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Middle school teachers' views and approaches to implement mathematical tasks

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This research examines middle school mathematics teachers' views regarding implementation of mathematical tasks and their enactments. We compare their views on tasks and their implementation, and determine the causes of difference between the two using qualitative research methods. We interview sixteen middle school mathematics teachers based on their professional experience and willingness, and observe task implementations of four. According to the data, we found that the teachers with the most professional experience have difficulty to implement the tasks, they have teacher-centred mentality and they make use of their own experiences as students than of the task itself.

Key words: Task implementation, number patterns, generalisation.

INTRODUCTION

Problem solving, mathematical reasoning and using mathematical knowledge in daily life are the teaching abilities that take part in many countries' mathematics curriculums. Getting these abilities requires accepting them as the appropriate classroom norms during the enactment of mathematical tasks, the purpose of which is "to focus students' attention on a particular mathematical concept, idea, or skill" (Stein et al., 1996). Tasks' is important likewise the abilities which has been touched upon in the literature as increasing the quality of mathematics education (Henningsen and Stein, 1997, p.528.) Horoks and Robert, 2007).

Effective teaching practices require student-centred discussions of problem solving tasks (Boaler and Humphreys, 2005). Especially tasks with real life situations increase student motivation significantly and push students to work (Stylianides and Stylianides, 2008). Recent studies have reported that tasks affect teachers' conceptual understanding and have positive effects both on a cognitive and affective level (Koichu et al., 2016). Although there is a tendency to view all classroom activities as "tasks" in the mathematics education, in fact an activity must meet certain requirements before considering it as a task. One of the most characteristic properties of tasks is that they must be prepared for in advance; in other words, one must plan for a specific educational purpose. Doyle (1988) has defined the required properties of tasks under four titles:

1. Must have a purpose to be fulfilled, or a result that must be reached (goals).
2. The students must be supplied with all of the tools

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required for the task to be achieved (resources).
3. The correct operations need to be applied to achieve the goal using the resources supplied (operations), and
4. The effect of the task on student achievement needs to be determined (responsibilities).

Looking at the necessary steps for planning a task, we can say that the first thing is to determine a purpose, such as solving a certain problem, or reaching a generalization. The next is the use of resources, such as study notes, books, solution models provided by the teacher, materials etc. Designing a task finalize by executing operations such as, going over the previous lesson to help with the completion of the task, applying a rule, going through the steps necessary to reach a generalization; and determining what percentage of the students' pass grade should be affected by the task.

When reviewed through these items, it is clear that task designing needs a pedagogical preparation and to keep the implementation process in mind. A task focusing on the cognitive domain can affect the student's learning in two stages: designing and implementation (Stein et al., 1996). The task design is the stage in which students obtain detailed explanations and the expectations. The main purpose here is explaining to the students how to approach the task obviously. Giving fruitful answers to the questions such as, "will there be any worksheet", "which documents will students work on" and "which goals are expected" are important for designing a task with which students will work on (Sherman, 2011).

However due to various classroom conditions, students and teachers may unable to reach the potential of a given task (Stigler and Hiebert, 2004). The implementation may diverge from designing, and a high-level cognitive skills demanding task can turn into lower level (Boston and Smith, 2009). Because of this reason, we need to consider designing and implementing together. Teacher enactment is important for revealing the mathematical tasks' potential. Students must have chance to find patterns and reach generalizations, as well as being in central role in order to verify the results and defend them (Yackel and Hanna, 2003). Posing questions about whether they gave correct answers or not, or if they are sure they have reached all potential results create opportunities to reason mathematically (Franke et al., 2009). Considering teachers' role to shape the tasks they enact, the implementation of tasks is at least as important as the designing them. Thus, it is important to determine factors that may affect the implementation during designing, and then implement the task accordingly.

Henningsten and Stein (1997) have examined classroom factors affect tasks that allow students to use high-level mathematical thinking and reasoning, and have summarized the relationship between the task and students' learning as seen in Figure 1.

