Full Length Research Paper

An analysis on the pattern generalizations of the Turkish pre-service Mathematics teachers that are presented in a different structure and presentation

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The pattern, which is a key concept in understanding the mathematical information and concepts, is the basis in comprehending mathematical relations and in understanding mathematical order and logic. The fact that students discover the relationships contained within the patterns and generalize them helps them develop their skills to better perceive the world around them. The major factors in the development of these skills are the math teachers. The objective of this study is to examine and analyze the pattern generalization performances and strategies of pre-service math teachers. A total of 221 pre-service teachers attending the secondary school mathematics teaching (1st, 2nd, 3rd, and 4th classes) participated in this study. It was found that in a pattern test prepared in different structures (linear, geometric, quadratic) and in different forms of presentations (table, figure, verbal, number), as the class level increased, so did the pattern generalization performances of the teacher candidates. It was also found out that asking patterns within different forms of presentations enabled the teacher candidates to understand the patterns better. Separately, rather than the forms of presentation, it can be stated that the performance to generalize patterns and the pattern structure are more effective.

Key words: Structure of pattern, presentation of pattern, pre-service math teacher.

INTRODUCTION

There is a system in a number of areas, such as the arrangement in the cones of pine trees, wall papers, tiles, artworks, paintings, or in the music tracks, in a leopard's spots, in the flow of water, in the rolling of a dice, in the earth's shape, in weather forecasts, etc. (Vogel, 2005; Yaman, 2010). It can be said that there are mathematical correlations and patterns within this set of rules and order (Devlin, 1998). The fact that nature is composed of several complex patterns and that the mathematicians have been trying to discover and identify such patterns throughout the history also put forward the progression of patterns in the field of mathematics. This situation has led the researchers of mathematics and math education to get interested in this subject (Orton and Orton, 1999). The reason for their interest is considered to be the fact that the mathematical correlations and patterns, after having been found, came to play the key role in understanding the mathematical concepts. The
relationship between tangible objects (Uygur-Kabael and Tanışlı, 2010); generalization, which is a cognitive state in concept formation and the concept of pattern regarding the understanding of the structure of mathematics have a significant place in the early stages of the primary education (Hargreaves et al., 1999).

There have been various definitions made on the concept of pattern by the educators of mathematics. Although Orton and Orton (1999) state that it is not so easy to define the concept of pattern, several math educators are seen to have made various definitions from their own perspectives. Patterns are the key concepts in understanding mathematical information and concepts. Studies conducted on patterns form the basis of comprehending mathematical relations and understanding mathematical order and logic (Burns, 2000). Patterns are particularly the basic elements in the development of little children’s mathematical thinking skills (Olkun and Toluk-Uçar, 2009). The fact that students discover the relationships contained within the patterns and generalize them helps them develop their skills to better perceive the world around them (Akkan and Çakıroğlu, 2012). In addition, the different forms of representation of patterns and their symbolic illustrations in particular contribute a great deal to the transition from arithmetics to algebra, algebraic thinking and the formation of the basic concepts of algebra (Schliemann et al., 2003). When the studies regarding the skills to recognize and use the mathematical patterns are reviewed, it is particularly seen in the studies conducted in recent years that the concept of pattern has been focused on at full length (Mason et al., 2005; NCTM, 2000). It is emphasized that recognizing and using patterns are important skills in the education of mathematics. It is also pointed out that at the end of the first grade of primary education, the students need to be able to identify and expand geometric and digital patterns and make generalizations over them; and express and analyze the patterns verbally, by using tables and graphics (NCTM, 2000).

THEORETICAL BACKGROUND

In order to be able to make sense of the patterns better, the types of patterns, their features and their mathematical analyses could be focused on. Patterns, according to the elements contained within them, can be classified as digital (numeric) and verbal patterns as well as figures and tables; on the other hand, they can be classified as linear, geometric and quadratic patterns considering the differences among them. Additionally, according to the relationships among the elements, they can be classified as the patterns with recursive and explicit relations (Threlfall, 1999; Feifei, 2005; Ley, 2005; Olkun and Yeşildere, 2007; Amit and Neria, 2008; Yeşildere and Akkoç, 2010). In general, patterns are seen to have gathered in two groups as the recursive and variable ones. The recursive patterns are those that are expressed as the units of recurrence by which several certain elements follow a cyclical progression. The variable patterns, on the other hand, are those that are formed in the way that the relationship between the terms follow an expanding or narrowing course, and they are grouped in three different forms as linear, geometric and quadratic patterns (Threlfall, 1999; Olkun and Yeşildere, 2007).

**Linear variable pattern:** This is the pattern in which a constant number is added to or substracted from the previous one of each succeeding term.

**Geometric variable patterns:** These are the patterns in which the succeeding terms vary in proportions.

**Quadratic patterns,** on the other hand, are those that are neither linear nor geometric, yet, they vary within a certain order (Feifei, 2005; Olkun and Yeşildere, 2007; Tanışlı and Özdəş, 2009).

It is seen in the studies concerned with pattern generalization that many varieties of patterns, such as linear and non-linear (quadratic and geometric) patterns, digital patterns, illustrated patterns, arithmetic patterns, geometric patterns and recursive patterns (Feifei, 2005; Rivera, 2007; Shute et al., 2005; Amit and Neria, 2008). Of these studies, those belong to Ley (2005) and Feifei (2005) in particular, which deal with the classifications over the formats of different types of patterns. NCTM standards on patterns in grades 6-8: all students should represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules (Feifei, 2005, p.62). Ley (2005) referred to the patterns of the first-degree in five different formats as figures, geometric and verbal problems, tables and sequence numbers, while Feifei (2005), in addition to Ley, mentioned the linear, quadratic and geometric sequence patterns (Akkan and Çakıroğlu, 2012). Bruner (1966) stated that applying the stages of different structural and presentational forms respectively provided convenience for students in their new learning materials. Therefore, the states of patterns used in different representations in the studies conducted by Ley (2005) and Feifei (2005) are considered to be of importance.

