,0,x<5,-x+5);
```

$$h := \begin{cases} x - 5 & 5 < x \\ 0 & x = 5 \\ -x + 5 & x < 5 \end{cases}$$

The pre-service teachers were requested to deal with the ordered pairs again and pay attention to the instructional need.

```
> for x from 5 to 10 do
print (x, f(x)) od;
```
5, 0
6, 1
7, 2
8, 3
9, 4
10, 5

```
> for x from -5 to 0 do
print (x,g(x)) od;
```
-5, 10
-4, 9
-3, 8
-2, 7
-1, 6
0, 5

With the above-mentioned commands, it was started to develop an understanding on the how the table came up and how the said ordered pairs determined the graph. Domain and range were worked on. The acquisition of a function as a real-value and single-variable function was discussed. The piecewise function was proceeded to. Graphs were drawn for two straight lines separately. Then such two graphs were turned into a single graph. In addition, representation was focused on. It was indicated that for making out the equation, the geometric approach could be used

```
> with(Student[Precalculus]):
> Line([5,0], 1)[1];
```
$y = x - 5$

```
> Line([0,5], -1)[1];
```
$y = -x + 5$

Or (veya)
```
> with(geometry):point(A,5,0),point(B,6,1):
> line(l,[A,B]);Equation(l,[x,y]);
```
$l \quad 5 - x + y = 0$

Different representations (example, the one provided above) could be employed through computer algebra systems.

```
> f:=x->abs(x);
plot(f(x),x=-1..1,y=-1..1,thickness=3);
```
$f := x \rightarrow |x|$

In the graph (Figure 1), based on the definition of $|x|$, the absolute value function, f should be included in the

Pretest sample answers

Posttest sample answers

Figure 2. Pretest and post test achievement scores

expression through the definition, $f(x) = \begin{cases} x & , & x > 0 \\ 0 & , & x = 0 \\ -x & , & x < 0 \end{cases}$.

Attention was drawn to algebraic representation, that is, how to write an equation.

In this expression which is a piecewise function, it should be emphasized that in the plane, $\square x \square$, the union $\begin{bmatrix} OA \cup \end{bmatrix} OB$ of the half-lines, $\{OA = \{(x, y): y = x, x > 0\}$ and $\{OB = \{(x, y): y = -x, x \le 0\}$ should be interpreted geometrically based on the following construction: "it is a curve, but looks like a wide V letter or a broken line". Since $y = f(x) = |x|$ is not addressed as a function, less attention is paid to $f : \square \to 0 \cup \square^+$ in terms of domains and ranges. Verbally, that should not require leaving out, in terms of geometric representation, the interpretation that the graph of $y = f(x) = |x|$ consists of the half of the straight line, $y = f(x) = x$ and the left half of the straight line, $y = f(x) = -x$, as showed in the Figure 2. Most of the pre-service teachers made such explanations in the pre-test. In Mathematics lessons, the outlooks of pre-service teachers on subjects should be strengthened in terms of different representations. That should be taken into consideration while covering the concept of relation in algebra courses. That is reinforced by the Maple as the following:

```
> f:=x->piecewise(x>0,x,x=0,0,x<0,-x);
```
$f := x \to \text{piecewise}(0 < x, x, x = 0, 0, x < 0, -x)$

```
> f:=piecewise(x>0,x,x=0,0,x<0,-x);
```
$f := \begin{cases} x & 0 < x \\ 0 & x = 0 \\ -x & x < 0 \end{cases}$

The function is symmetric in view of the axis y (straight line of symmetry). It is associated with reflection in secondary education. The symmetry is based on the axis y. Since $f(x) = f(-x)$ for $f(x) = |x|$, f is the even function. Thus, its standing as an even function should be stressed. The origin (0,0) is an intersection point of the function. It is a critical point. The derivative of an absolute value function reveals that it must be treated as a piecewise function, and must be constructed very well due to the whole angle coming into existence at 0. The primary concept needed for mathematical analysis is function. This is because we cannot initiate an analysis, or we have difficulty initiating it without understanding it. If a student studying in the field of sciences starts mathematical analysis without understanding this concept, s/he may have difficulty in understanding other subsequent concepts and even other mathematical objects (Schwarzenberger, 1980; Tall, 1992).