According to the schema, the mathematical task leads to learning outcomes in three stages: "first as curricular or instruction materials; second, as set up by the teacher in the classroom; and third, as implemented by students during the lesson" (Henningsen and Stein, 1997). In that case, teachers set up mathematical tasks that are designed by curriculum planners in order to enact in the classroom. The objectives set by the teacher, and the teachers' knowledge of subject matter and student affect this process. The teacher may give explanations that increase student motivation.

One other dimension is cognitive requirements. Cognitive demands involve the explanations given to lead the students through the thought process leading to solving the task. The factors affecting the task implementation process are classroom norms and task conditions, the teacher's instructional dispositions and the students' learning dispositions.

Classroom norms include by whom and how will the task be executed, and the qualities and responsibilities expected for execution. Task conditions are a factor involving properties such as what previous learning students must have, and how much time is necessary for the completion of the task. The factor containing how the pedagogic and learning approaches of the teacher and the students affect the execution of the task in the classroom is student and teacher tendencies. All of these stages and factors have influence on the quality of learning outcomes.

Although differently categorized by different research, there is a consensus within the literature regarding the principles of task design (Ainley, 2006; Bell, 1993, Liljedahl et al., 2007). At this point, the question seems to be do tasks designed according to these principles have the desired effect on students. Put differently, to what extent does the consistency between the designing objectives of tasks and their modes of implementation affect the mathematical learning? This question is important for defining the problems faced during the implementation process of tasks and for reflecting the theoretical task principles in the classroom implementation level. When looked at through the perspective of mathematics education in Turkey, the efficiency level of tasks used in mathematics are very low and the consistency with task planning principles is less than expected (Kerpiç and Bozkurt, 2011).

One reason for classroom task implementations mistakes that occur is that curriculums and textbooks fail to explain implementation clearly. Lack of explanation is one of the several issues that generate mistrust and negative attitudes toward using tasks to teach mathematics. Correct definitions and explanations of classroom task implementation are equally important as determining task planning principles.

Some research has been conducted on the task design and implementation. Silver et al. (2005) conducted a study with experienced teachers to identify the reasons