It was found in the conducted studies that the patterns in different presentational forms were aimed at a certain objective. The purpose of figure patterns is to promote the students to think through visual approaches and, by starting from a visual approach, to enable them to be able to find an alternative way for numbers (Orton, Orton and Roper 1999). The purpose of the patterns presented in the form of tables or graphics is to play a critical role in the students' process of recording systematically all the outputs in each line and, depending on this, searching a pattern over the outputs they find (Carrarher et al., 2008). On the other hand, the purpose of the patterns presented in the form of a sequence of numbers is to ask the students to determine the relationships between the given terms of a pattern and write down the numbers that are not given right in their places. Separately, they are also
expected to express the rule of the pattern by starting from the correlation they have found (Feifei, 2005; Ley, 2005). The patterns presented in the form of verbal problems, on the other hand, can be presented as verbal problems or short stories (Van De Walle, 2004).

One of the basic steps of patterns is the generalization. The standards of the NCTM (2000) refer to generalization as one of the main objectives of math education. Patterns can be considered as one of the building blocks of generalizations and generalizations as the building blocks of algebra. Zazkis and Liljedahl (2002) refer to patterns as the heart and soul of mathematics. Since generalization is accepted as a top-level cognitive skill (Krutetskii, 1976), it is quite a significant action in math education (Yeşildere and Akkoç, 2011, p.142).

In the literature, there are studies encountered for determining the strategies used in the process of generalization relative to the structural and presentational forms of pattern (Garcia-Cruz and Martinon, 1997; Orton et al., 1999; Feifei, 2005; Ley, 2005; Becker and Rivera, 2005, 2006; Barbosa et al., 2007; Carraher et al., 2008; Yeşildere and Akkoç, 2010; Tanışlı and Yavuzsoy-Köse, 2011; Akkan and Çakiroğlu, 2012; Akkan, 2013). Although there is a research subject dealt with by various researchers as to the generalization process of the pattern, it was found that there were few researches performed on the generalization processes of teachers and teacher candidates.

There are several attempts to develop students' pattern finding strategies at different levels, from nursery school to secondary school (Zazkis and Liljedahl, 2002). When the researches carried out with respect to the strategies applied in the pattern generalization process are reviewed, Stacey (1989) reported that in the linear patterns, the students had used counting or modelling, whole-object or direct proportion, difference and linear generalization strategies. Tanışlı and Yavuzsoy-Köse (2011) found that while determining the rule of pattern and extending the linear shape pattern to next/far step, some pre-service teachers adapted the numerical approach through which the visual and figural patterns that focus on only shape are conveyed into numerical pattern. Orton and Orton (1999) stated that the students had usually preferred the recursive strategy in the linear and quadratic pattern problems. Yeşildere and Akkoç (2010) found that four categories of strategies (pre-service teachers): examining the relationship between consecutive numbers, preparing tables of values, constructing models, trial and error. They has also been found out that pre-service teachers have had difficulties in finding the rules of "patterns" reported in the literature. Ley (2005) highlighted that the students of primary education had used the recursive, proportional and explicit strategies. Lanin (2005) divided the strategies of the linear pattern generalization in two as non-functional (counting pieces or modelling, recursive) and functional (proportion, guess and check, contextual). Akkan and Çakiroğlu (2012) reported that most of the students used recursive or additive strategies, but only a few number of students used explicit strategies.

As is also significantly emphasized in the literature, it is required that pattern activities be focused on in the pre-school and primary education (primary school and secondary school) math curriculum for the purpose of ensuring the students' development of algebraic thinking, preventing them from a number of hardships they may confront in the field of mathematics in their later years and forming a good foundation for secondary education/higher education mathematics.

On the other hand, since the secondary school math teachers and the teacher candidates take part as the chief people responsible for the generalization of pattern and patterns in the primary education, it is of great importance to know how these people generalize patterns. It is also important to know whether or not they use various efficient strategies or not, or how they use these strategies in this process.

It is considered that this situation will increase the pedagogic field of knowledge of teachers and teacher candidates (Shulman, 1986).

**Purpose of the study**

Patterns are one of the main subjects in the curriculum of primary math education in many countries. NCTM (2000) regards the patterns and understanding relations as one of the standards of algebra for all class levels. Some educators are of the opinion that teachers should have a profound knowledge of certain math subjects that they teach with the philosophy, "one cannot teach unless he himself knows" (Post et al., 1991). In this context, the purpose of this study is to determine the pattern generalization performances of the secondary school math teacher candidates and the strategies used by them in the process of generalizations.

**The research questions**

Within the scope of this general objective determined, answers to the following questions are sought:

1. How are the generalization performances of secondary school math teacher candidates on the pattern problems of different structure?
2. How are the generalization performances of secondary school math teacher candidates on the pattern problems in different forms of presentations?
3. What generalization strategies do the secondary math teacher candidates apply in the patterns presented in different structures and presentations?