In the later 17th century, Leibniz became the first mathematician who used the notation of function in his

Table 1. Question 1- In-group comparison of the algebraic representations of the function, $f(x) = |2x - 2|$ (pretest-posttest).

MD-C	$\overline{X}$	df	t	P
Pretest	1.96	25	-2.601	0.015
Posttest	2.42			

writings in order to explain a varying *y* value depending on the *x* variable (Tall, 1997). The concept was presented with a specific formula and the general use of $y = f(x)$ in the following centuries. The review of the related literature shows that teachers turn to conceptually understanding various Mathematics concepts via alternative approaches without losing basic procedural skills (Hiebert, 2003; Hiebert and Wearne, 1996; Kamii and Joseph, 1989; Wood and Sellers, 1996). Moreover, the process standards suggested by NCTM (problem-solving, reasoning and proving, interaction) are more careful than those provided by traditional approaches (Hiebert, 1986, 2003; NCTM, 2000). The pre-service teachers were not interested in tabular representation and algebraic representation in the first graphical drawing. The absolute value function is a function that must be written algebraically, that is, as a piecewise function. 3 students solved the equation, $2x - 2 = 0$, and showed that the point, $x = 1$ was the critical point. Since they were the pre-service teachers studying in the department of Mathematics, they did not employ tabular representation. In addition, they did not think that there was a need to determine the domain and the range of the function. In the pre-test, none of 26 students emphasized that the domain and range of the function should be determined.

Thus, with the question, "place the natural numbers whose sum and difference are 5 in the coordinate plane" at the beginning of the lesson, the determination of the points, the extension of a straight line on two points, the acquisition of the equation of the straight line, and the absolute value function (on the arising graph) were worked on. In this way, the absolute function was included in the equations of straight lines on the points, and thus the expression of a piecewise function.

METHODOLOGY

The research participants were 2nd grade prospective teachers attending a state university located in the Western Black Sea Region of Turkey in the fall semester of the 2012 to 2013 academic year. None of the pre-service teachers participating in the study had experienced a learning environment where they had faced any CAS software or graphing calculator in textbooks or other printed sources beforehand. None of them had used any graphing calculator, CAS, or dynamic graph calculator (DGS). Furthermore, it

was seen that the students did not have much knowledge and ability to write math signs on computer.

Many Mathematics educators expect students to employ tabular (numerical), algebraic, and graphical approaches and move between such representations flexibly so that they can understand the subject of function (NCTM, 2000). Before the study began, three open ended questions, as part of pre-test, were addressed to the pre-service teachers. The questions were taken from Monaghan and Özmantar (2007) on abstraction. The researcher gave the function of $f(x) = 2x - 2$, and examined how the pre-service teachers dealt with the functions of $|f(x)|$, $f(|x|)$ and $|f(|x|)|$. Then the pre-service teachers studied in the computer laboratory for 4 course hours in order to learn how to use Maple 9. After that, the lessons were launched. The study was carried out by the researcher in the computer laboratory in 4 course hours for 2 weeks. The post-test was administered to the pre-service teachers at the end of 4 course hours. In the present study, absolute value function was examined in terms of:

1. MDF-C: algebraic (symbolic) interpretation (domains and ranges, the solution of equation)
2. MDF-T: tabular interpretation
3. MDF-G: the graphic drawing of function. By integrating technological knowledge into pedagogical content knowledge.

The answers of the pre-service teachers to the questions were charted. Based on the research of Weber (2008), the chart coded the answers of the pre-service teachers as follows:

1- Correct (pre-service teacher made explanations by indicating the necessary information – capability to make proper mathematical explanations),
2- Deficiency in resorting to the knowledge (pre-service teacher made some correct explanations without displaying the necessary information),
1- No comment (explanation)
0- Incorrect.