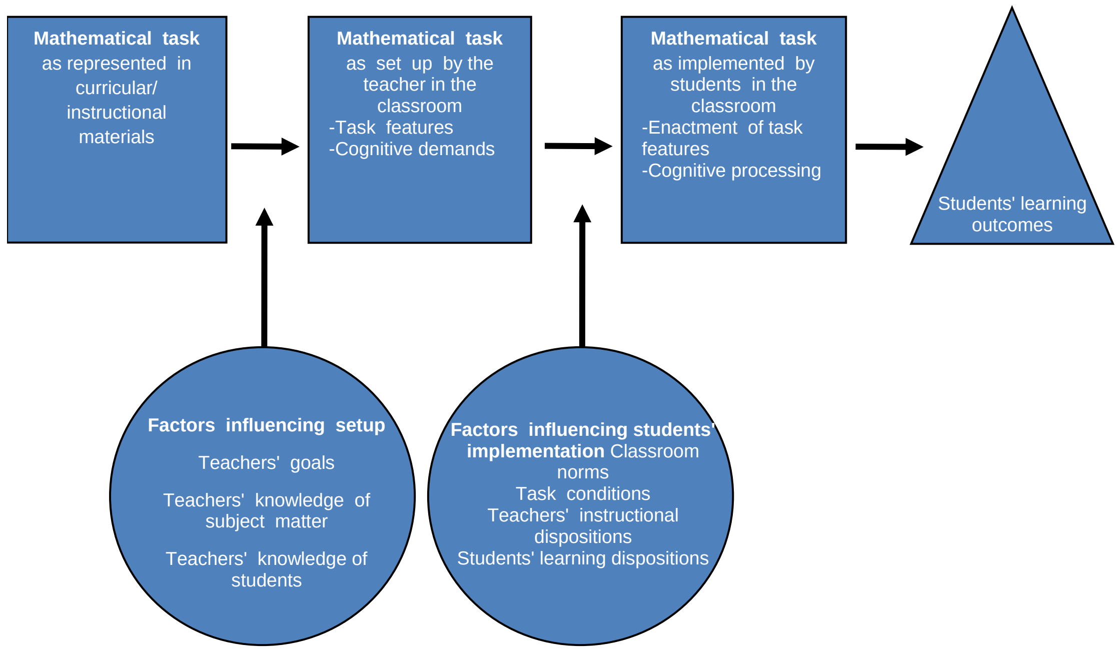

Figure 1. Relationships among various task-related variables and students' learning outcomes (Henningsen and Stein, 1997)

why tasks could not reach their aims. Klusmann et al. (2008) investigated the relationship between teachers' instructional performance and occupational well-being. Stigler and Hiebert (1999) interviewed the teachers to mention the differences between the implementations of tasks. Berg (2012) examined two teachers' task implementation processes and the differences between them, and emphasised the possible changes according to teachers' aims in the study. In an attempt to contribute to this growing literature, the aim of the research is to find out the factors that affect the task implementation of middle school mathematics teachers. Research questions of the study are:

1. What are the views of middle school teachers about designing and implementing mathematical tasks?
2. What are the approaches of middle school teachers to implement mathematical tasks?

Generalizing number patterns

Generalizing number patterns is an important concept for developing students' algebraic thinking. In order to help students to develop their algebraic thinking it is crucial to focus on algebraic form through numeric relations rather than using trial and error method while finding the general term of a number pattern.

According to Radford (2008) who approaches number patterns with this perspective poses three kinds of generalizations: algebraic generalization, arithmetic generalization and naïve induction. Radford (2008) suggests finding the rule of a number pattern using algebraic generalization. The algebraic generalization process is comprised of the following stages: noticing a commonality of the number pattern, examining whether or not the commonality is consistent in other terms of the pattern, and composing a general rule for finding any term within the pattern. Figure 2 shows the architecture of algebraic pattern generalizations.

When the focus is on a certain part of the number pattern, instead of the whole, the generalization reached does not serve to find any term within the pattern and does not have an algebraic structure. The arithmetic generalization process consists of noticing the common trait of the number pattern, and examining whether or not it exists in other terms of the pattern. The last stage of the algebraic generalization (reaching the expression p_n) does not exist in the arithmetic generalization. Pointing out relations between terms using numbers and finding the consecutive term are the examples of arithmetic generalization.

Naïve induction expresses reaching a generalization by trying out any rule, rather than reaching a rule through generalization. The naïve induction process includes the first and last stages of the algebraic generalization process. Finding the general term of a number pattern through trial and error, without examining the relations between the terms within the pattern is an example of naïve induction. In this regard, Radford (2008) expresses

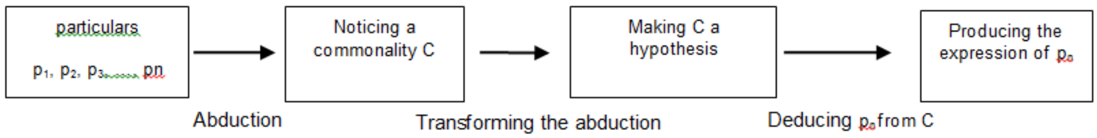

Figure 2. The architecture of algebraic pattern generalizations (Radford, 2008)

that naïve inductions are different from generalizations in that way.

If an example is to be given; a student who discovers the relation between the numbers when trying to find the general term of the number pattern 3 5 7 9… and reaches the rule 2*n*+1 makes an algebraic generalization. During this process, the student may use visual aids, like tables and models. A student, who focuses on the relation between the numbers and is able to point out that they increase two by two, reaches an arithmetic generalization. If someone, who does not conduct any examination of the number pattern, but instead tries to determine the correctness of certain algebraic expressions through trial and error uses naïve induction.