**METHOD**

**The research design**

In this research, a mixed method in which quantitative and
qualitative techniques are included together has been used. The quantitative data obtained from the test comprising the patterns presented in different structures and representations were collected and analyzed, and then the qualitative information was utilized in order to be able to get detailed information and to support the quantitative findings regarding the generalization processes of pattern problems contained in the tests. The mixed method performed in this way is referred to as “an explanatory design” (Creswell and Plano-Clark, 2007, p.71-74; Fransenkel and Wallen, 2006, p.443).

In the first part of the research, the participants were asked to solve the “Pattern Test” presented in different structures and representations, and the quantitative data obtained from these solutions were subjected to statistical analyses. In the second stage, interviews were made with the teacher candidates along with the papers containing the answers given in the tests, and the qualitative data were collected, as well. The quantitative method of this research is to be able to get involved in the group of descriptive researches as the scores obtained from the pattern test are tried to be described. For this reason, the relational survey method which aims to reveal the relationships among the variables and which is within the scope of the descriptive research was used (Fraenkel and Wallen, 2006). In order to analyze whether the scores they got from the pattern tests differed in terms of class levels, the comparative type of relational survey method was used. On the other hand, in the qualitative part of the research, a semi-structured interview along with the document analysis was performed in addition to the answers in the test papers comprising the pattern problems of the teacher candidates according to different forms of structure and representation. In this application, however, the data regarding the strategy/approach used by the teacher candidates were collected in order to find the expression of generalizing patterns contained in the tests.

Participants

A total of 221 teacher candidates studying in the department of primary mathematics teaching (secondary school) of a state university in Turkey voluntarily participated in this research. 62 of the teacher candidates were from the 1st grade, 58 of them were from the 2nd grade, 54 of them were from the 3rd grade and 47 of them were from the 4th grade. All of the teacher candidates who participated in the practice answered the pattern test and quantitative data were obtained. A document analysis was performed in order to determine the strategies/approaches used in the generalization of patterns. By applying the document analysis technique on the answer sheets of the teacher candidates, the strategy used was ensured to be understood. In the event that it was not comprehended, some of the teacher candidates were interviewed. The teacher candidates were selected through a sampling strategy referred to as convenience strategy (Cohen et al., 2002).

The subject of pattern plays a key role in understanding and conceptualizing the mathematical information in primary and secondary schools (NCTM, 2000). Since the greatest responsibility was on the secondary school math teachers in transferring this subject to the students, it was decided that the study be conducted with these candidates. One of the undergraduate programs in Turkey is the department of primary mathematics teaching. After this group of teachers graduate, they can only take classes in the secondary schools (5th, 6th, 7th and 8th class). Therefore, the undergraduate curriculums have been formed in this way. The pedagogic field of knowledge they acquired in the classes throughout their undergraduate education is of great importance in their professional experiences in later years. In this context, the main reason why math teachers of primary education were selected in this study was to test the pedagogic field of knowledge of pattern generalization.

Data collecting materials

The subject of pattern was selected in this study since the importance of describing and using the mathematical patterns and relations was stressed while analyzing a mathematical case in the 5th, 6th, 7th and 8th class educational program in the NCTM (2000) and MEB (2013) reports. Separately, the fact that there are acquisitions like building up the desired steps of digital patterns and figures, the formula/rule of which is given in the 5th class, and including the studies aiming at finding out the rule by turning the relations in the daily life situations or figure patterns into an arithmetic sequence in the 6th class are the greatest factors in the selection of this subject.

Taking the objective of this study into consideration, a “Pattern Test” and semi-structured interview questions consisting of pattern questions were prepared. The questions contained in the test were prepared by the support of both literature and teachers (Akkan, 2013; Akkan and Çakıroğlu, 2012; English and Warren, 1998; Feifei, 2005; Ley, 2005; Orton and Orton, 1999; Yeşildere and Akkoç, 2011). In the test, there are pattern problems in 3 structures as linear, quadratic and geometric ones. The problems within each structure were formed in shape, sequence of numbers, tables and verbal forms. 4 questions were prepared for each structure, which consisted of different forms of presentations. Consequently, this pattern test comprised 12 questions. Each pattern problem asked in the test was prepared by getting support from the literature. In the preparation of pattern problems, the studies of Akkan and Çakıroğlu (2012), Yeşildere and Akkoç (2010, 2011), English and Warren (1998) and Ley (2005) were benefited from according to the forms of presentation, and also the studies of Lannin (2003), Looney (2004) and Feifei (2005) were utilized according to the pattern structure. The complete pattern problems contained in the test were prepared to find the expression of generalizing the problem. Whether or not the problems within this arranged data collecting material were in compliance with the purpose of measurement, and whether they represented the field required to be measured were performed by receiving an expert’s opinion (Karasar, 2005). For this, a group of experts (two math educators and an assessment-evaluation expert) performed compliance tests for the purpose of measurement and content analyses required by these objectives. Separately, whether the problems within the test could represent the purposes of measurement and content was discussed. The significance, language rules, level and scope validity of the problems in the test prepared in this respect were ensured. Arrangements were made in line with the suggestions by the group of testing experts. The prepared test was performed for a pilot scheme with 45 teacher candidates of the same age studying in the same department of a different state university. As the result of the pilot scheme, the Croanbach alpha (α) reliability co-efficient of the test was found to be .85. The questions belonging to the sample structure and forms of presentation in the pattern test are given in Table 1 and Figure 1.