Research design

When the same subjects are measured by a dependent variable before and after quasi-experimental process, the subjects are exposed to repeated temporal measurements, and the obtained results are interrelated. It is called repeated measures design. It is an in-group and one-factor design (Büyüköztürk, 2001).

FINDINGS

According to Table 1, there was a significant difference (t(25)=-2.601 p<.015) between the pretest and posttest achievement scores of the pre-service teachers learning via CAS (Maple 9 software) in the algebraic expression of the absolute value function ($f(x) = |2x - 2|$). Thus, it was understood that the CAS was influential on the algebraic expression of the absolute value function. As shown in Figure 2.

According to Table 2, there was a significant difference (t(25)=-12.810, p<.000) between the pretest and posttest achievement scores of the pre-service teachers learning via CAS (Maple 9 software) in the tabular expression of

Table 2. Question 2- In-group comparison of the tabular representations of the function, $f(x) = |2x - 2|$ (pretest-posttest).

MD-T	$\overline{X}$	df	t	P
Pretest	1.19	25	-12.810	.000
Posttest	2.65			

Table 3. Question 3- In-group comparison of the graphical representations of the function, $f(x) = |2x - 2|$ (pretest-posttest).

MD-G	$\overline{X}$	df	t	P
Pretest	2.69	25	-1.806	0.083
Posttest	2.92			

Table 4. In-group comparison of the algebraic representations of the function, $f(x) = 2|x| - 2$ (pretest-posttest).

MD- C	$\overline{X}$	df	t	P
Pretest	1.46	25	-2.206	0.037
Posttest	1.92			

Table 5. In-group comparison of the tabular representations of the function, $f(x) = 2|x| - 2$ (pretest-posttest).

MD-T	$\overline{X}$	df	t	P
Pretest	1.81	25	-2.379	0.025
Posttest	2.27			

Table 6. In-group comparison of the graphic representations of the function, $f(x) = 2|x| - 2$ (pretest-posttest).

MD-G	$\overline{X}$	df	t	P
Pretest	2.15	25	-3.143	0.004
Posttest	2.85			

the absolute value function ($f(x) = |2x - 2|$). Thus, it was understood that the CAS was influential on the tabular expression of the absolute value function. As shown in Figure 3

According to Table 3, there was no significant difference (t(25)=-1.806 p<.083) between the pretest and posttest achievement scores of the pre-service teachers learning via CAS (Maple 9 software) in the graphical expression of the absolute value function ($f(x) = |2x - 2|$).

Thus, it was understood that the CAS was not influential on the graphical expression of the absolute value function. Although, most of the pre-service teachers had a concept image for graphical drawing, they had difficulties in algebraic and tabular expressions. CAS helped them in that matter.

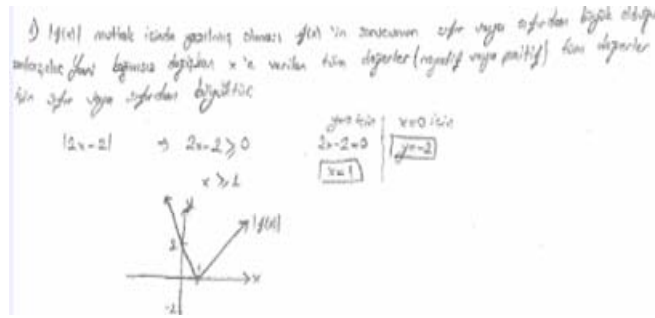According to Table 4, there was a significant difference (t(25)=-2.573, p<.016) between the pretest and posttest achievement scores of the pre-service teachers learning via CAS (Maple 9 software) in the algebraic expression of the absolute value function ($f(x) = 2|x| - 2$). Thus, it was understood that the CAS was influential on the algebraic expression of the absolute value function. As shown in

Figure 4.