METHODOLOGY

The research model

Yin (1994) suggests determining research model regarding researchers' control over research situations, and the focus. Accordingly, we designate qualitative research model according to our research problems. Case study is the research strategy selected which is based on the detailed examination of a specific case in order to examine how mathematics teachers apply tasks in classrooms in detail (Wiersma, 2000). Since it is in the nature of case studies to understand social phenomena, interview and observation are two of the most commonly used qualitative methods. In order to gather information on teachers' views on the task design and implementing, interviews was conducted. Yet, as Patton (2014) has pointed out, we think that the participants may be unqualified to give adequate information. Because we think that the teachers may not be able to identify their in-class behaviours that have become habitual, and we utilised the observation method to assess their behaviour while implementing tasks.

Participants and setting

Due to the nature of qualitative studies, we regard participants not as a source in which equal information is gathered from each of its members, but as a source from which information on particularly rich situations can be gathered (Wiersma, 2000). Since this situation requires purposeful sampling, we select the participants on purpose. We used the criterion sampling that is one of the purposeful sampling methods in qualitative studies (Yıldırım and Şimşek, 2008).

The criterion specified in selecting the participant teachers are; willingness and professional experience. We group the teachers as less experienced than fifteen years and those with more experienced than fifteen years. We talk to the teachers individually to specify those who would voluntarily participate in the study. We continued the research with voluntary teachers. Table 1 presents the participants in-group.

Ministry of National Education and the administration of the middle schools give necessary ethical permission for research. Sixteen middle school teachers first interview about task and task enactment using "Pre-Interview Form". Afterwards we chose four of them based on their answers in pre-interviews and they implemented readily given tasks to the thirteen years old students. The classrooms consist of about thirty-five students. Students have experience about number pattern but not have any knowledge about the effective use of pictorial representation of a pattern. The observations were conducted with care in order not to disturb the teachers' and schools' routine.

Data collection tools

There are two data collection tools in the research. These are "Pre-Interview Form" and "Generalization Tasks (GT)". The main objective of the pre-interview form is to learn about mathematics teachers' views on task design and task implementation. We utilized the analytical framework designed by Stylianides and Stylianides (2008), who have done work on the classroom implementation of mathematical tasks, and the theoretical framework provided by Henningsen and Stein (1997) in designing the pre-interview form. We also used GT, which aims to help students to learn number pattern generalization with multiple representations. Aslan (2011) developed GT considering task design principles and student difficulties reported in the literature based on Swan (2007) theoretical framework as part of her master thesis.

Data analysis

All interview records were transcribed word for word. As Yıldırım and Şimşek (2008) have suggested for qualitative data analysis, we segmented and examined the data gathered, and determined the categories required for comparison and conceptualization. Categories for pre-interview form are; *attention points during task design, purpose of task implementation, use of tasks in lessons, points of attention during implementation, difficulties during implementation* and *effect of task implementation on learning*. Categories for GT in terms of the factors affecting students' implementation of tasks in the classroom are *task conditions, classroom habits and teaching approaches.*

Validity and reliability

A pilot study for pre-interview form with six teachers and edit accordingly misunderstood questions was conducted. Aslan (2011)

Table 1. Participants grouped according to their professional experience.

Less experienced than fifteen years	More experienced than fifteen years
K_5 K_6 K_7 K_8 K_9 K_{10} K_{13} K_{14}	K_1 K_2 K_3 K_4 K_{11} K_{12} K_{15} K_{16}

who designed the GT presents detailed validity and reliability in her master thesis. For the validity of the data analysis, the criteria considered are credibility and transferability. For credibility, we used researcher triangulation. The coding acquired during the data analysis conducted by the two researchers separately and later compared in order to reach a mutual categorization. The detailed description strategy was utilized in order to achieve validity of the analysis of the interview and observation data.