A document analysis along with the semi-structured interview was performed in order to determine the generalization strategies used by the math teacher candidates in the problems comprising the patterns of different structures and presentations. The semi-structured interview is a type of interview technique which is generally used in the studies conducted in the field of education (Patton, 2002). In this type of interview, the researcher may allow one to make their answers more clear and in detail, depending on the course of the interview. The basic questions to be asked during these interviews are type of questions, such as “Could you tell us how you found out the expression of pattern-generalization?” or “How did you come to the conclusion that the formula/rule was...?”. The interviews were performed in a quiet atmosphere in the way
Table 1. An example to the pattern problems of different structures within the pattern test.

<table>
<thead>
<tr>
<th>Types of test</th>
<th>Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>A sequence of numbers are given below. 6, 11, 16, 21, 26, … Let n be any given number. Write down an expression to find the n th term in the number sequence.</td>
</tr>
<tr>
<td>Geometric</td>
<td>Below is a sequence of numbers. 3, 9, 27, 81, … Let n be any given number. Write down an expression to find the n th term you used in the number sequence.</td>
</tr>
<tr>
<td>Quadratic</td>
<td>Below is a sequence of numbers. 1, 5, 12, 22, 35, … Let n be any given number. Write down an expression to find the n th term in the number sequence.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Presentation types</th>
<th>Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number (Linear)</td>
<td>Below is a sequence of numbers. 6, 11, 16, 21, 26, … Let n be any given number. Write down an expression to find the n th term in the number sequence.</td>
</tr>
<tr>
<td>Shape (Quadratic)</td>
<td>You see the series of three similar triangles. All small triangles are in compliance with each other. Write down an expression indicating how many small triangles are needed in the n th figure.</td>
</tr>
<tr>
<td>Table (Quadratic)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Verbal (Linear)</td>
<td>Ufuk has a customer card he prefers for shopping. When Ufuk used the card for the first time, the market put 3 credits into his card. Later, Ufuk gets 2 more credits into his card each time he shops. Ufuk has collected 15 credits in his card as the result of the shoppings he has done. So, how many shoppings, do you think, Ufuk did in total? (Write down an expression that generalizes the shopping done by Ufuk).</td>
</tr>
</tbody>
</table>

Figure 1. An example to the pattern problems in a different representation within the Pattern Test.
Table 2. The scoring key of the pattern.

<table>
<thead>
<tr>
<th>Categories</th>
<th>Description</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>L0</td>
<td>No response; irrelevant answer, illegible answer. The fact that it only includes a piece of information as to the expression of generalization.</td>
<td>0</td>
</tr>
<tr>
<td>L1</td>
<td>Failure in significant points related to solution. False Reasoning.</td>
<td>1</td>
</tr>
<tr>
<td>L2</td>
<td>Providing some supportive evidence; yet, some part of the evidence is missing. The fact that it comprises minor errors in calculations or reasoning processes.</td>
<td>2</td>
</tr>
<tr>
<td>L3</td>
<td>Ensuring accurate evidence that shows the expression of generalization.</td>
<td>3</td>
</tr>
</tbody>
</table>

*L0, L1, L2, L3: Level of scores*

that the teacher candidates would feel themselves comfortable. It was pointed out that the important point for the teacher candidates was not reaching an accurate or an inaccurate answer but the way they achieved that answer. While they were answering the questions addressed to them, they were asked to think loudly and explain their solutions.

**Data analysis and application**

The pattern-generalization performances of the teacher candidates were evaluated by performing scoring over the answers given by them in the tests. The accuracy or inaccuracy of the expression of generalization was done according to the scoring key in Table 2. This scoring key utilized in this process was that which had been developed by Feifei (2005).

The researchers and field experts formed the generalization strategies over the research data. In this context, the data within the research were collected together and analyzed, and the common aspects were tried to be found among them. Each datum with common aspects was grouped under a strategy heading. These strategies were dealt with as the generalization strategies of the pattern problems contained within the research. In forming the strategies, the pattern-generalization strategies included in the formerly-conducted studies in the literature were taken into consideration. The classification information in relation to these strategies is given in Table 3.

Along with the scoring of the answers in the pattern test given by the teacher candidates, the encoding and scoring processes regarding the strategies used in the generalization of patterns in this test were performed by three field experts (including the researcher) working independently in the process. While the data concerned with the generalization strategies were being encoded in the scoring process of the answers obtained from the tests, the conceptual framework prepared previously by performing a literature review was taken into consideration. One of the reliability studies to be made at the stage of the data analysis is the reliability of encoding process. In this research, the following compliance percentage proposed by Miles and Huberman (1994, p. 64) was used for the calculation of encoding reliability. For this, “consensus” and “dissensus (difference of opinion)” numbers were specified for the scores and codes determined by the researcher and two field experts, and as the result of the calculation, the compliance percentage for the scoring (Table 2) was found to be 94%, whereas the compliance percentage for the encoding (Table 3) process was found as 86%.

\[
\text{Reliability} = \frac{\text{Consensus}}{\text{Consensus} + \text{Dissensus}} \times 100
\]

Since a separately normal distribution was seen for each group, a one-way ANOVA was used in the cases in which comparisons were made between the class levels according to the pattern structure and the form of representation of the teacher candidates’ performances on being able to generalize the arranged pattern problems. In order to express the source of the difference between groups, the LSD test, one of the post-hoc tests, was applied. In determining the effect values in comparisons, the eta-square scores were utilized. According to Cohen (1988); if \( \eta^2 \) between .01-.06, it is considered as a “minor” effect, whereas if \( \eta^2 \) between .06-.14, it is considered as a “medium” effect, and if \( \eta^2 \) is an eta-square value above that, it is regarded as a “major” effect.