According to Table 5, there was a significant difference (t(25)=-2.379, p<.025) between the pretest and posttest achievement scores of the pre-service teachers learning via CAS (Maple 9 software) in the tabular expression of the absolute value function ($f(x) = |2x - 2|$). Thus, it was understood that the CAS was influential on the tabular expression of the absolute value function. As shown in Figure 5. For example, lack of algebraic and tabular interpretation in the pretest caused a failure in writing that the graphical drawing consisted of two separate equations.

For example, lack of algebraic and tabular interpretation in the pretest caused a failure in writing that the graphical drawing consisted of two separate equations.

According to Table 6, there was a significant difference (t(25)=-3.143, p<.004) between the pretest and posttest achievement scores of the pre-service teachers learning via CAS (Maple 9 software) in the graphical expression of the absolute value function ($f(x) = |2x - 2|$). Thus, it was understood that the CAS was influential on the graphic expression of the absolute value function. As shown in Figure 6.

While most of the pre-service teachers were successful in solving algebraic equations, they had difficulty in drawing the graphs of such equations (as a straight line). That was more apparent in the case of absolute value function. In high school mathematics, the equation presented with the rule, $y = ax \pm b$ is drawn in the coordinate system by finding two points providing it. However, an understanding should be developed on not only drawing a straight line by finding two points but also directions from southwest to northeast and from northwest to southeast.
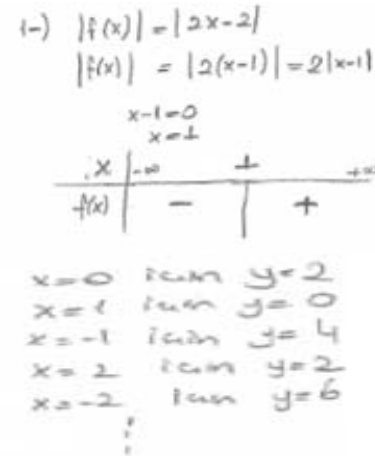
Pretest sample answers

Posttest sample answers

Figure 3. Difference between the pretest and posttest achievement scores of the pre-service teachers learning via CAS

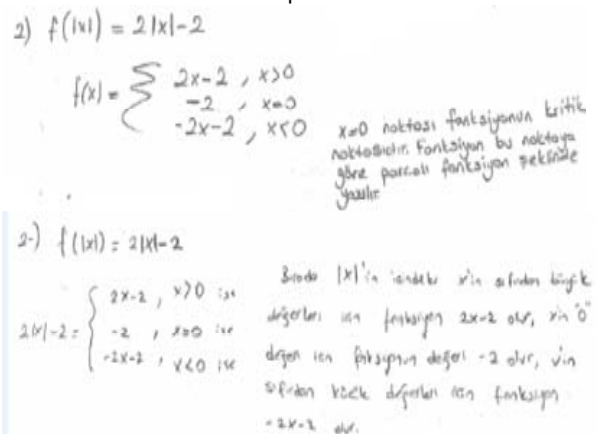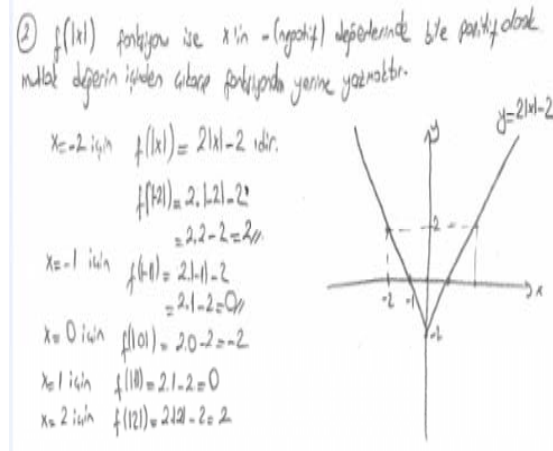Pretest sample answers

Posttest sample answers

Figure 4. Algebraic expression of the absolute value function.