FINDINGS

In this section, we present the data gathered with the titles "views of mathematics teachers" and "classroom observations."

Views of mathematics teachers

According to the analysis of the data obtained through the pre-interview form on teacher processes of task planning and implementation, we emerge the following categories: points to consider in task design, the purpose of task implementation, place of tasks in lessons, points of attention during implementation, difficulties during task implementation, and effect of task use on learning. Categories and sub categories along with their frequencies are presented in Table 2.

As seen in Table 2, we collect the data on task designing under four categories. One of the sub categories is learning outcome. Participants who think tasks should focus on learning outcomes have expressed that tasks toss the topic into different places, and do not help to reach a conclusion otherwise. Although teachers agreed that tasks should focus on student outcomes, their opinions on outcomes differ on what is the extent of outcome. The number of learning outcomes seems to be the main point of difference among teachers: one single task for each learning outcome or a task for more than one learning outcome. K_2 stated his opinion as follows:

"… [The task] should not complicate the concept too much. I mean, the task needs to focus on a certain outcome you need to give. It shouldn't be too detailed."

Another participant (K_5) pointed out that the task should aim to teach the whole of the outcome, and explained:

"… Tasks are good but they handle the concept at all points. They remain on a single, small piece. Therefore 40 minutes flies with a particular part of the concept, we don't have time to emphasize the other features of the concept."

Well-structured tasks are another emphasize of teachers. Some of the teachers touched on the importance of well-structured tasks and underlined that teacher must prepare it before the lesson before implementation. K_6 explained his/her point of view as:

"… The task needs to be very well structured. If a teacher uses a task, then s/he will plan it right from the beginning to the end. S/he must consider every stage of the task for the duration of 40 minutes. Of course spontaneous situations might happen during lesson but the teacher must plan most of it and be prepared accordingly."

Similarly, K_7 has expressed the importance of clarity of the stages of the task:

"It needs to be well communicated. For instance, when I give a performance task to students, I explain it step-by-step and give them a photocopy. The tasks should be worked on like this."

Some of the teachers found the visuality and interestingness of the task as important as the content. K_1 explained this opinion as:

"… [The task]... must be colourful, because when the child looks at the colours their colour perception becomes much higher. I'm not talking about painting or drawing, because just merely drawing is no use."

Some of the participants have pointed out that the task must both have conceptual and interesting aspects. One of them, K_9, said this:

"… some [tasks] are really too simple and aren't really aim to make the topic understandable […] being fun is a must, but well, also it has to be in a way that makes the child constantly curious about the next step. Students just glance over the task and say, I already know this. Tasks could be more interesting, more intriguing."

The two prominent sub-categories that stand out for first category are learning outcome based and well planned. We observed that the teachers mostly expressed that a task should have these two features. The second

Table 2. Participants' views on task design and implementation.

Categories		Participants	Frequency (f)
Points to consider in task design	Being visual	K_1, K_{12}	2
	Focused on learning outcome	K_2 K_3 K_5 K_8 K_9 K_{10} K_{11} K_{12} K_{13} K_{14} K_{16}	11
	Being well planned	K_5 K_6 K_7 K_9 K_{10} K_{11} K_{12} K_{13} K_{14} K_{15}	10
	Being interesting	K_6 K_8 K_9 K_{11}	4
Purpose of task implementation	Motivation	K_4 K_2 K_{13}	3
	Conceptual learning	K_2 K_5 K_{13} K_{14}	4
Place of tasks in lessons	Use	K_2 K_3 K_5 K_6 K_7 K_8 K_9 K_{10} K_{11} K_{12} K_{13} K_{14} K_{15} K_{16}	14
	Not use	K_1 K_4	2
Points of attention during implementation	Active participation	K_3 K_6 K_9 K_{10} K_{12} K_{13} K_{14}	7
	Preparing the materials beforehand	K_6 K_7	2
	Coming prepared (for students)	K_8 K_9 K_{15}	3
	Teacher guidance	K_2 K_3 K_{10} K_{11} K_{14}	5
	Spontaneous response	K_3 K_6 K_{11} K_{13}	4
Difficulties during implementation	Inconvenient classroom conditions	K_1 K_3 K_4 K_5 K_7 K_8 K_9 K_{10} K_{12} K_{14} $K_{15}K_{16}$	12
	Insufficient time	K_3 K_4 K_5 K_6 K_8 K_{12} K_{13} K_{14} K_{15}	9
	Different success levels	K_5 K_6 K_{10}	3
	Classroom management	K_2 K_6 K_{15}	3
Effect of task use on learning	Effective	K_5 K_6 K_7 K_9 K_{10} K_{11} K_{13} K_{14} K_{16}	9
	Not effective	K_4 K_8	2