**FINDINGS**

In this section, the findings regarding the pattern test of the primary math teacher candidates were analyzed in two main topics. In the first one, the findings with respect to the generalization performances in the pattern problems prepared according to the structural and presentational forms of the teacher candidates were presented, whereas in the second one, the findings as to the generalization strategies used in the generalization within the pattern test were presented.

**Generalization performances of the teacher candidates according to the pattern structure and the form of presentation**

The performances of the teacher candidates in the ‘Pattern Test’ involving differences according to the structural forms of patterns were examined and analyzed. From the obtained data, the performances of the scores of each structure achieved are shown in Figure 2 according to their levels and class levels.

It is seen that there are differences among the class levels according to the structural state of patterns. It is
### Table 3. The Generalization strategies obtained from the conducted studies (Akkan, 2013, p.712).

<table>
<thead>
<tr>
<th>Strategies</th>
<th>Characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Counting and Modelling</td>
<td>This includes calculating the number of the pieces that form a figure or a shape, or building up a model that illustrates the state in order to calculate the desired quality, or it comprises drawing a figure.</td>
</tr>
<tr>
<td>Recursive or Additive</td>
<td>This includes the use of the former term within the pattern in order to find the succeeding term or terms. Students usually try to find the difference between two terms and add the final difference obtained to find out the succeeding term to the final term. Since this process proceeds in a recursive or additive fashion, it is also referred to as the additive strategy.</td>
</tr>
<tr>
<td>Multiplying by the Difference</td>
<td>This includes the multiplication of the difference between the two consecutive terms in the sequence. In this case that occurs in the generalization of linear relations in particular, the student is aware of the constant difference between the terms.</td>
</tr>
<tr>
<td>Whole-Object or Proportion</td>
<td>This strategy includes the use of proportional reasoning process in solving the pattern problems.</td>
</tr>
<tr>
<td>Guess and Check</td>
<td>It comprises the guessing of a formula/rule, regardless of whether that formula/rule is functioning or not. An algebraic relation (formula) is put forward to represent the problem case.</td>
</tr>
<tr>
<td>Contextual</td>
<td>This includes building up a formula or a rule focusing on the information that provides the case, which is the content. This rule or formula is the principle that is associated with the rule or formula-calculating technique.</td>
</tr>
<tr>
<td>Functional or Precise (Explicit)</td>
<td>This strategy involves generalizing the relationship between two variables in order to be able to determine any given value. This is the first step of a phased progress towards determining the functions by using equations and formulas. This strategy also allows for finding out the n. term and writing out a general formula.</td>
</tr>
</tbody>
</table>

**Figure 2.** The generalization performances of the teacher candidates according to the structure of patterns.

It was also seen that as the class level increases, so does the generalization performance of the pattern. It was seen that the average scores acquired by the teacher candidates from the pattern test according to the
Table 4. The one-way ANOVA results for the generalization performances in the pattern tests according to the class levels and the status of teacher candidates.

<table>
<thead>
<tr>
<th>Types of Pattern</th>
<th>Source</th>
<th>Sum of Squares</th>
<th>df</th>
<th>Mean Square</th>
<th>F</th>
<th>$\eta^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>Between Groups</td>
<td>15.87</td>
<td>3</td>
<td>5.29</td>
<td>2.652</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Within Groups</td>
<td>432.698</td>
<td>217</td>
<td>1.994</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>448.568</td>
<td>220</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geometric</td>
<td>Between Groups</td>
<td>13.23</td>
<td>3</td>
<td>4.41</td>
<td>3.235</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>Within Groups</td>
<td>295.771</td>
<td>217</td>
<td>1.363</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>309.001</td>
<td>220</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quadratic</td>
<td>Between Groups</td>
<td>12.97</td>
<td>3</td>
<td>4.32</td>
<td>4.038*</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>Within Groups</td>
<td>231.973</td>
<td>217</td>
<td>1.069</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>244.943</td>
<td>220</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*p<.05

Table 5. The results regarding the LSD test applied for determining the source of the difference among the scores of the pattern problems of quadratic structure in the pattern tests.

<table>
<thead>
<tr>
<th>Class levels (I)</th>
<th>Class levels (J)</th>
<th>Mean difference (I-J)</th>
<th>Standard error</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>0.56</td>
<td>.84</td>
<td>.000*</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.51</td>
<td>.79</td>
<td>.000*</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0.55</td>
<td>.82</td>
<td>.000*</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.50</td>
<td>.80</td>
<td>.000*</td>
</tr>
</tbody>
</table>

structural patterns were ranged from top to bottom as “Linear”, “Geometric” and “Quadratic”, respectively. The performance of a quadratic structure is seen more in upper classes. The performances of the teacher candidates in the “Pattern Test” involving differences according to the structural forms of patterns and the differences among class levels are presented in Table 4.

When Table 4 is examined, it was found that there were statistically significant differences among the generalization performances of secondary school math teacher candidates in the problems showing differences according to the structure of the pattern. This difference was found in the pattern problems of quadratic structure. (F(3,221) = 4.038; p< .05). Also according to the eta-square result, this difference can be said to be effective at a lower level ($\eta^2=.05$). In order to decide as to which post-hoc test would be applied to determine the source of this difference that shows up in pattern problems of quadratic structure, the variance homogeneity between groups was examined, and the variance between groups was seen to be homogenous. For this reason, in order to determine the source of the differences between groups, the LSD test, which is one of the post-hoc tests used in the cases where the variances are homogenous, was applied. The results of the LSD test are presented in Table 5.