Table 7. In-group comparison of the algebraic representations of the function, $f(x) = 2|x| - 2$ (pretest-posttest).

MD- C	$\overline{X}$	df	t	P
Pretest	1.31	25	-2,184	,039
Posttest	1.69			

According to Table 7, there was a significant difference (t(25)=-2.184, p<.039) between the pretest and posttest

achievement scores of the pre-service teachers learning via CAS (Maple 9 software) in the algebraic expression of the absolute value function ($f(x) = |2|x| - 2|$). Thus, it was understood that the CAS was influential on the algebraic expression of the absolute value function. As shown in Figure 7.

According to Table 8, there was no significant difference (t(25)=-1.690, p<.103) between the pretest and posttest achievement scores of the pre-service teachers learning via CAS (Maple 9 software) in the tabular expression of the absolute value function ($f(x) = |2|x| - 2|$).

Pretest sample answers
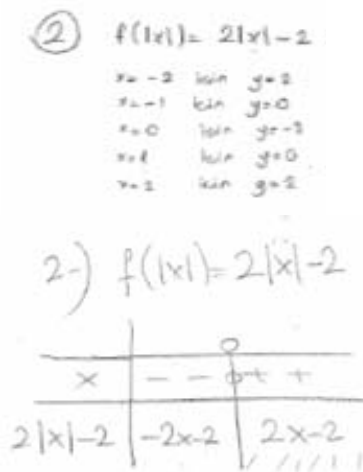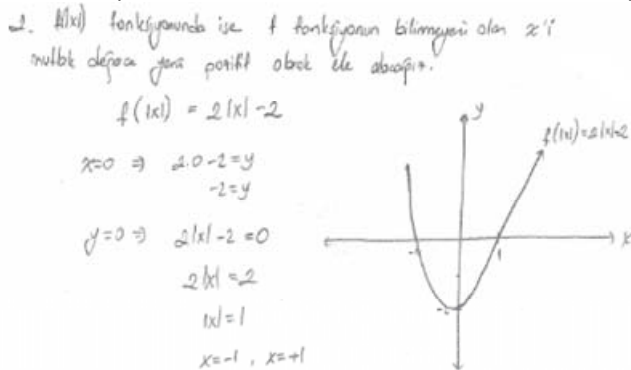
Posttest sample answers

Figure 5. CAS influential on the tabular expression of the absolute value function.

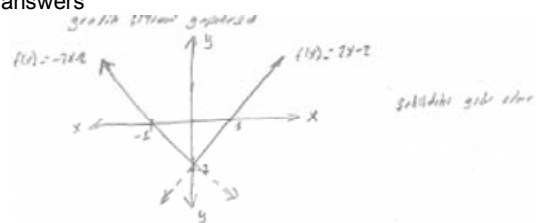Pretest sample answers   Posttest sample answers

Figure 6. CAS was influential on the graphic expression of the absolute value function.

Table 8. In-group comparison of the tabular representations of the function, $f(x) = \big|2|x| - 2\big|$ (pretest-posttest).

MD-T	$\overline{X}$	df	t	P
Pretest	1.62	25	-1.690	.103
Posttest	1.92			

Thus, it was understood that the CAS was not influential on the algebraic expression of the absolute value function.

According to Table 9, there was a significant difference (t(25)=-2.961, p<.007) between the pretest and posttest achievement scores of the pre-service teachers learning via CAS (Maple 9 software) in the graphical expression of the absolute value function ($f(x) = \big|2|x| - 2\big|$). Thus, it was

Pretest sample answers   Posttest sample answers

Figure 7. CAS was influential on the algebraic expression of the absolute value function.