category is teachers' objectives for task implementation, which has two subcategories. One of them is conceptual learning. On this subject, K_{13} expressed:

"… [The task] extends my lesson plan three minutes more, but students understand the concept better. This is what a task means to me…"

Similarly, K_2 has also expressed the reason for using tasks as it increases student understanding:

"… when [the task] implemented, students understand the topic better. […] I really prefer using tasks in that regard."

The other subcategory of teachers' purpose is motivation. Some of the teachers expressed that tasks increase student motivation. One example is K_2's opinion:

"Tasks motivate the students. Because they get bored of constantly writing, they would like to work by themselves. Because students practice and reach conclusions by themselves with the use of tasks, I think they really support the learning"

K_4 has also explained the reason for implementing tasks as:

"…this is also about the children's motivation. Believing in what they will learn, rather than what the teacher will teach them gives them pleasure."

The third category is place of tasks in lessons. Two of sixteen teachers stated in interviews that they did not make use of tasks in maths lessons. One of them, K_4, explained the reason as:

"Up till now I haven't [enacted tasks]. The reason is my classes' size; very crowded. I give tasks as homework to the students."

Contrary to the teachers who prefer not to implement tasks, K_{13} stated that they prefer to prepare tasks, especially the tasks designed by him/her:

"I use tasks as often as possible in my lessons […] and I prepare most of my tasks myself."

We group the forth category, points of attention during implementation, into five subcategories. The first of these is active participation. One of the teachers who find active student participation important is K_3, who has explained this with the following words:

"...student's participation affects all of the students. I always pay special attention to include all of the students in the classroom."

Some of the teachers, who find the active participation important, have also emphasized that students must understand the task completely. K_6's view is an example of this:

"If students did not understand the task, I'm saying, let's question this... maybe some of the students did not understand something... We need to notice these and observe the students well."

The second subcategory is the teacher's guidance during implementation of tasks. Teachers pointed out the importance of guidance, and being aware of and acting according to students' individual differences. K_{10} shared ideas with the following words:

"It's very important do draw attention of students. When we focus on children with different learning levels, or students with high intelligence, or high perceptive capacity, who can't answer the simplest questions, or have difficulty understanding the simplest examples, who's attention we can barely get, who have economic hardships, or are afraid of either the lesson or teachers, when we consider them it becomes easier."

In addition, K_{13} has explained the importance of the teacher's guidance within student-teacher interaction:

"First of all the eye contact with the students is very important […] they give you the signals. You act according to the reactions you get from the students."

The third and fourth subcategories are "coming prepared for the task (for the students)" and "preparing the necessary materials beforehand". The participants believe it is better for the students to come prepared for the task because this makes the implementation easier. K_8 explains:

"… I suppose it is better coming prepared for the task. I think if teacher explains and students study on it at home before the lesson, task will be over sooner."