According to Table 5, when the scores of the 3rd and 4th grade teacher candidates and those of the teacher candidates from the other class levels were compared, it was found that there was a significant difference in comparison to the 1st and 2nd class levels. The performances of the teacher candidates in the “Pattern Test” involving differences according to the presentational forms of patterns were examined. From the obtained data, the degree of the scores acquired from each way/form of presentation and their performances according to the class levels are given in Figure 3.

When Figure 3 is examined, it is seen that the performances of the teacher candidates in the pattern problems with different presentational forms differ according to the class levels. The average scores acquired by the teacher candidates from the tests were seen to have ranged, according to the ways of presentation, as “Number Sequence”, “Figure”, “Table” and “Verbal”, respectively from top to bottom. The 3rd and 4th class teacher candidates are seen to have been more successful in the verbal and table problems. It was found that there was no difference among the class levels in the performances of digital and figure representations. However, in general, it can be stated that the performance increases in parallel to the increase in the class levels and that the forms of
presentation affect these performances. Whether there was any difference among the performances of the teacher candidates in the "Pattern Test" involving differences according to the forms of presentation of patterns is presented in Table 4.

When Table 6 is examined, it is seen that there are statistically significant differences among the generalization performances of primary education (secondary school) math teacher candidates in the problems showing difference according to the forms of representation of patterns. This difference was found to have occurred in the pattern problems in the form of verbal and table representations. \( F(2,219) = 2.425, F(2,219) = 4.844; p<.05 \). According to the eta-square result, it can be stated...
that this difference is effective at a lower level according to the form of the table representation ($\eta^2 = .03$), whereas the difference in the pattern problems in the form of verbal representations is effective at a medium level ($\eta^2 = .06$). In order to decide on which post-hoc test would be performed for determining the source of this difference that showed up among the class levels, the variance homogeneity between the groups were examined, and the variance between groups was seen to be homogenous. For this reason, an LSD test, one of the post-hoc tests applied in cases where the variances are homogenous, was used to determine the source of the differences between groups. The results of the LSD test is seen in Table 7.

According to Table 7, when the scores acquired by the primary education (secondary school) math teacher candidates from the pattern problems prepared according to the forms of presentation were compared by considering the class levels, the 3rd and 4th class teacher candidates were found to be significantly different in comparison to those in the other class levels.

### Table 7. The results regarding the LSD test applied to determine the source of the difference between the scores in the pattern test according to the table and verbal presentational forms.

<table>
<thead>
<tr>
<th>Types of pattern</th>
<th>Class levels (I)</th>
<th>Class levels (J)</th>
<th>Mean difference (I-J)</th>
<th>Standard error</th>
<th>p</th>
</tr>
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<tr>
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<td>.34</td>
<td>.000*</td>
</tr>
<tr>
<td></td>
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<td></td>
<td>2</td>
<td>0.36</td>
<td>.33</td>
<td>.002*</td>
</tr>
<tr>
<td>Verbal</td>
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<td>.30</td>
<td>.004*</td>
</tr>
<tr>
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<td>2</td>
<td></td>
<td>0.27</td>
<td>.31</td>
<td>.000*</td>
</tr>
<tr>
<td></td>
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<td>0.27</td>
<td>.29</td>
<td>0.01*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>0.29</td>
<td>.30</td>
<td>0.00*</td>
</tr>
</tbody>
</table>

*p<.05.

The strategies used by the teacher candidates in generalizing the pattern problems

As the result of the interviews and the answers given in the test by the teacher candidates, the pattern-generalization strategies are given in Table 8. When Table 8 is examined, it was found that the strategies used in the pattern problems according to the linear, geometric and quadratic structure showed difference. As the class level increases, the applied strategies become different. It was found that the applied strategies had particularly differed in the 3rd and 4th classes when compared to those applied in the other classes. It was also discovered that the teacher candidates had used the multiplying by the difference strategy in the pattern problems of linear structure according to the form of presentation and without any difference. It was seen that the teacher candidates had used the contextual strategy instead of this one as the class level kept on increasing.

In the pattern problems of geometric structure, however, they were seen to have used the proportion strategy. The fact that this strategy was used by the teacher candidates is considered to have resulted from the nature of the problems. Separately, it was found that the use of the proportional reasoning strategy in the differentiation of class levels did not differ. When the teacher candidates' pattern-generalization strategies of quadratic structure were analyzed, it was found that the 1st and 2nd class teacher candidates has mostly used the trial-error (guess-check) strategy, whereas the 3rd and 4th class teacher candidates had used the explicit and "contextual" strategies rather than the trial-error one. Some of the teacher candidates expressed that they had to pick the trial-error strategy since they could not find a common inter-term feature in the patterns of quadratic structure.

When the strategies they used in the generalizations of the pattern problems showing difference according to the forms of presentation were analyzed, it was seen that there were distinctions among the classes, yet, the strategies used in the form of different presentations in the same class levels remained unchanged. At the end of the interviews performed with the teacher candidates, they stated that the way of the solution to the problems did not change despite the fact that they were asked in different presentations. For instance, it was observed that they discovered an expression of a generalization by writing out in digital form (transforming into a number sequence) the pattern problem asked by means of a figure. Among the results obtained, there is the fact that the forms/ways of presentations facilitated the generalization processes in obtaining the expressions of pattern-generalization. Separately, in the light of the findings obtained, it can be said that the ways of presenting the patterns are quite effective in enabling the teacher candidates to perceive and understand the patterns.