Pretest sample answers   Posttest sample answers

Figure 8. CAS was influential on the graphical expression of the absolute value function.

Table 9. In-group comparison of the graphic representations of the function, $f(x) = \left|2|x| - 2\right|$ (pretest-posttest).

MD- C	$\overline{X}$	df	t	P
Pretest	1.92	25	-2.961	.007
Posttest	2.62			

understood that the CAS was influential on the graphical expression of the absolute value function. As shown in

Figure 8.

CONCLUSION

In the study conducted by Even (1998) with 152 university students, it was found out that 14% of the students succeeded in establishing associations between the algebraic and graphical representations of functions. This study made an attempt to bring forward a solution, via CAS, for the difficulty in expressing the absolute value function algebraically, tabularly and graphically. Functions
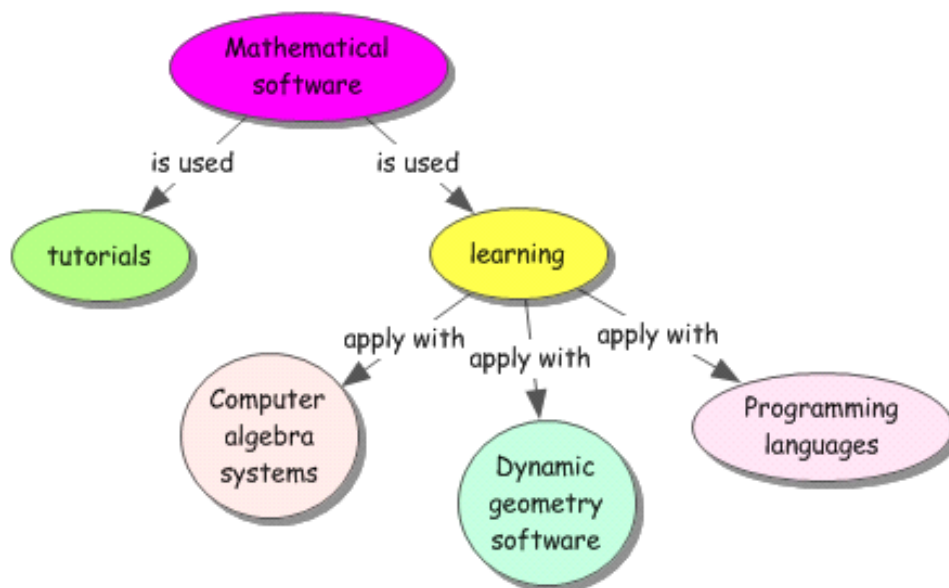
Figure 9. Analysis and General Mathematics courses supported by software fit.

can be examined in many ways including formulas, graphs, ordered pairs, arrow diagrams, tables, etc. (Eisenberg, 1992). That confronted us as a difficulty in the algebraic, tabular, and graphic representations of the function, $|f(x) \pm a|$ (Vinner, 1983, Leinhardt et al., 1990). The pre-service teachers had difficulty in the graphical representation of the piecewise functions composed of curves and straight lines. CAS software can be helpful in this matter. In general, function is regarded as a formula (Graham and Ferrini-Mundy, 1990). That makes it difficult to acquire the concept (Eisenberg, 1991). A large majority of students consider the function an arithmetic or algebraic rule (Vinner, 1983; Sfard, 1992). CAS software may be helpful in the case of the absolute value function, too. General Mathematics courses can be treated as part of preparation for courses on mathematical analysis. The structures of such courses and the use of technology in the conduct of these courses need to be researched and discussed in Turkey. Analysis and General Mathematics courses can be supported by software fit for the purpose, as indicated in the Figure 9.

The present study showed that the pre-service teachers' learning of the absolute value function was improved by interactive software (example, Maple). It is evident that educators should employ such software besides tradi-tional approaches.

Conflict of Interests

The author have not declared any conflict of interests.

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