Teachers argued the implementation becomes difficult if the necessary material is not ready. K_7 has expressed their opinions thusly:

"Firstly, attention needs to pay providing the children materials. I mean, the class's economic situation. If material is not provided it is not possible to implement the task successfully."

Similarly, K_{10} has pointed out the importance of having the material pre-prepared, and mentioned that there should be material preparation rooms in schools.

"… Most importantly, if only, there were material preparation rooms in every school with let us say rulers, craft knives, carton, and glue. In order to prepare we need extra time and elbowroom."

The final subcategory is spontaneous response, indicating that teachers should be able to change up the lesson according to their requirements in the classroom. K_{13} states:

"… The task might not go as you imagined in your head once it starts. It begins in your head, but then depending on the condition of the class, students might get bored. You need to quit the task because it did not work at that point. You must not insist on continuing the task implementation just because you started. In addition, if you complain to the children that you implemented this task with the students from different class and it worked, you will label them and the task implementation will go in a worse direction. The teacher must be able to discern these and know when to stop himself or herself".

This is an important aspect of task implementation, however very few teachers underlined this statement. The fifth category is about the difficulties encountered during implementation of tasks. We group this category into four subcategories. These are inconvenient classroom conditions, lack of time due to curriculum, different success levels, and classroom management. Twelve of the sixteen teachers said that inconvenient classroom conditions have led to the implementation difficulties. K_9 has said that the number of students is also a problem during implementation:

"… The sizes of classes I teach are always around 35 people. It is difficult to control students because of crowded. I mean, it is not easy to check their under-standing in the beginning and during of the task one by one. I have no other problems to implementation of task apart from that."

Another classroom difficulty teachers touch upon is the lack of special mathematics classrooms in schools. K_1's view is as follows:

“It’s good to implement tasks, but this necessities maths classroom. Students need to work in groups […] But because classes are overcrowded it takes time to come and get them into a certain sitting arrangement, getting the belongings, students, desks in a certain order every time. I mean if there was a mathematics classroom, where the students would be readily prepared and these weren’t required, than it would be done easier…”

In addition, the teachers emphasized that mathematics curriculum had so many subject to teach. Therefore, they could not give enough time to students for tasks. One of the teachers who expressed difficulties with time management due to the curriculum is K_5, who said;

“Implementing tasks is actually positive, in theory. In fact, it is also good for some of the topics. But crowded classes and the volume of topics make it impossible to implement the given task for each lesson.”

Similarly, K_8 has explained the difficulties due to curriculum:

“I admit that tasks are helpful, but because of time restrictions we don’t always implement tasks. OK it is helpful, the children are entertained and tasks make them happy but we also have topics to cover. There are so many topics to teach in the maths curriculum. I mean if it becomes simpler in the future, then maybe we can implement the tasks in a calmer and more comfortable manner.”

Three teachers mention that the different success levels of students complicate task implementation. K_{10} who is one of these teachers explains:

“When some of the students get the task quickly and express the opinions rapidly, implementation can be negatively affected.”

Some teachers have pointed out that they have difficulties managing the class during task implementation. For example, K_2 has spoken on having difficulty to control communication between each other in the classroom:

“Well yes, there is some noise. Some experts say, learning creates some noise; but also a good learning happens in a noiseless environment. We try to keep it balanced.”

K_6 has emphasized the difficulties that arise when the students work in not properly arranged groups, and expresses his/her views as such:

“*In very crowded classes task implementation can lead to noise, commotion and disorder. In that sense, the students need to work in groups; we must organize the groups carefully. Sometimes one student in the group dominates the other members, and none of the others does any work. Then we must organize the groups carefully.”*

The sixth category is the effect of task use on learning. Nine of the teachers have argued that it does matter. K_{13} explained his/her views as:

“… It certainly is [effective]. As previously stated, teachers’ activeness, control over the students, command over the task and problems you may encounter, you have to foresee these things before the implementation. You need to know the students well. Without these the task won’t achieve its purpose anyway.”