When the strategies of the teacher candidates used by them in the pattern generalization process were analyzed, the structure of the pattern was seen to have been more...
important than the ways or forms of presentation of patterns. On account of the fact that the forms of presentation differ in the representation form of the pattern structure, it was stated by the teacher candidates that the structure was of more importance. Separately, when the generalization process according to the presentational forms were examined, the strategies used were seen to have changed in some of the presentational forms. It was found out that the teacher candidates generally used the numeric sequence in the forms of figures, tables and verbal presentations and started the generalization process. Hence, the forms of presentations were found to have been efficient in understanding the pattern problems prepared according to the pattern structure. It was seen that the teacher candidates were challenged during the generalization process, particularly in the pattern problems with verbal content.

**DISCUSSION AND CONCLUSİON**

In this study, the pattern-generalization performances of secondary school math teacher candidates from different class levels in linear, geometric, quadratic and different forms of
presentation along with the applied generalization strategies were examined and analyzed. In this section, a general evaluation of the obtained findings along with the suggestions were made. It was found, according to the findings of the research, that there was a significant difference between the pattern-generalization performances in the pattern structure and in the forms of presentation according to the class levels of the teacher candidates. The fact that the difference is seen in different structures and presentational forms of patterns can be said to have affected the performance of teacher candidates regarding the patterns. The same findings were encountered in the study conducted by Looney in 2004.

Perceiving the patterns accurately forms an important basis for the algebra classes and generalizations to be taken and made by the students of primary education in particular. Armstrong (1995) emphasized the fact that discovering patterns would develop the algebraic thinking talents of the children of early ages, and drew attention to the fact that making generalizations by making use of patterns was significant for algebra. Separately, the fact that children can develop the algebraic concepts at early ages and studying with patterns and numbers may help form the foundation for the algebraic thinking required in the succeeding classes (NCTM, 2000). Since pattern problems are the main themes of several subjects like algebraic operations, they are regarded as quite significant in the teaching of mathematics. It is considered that providing this subject in a conceptual form rather than an operational form during primary education will prepare the sub-structure of several other subjects. The teachers of mathematics in secondary school who are involved in this subject are of the opinion that this subject is a major factor in the performances of the students towards pattern problems.

Baxter and Lederman (1999), who discussed what methods could be applied to examine the pedagogic field of knowledge, advise that observing the teaching practices and performing interviews would be efficient methods to use in this respect. Therefore, from this perspective, the lack of teaching experiences of the teacher candidates of primary education mathematics regarding the concept of pattern suggest the fact that the teacher candidates may have incompetencies in their own field of knowledge. Thus, in this study, it was also observed in the literature researches that some of the teacher candidates had difficulty in finding out the formula or rule of patterns. The teacher candidates may confront the topics they had never learned before in the curriculums of math education during their educational years in Turkey. Patterns and making generalizations are topics found among these subjects. The teacher candidates themselves also gain experiences by going through similar challenges exactly in the same way as those that are learning this subject for the first time in the primary and secondary education.

It is seen in the wake of the literature research that the structural types of patterns are in linear, geometric and quadratic forms. In this research, it was found that the generalization performances of the teacher candidates, according to these structural differences, had differed according to the class level. The structure in which the teacher candidates showed high performances was found to be linear, whereas the structure in which they showed lower performances was found to be quadratic. Besides, when the findings were analyzed, there was no difference among the class levels and among the generalization performances in the patterns comprising linear and geometric structures. However, in the pattern problems of quadratic structure, it was found out that there was a significant difference among the performances of 3rd and 4th class teacher candidates when compared with the other class levels, which could be attributed to class levels. The reason why the performance in the pattern problems of quadratic structure are different from those of the linear and geometric structure is considered to be possibly due to the fact that it requires a more complex way of thinking rather than a linear or proportional one. And because the cognitive difference changes as the class level rises up (Akkan, 2013; Dündar, 2013), it can be said that a difference in such performances occurs. Separately, as is also seen in many research findings in which the teacher candidates were determining the formula or the rule of the generalization strategies that changed according to the pattern structure, it was discovered that they specified the formation of the expression of generalizations in patterns by examining the structural features of patterns (Becker and Rivera, 2005; Orton ve Orton, 1999). In this respect, when the teacher candidates’ generalization performances of problems regarding the structure of patterns were analyzed, it was found that 3rd and 4th classes became more successful than the other classes. The success of the 3rd and 4th classes can be attributed to the fact that they took the courses like math teaching during their undergraduate classes in comparison to the other classes. Since the 1st and 2nd classes take rather field courses, the teacher candidates of these class levels can be said to have had lower performances in pattern problems. In a study conducted by Dündar and Yaman (2014), it was reported that the education received by the teacher candidates during their undergraduate periods had quite an impact on their thinking levels. Since the undergraduate lessons, such as math teaching, specific methods of teaching, the history of mathematics, problem solving and the art of mathematics, are taught in the 3rd and 4th classes in the faculties of math education in Turkey, the views of the teacher candidates towards the subjects are thought to have changed due to the education they received. As was also stated by Radford (2008), the teacher candidates mostly used the trial-error strategy in using the generalization strategies of the pattern problems of quadratic structure. Also in this study,
it was found that the teacher candidates of 1st and 2nd class levels had used this strategy more in the pattern problems of quadratic structure when compared with the other class levels. Although this application may help find out the expression of generalization, it may still not always be the accurate use in finding the correct algebraic expression (Radford, 2008). It was determined that the mean scores received by the teacher candidates from the tests according to the forms of structures in the pattern test were ranged from top to bottom as “linear”, “geometric” and “quadratic”, respectively.

It was seen in the conducted studies that the pattern problems in different forms of presentations (table, figure, verbal and number sequence) affected the performances of the students or the teacher candidates. When the findings of this study were analyzed, it was found that there was a significant difference among the pattern-test performances according to the forms of pattern presentations in terms of the class levels of students. The performances of the teacher candidates, according to the class levels and the forms of pattern presentations, were determined to be high in the 3rd and 4th classes. In terms of the class levels, there was a significant difference among the performances in the pattern problems in the form of verbal and table representations. It was found that there was no significant difference among the class levels of the teacher candidates in their forms of presentations as number sequences and figures. The differences among these performances of the teacher candidates according to the forms of presentation of patterns can be attributed to their cognitive levels and the education they received. Since the teacher candidates, during their undergraduate period, use different representations in solving the problems that require reasoning and questioning, it is considered that their performances differ in the 3rd and 4th class levels. When the strategies used by the teacher candidates in finding out the expressions of pattern-generalizations involving differences according to the forms of presentations were analyzed, some of them used the numeric approach as had also been in the researches carried out by Orton and Orton (1999), Becker and Rivera (2006) and Lan Ma (2007). As is also seen in several research results, there was the process of transformation from the figure pattern into the numeric pattern in the first place (Lan Ma, 2007; Orton and Orton, 1999). Among the results obtained, there is the fact that the forms of presentation facilitate the generalization process in acquiring the expressions of pattern-generalizations. Separately, in the light of the obtained findings, it can be said that the forms of pattern-presentations have been effective on the pattern perception and understanding of the teacher candidates. The mean scores received by the teacher candidates from the tests according to the forms of presentation were seen to have ranged from top to bottom as “Number Sequence”, “Shape”, “Table” and “Verbal”, respectively. Pegg and Redden (1990), in a study they conducted, found that the students had answered the patterns presented in the form of “shapes” more easily than those presented in the form of “tables”; therefore, they got higher averages in the patterns presented in the form of “shapes”. Also as the result of their study, claimed that the tables were perceived by the students as more abstract than the figures. Looney (2004), in a research he performed, found out that the students got the highest average in the patterns presented in the form of “Tables”, that they got the secondary higher average in the patterns in the form of “Shapes” and that the lowest average was received from the patterns presented in the form of “Number Sequence”. Separately, the teacher candidates tried to reach the result in the pattern-generalization process by benefiting from numerical writing. This finding had also been revealed in Rico (1996) and Stacey (1989)’s studies, and the students had used the figures or shapes to transform them into a pattern of numbers instead of examining them in the direction of finding a correlation.

The major factor hampering the realization of the generalization process is to stick to the trial-error process only without ever exploring a common feature among the terms of the patterns. (Akkan, 2013). Another point we were faced with while examining the generalization processes of the teacher candidates was associated with finding a common feature among the pattern terms. In the process of making generalizations, the teacher candidates are considered to experience troubles due to the fact that they fail to form a hypothesis in the way that they will be able to find all the terms of patterns. The main reason for this is the fact that the common feature they notice within the pattern does not help them do it. For instance, while the teacher candidates were researching into the formula or the rule of the pattern, they tended to find out the succeeding terms by starting from the terms of the pattern already given. When the common feature seized between terms is towards finding the next term by starting from the previous one, such a case does not take one to an algebraic generalization but an arithmetic one, because it is required that rather than focusing on the next term for an algebraic generalization, a relationship between the terms and the number of terms be investigated, and a common feature in this direction be discovered. While the teacher candidates were forming the common feature in the linear patterns in which the relationship between the term and the number of terms are more clearly seen, they focused on the trial-error process in more complex (quadratic) patterns. It was seen that in the strategies used by the teacher candidates in the process of pattern-generalization, the 3rd and 4th class teacher candidates had used the guess strategy less than those in the other class levels. Yeşildere and Akkoç (2010), in the studies where the pattern-generalization processes of math teacher candidates were investigated, touched on the fact that the teacher candidates had focused on only the next step
of the pattern and jumped into making generalizations, which, therefore, led them to make inaccurate generalizations. Tanışlı and Köse (2011) mentioned similar cases in the studies they conducted with class teachers. English and Warren (1998), in their study where they measured and assessed the students’ skills to transform the verbal expressions into algebraic notations, also mentioned a similar situation and stated that the transformation of the verbal demonstrations into symbolic ones had been a challenging situation among the students. In this study, the low performances of the teacher candidates in their verbal forms of presentations could be attributed to this case, as well. It can be stated that the incompetencies of the teacher candidates which have been put forward in terms of various variables indicate a gap or insufficiency in their pedagogic field of knowledge. Similar findings are seen in various researches in the literature. The researchers suggest that the inexperienced teachers have great problems with their pedagogic field of knowledge, and that they have difficulty in particularly presenting the concepts and ideas in the way that they will be of significance for the students (Onslow et al., 1992).

In this context, in the teacher training programs, it can be recommended that the topics learnt by the teacher candidates for the first time in their teacher training programs in particular be focused on more in order for them to be efficient in this field. On the other hand, considering the students’ development in algebraic thinking, it becomes quite important to educate and train particularly the math teacher candidates of the primary education who are supposed to teach these students in this field.

Thus, it is of great significance to question the field knowledge of the math teacher candidates of primary education with respect to this matter and also to reinforce their pedagogic field of knowledge in this sense.

**Conflict of Interests**

The author has not declared any conflict of interests.

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