When we evaluate the findings from the teacher interviews, we conclude that they mostly dwell on good planning and learning outcome based approach in task design. About the purpose for implementing tasks, there were no views that particularly stood out, but some stated that their objective was to motivate or achieve conceptual learning. Most of the teachers mentioned that they implement tasks, and during implementation, their focuses were to get active student participation and supply effective guidance. While they think that, the ways of task enactment affect the learning due to curriculum and inconvenient classroom conditions they have problems with implementation of tasks.

Classroom observations

We examine the data gathered on the teachers’ task implementation based on two headings in line with Henningsen and Stein (1997) framework:

1. Factors effecting teachers’ (re)composition of tasks
2. Factors affecting students’ implementation of tasks in the classroom.

Factors affecting teachers’ (re)composition of tasks

According to the theoretical framework subject matter knowledge, student knowledge and objective affects teachers’ (re)composition of tasks. When the data were analysed considering these three factors in mind, the categories here show up as use of strategies, use of multiple representations, and use of explanations and justifications.

Radford (2008) defined the category for strategies addressed with “architecture of algebraic generalizations” conceptual framework. As such, the subcategories for use of strategy are algebraic generalization, arithmetic

Table 3. Factors affecting teachers' implementation of tasks in the classroom.

Categories		K_{13}	K_5	K_6	K_2	K_{15}	K_4
	Algebraic generalization	√	-	-	-	-	-
Use of strategies	Arithmetic generalization	-	-	√	-	√	-
	Naive induction	-	√	-	√	-	√
Use of multiple representations	Tables	-	-	-	-	-	-
	Visual models	√	√	√	√	√	√
	Verbal	√	√	√	√	√	√
Use of explanations and justifications	Numerical	√			√	-	-
	Algebraic	√	√	√	√	√	√

generalization and naïve inductions. The subcategories of multiple representations are tables, and visual models.

Finally, subcategories of explanation and justification are verbal, numerical and algebraic. Table 3 summarizes these categories and subcategories. In terms of use of strategies, we expect to generalize patterns with the aid of visual models provided, and to reach an algebraic generalization using the using the *n* notation. As seen in Table 3, three of the teachers' approach was naïve inductions; two of them made arithmetic generalizations and one of them preferred algebraic generalizations. One of two teachers who make use of arithmetic generalizations, K_6, stated his/her point of view as follows:

"…Are there unchanging or stable parts in the visual models in every step? […] on the first step I added one to each, on the second step I added two to each, on the third step I added three to each; so then can there be a relationship between them? You've already found that as many squares as the number of steps will be added next to the white model."

Yet, K_5 has only expected the students to state the rule, which means K_5 made use of naïve induction when generalizing patterns. K_5 has explained his/her strategy as:

"We increased the visual models by adding four to each, but look, in the first on there are five squares, in the second there are nine. If it goes on like this, how many squares would be in the ninth step?"

According to an observation, only K_{13} used algebraic generalization strategy. K_{13} made the following explanation in class:

"… We added two bars in every step. Then in order to find the tenth step, we multiply ten by two. Therefore, we add twenty bars. However, here in the first step there are additional three bars, so we add one."

The tasks given to the teachers had both tables and visual models. Nevertheless, all of the teachers preferred to generalize the patterns with the aid of visual models alone. In terms of the use of explanations and justifications category, all of the teachers participating in the study have supplied algebraic explanations and confirmations. K_{13} and K_2 have used all three types of explanations. In addition, K_{13}, not only explained the relation between steps but the relation of the number of the step to the step itself.

"In the first step there are 5 square units, in the second step there are nine square units. Therefore, there is a four square unit increase between each step. Then we multiply the number of the step by four, and then add one (4.1+1). Now let us see if we can control all other steps like this (4.2+1=9). Then how can we find the 75th step? (75.4+1)"

In sum, factors affecting teachers' (re)composition of tasks class are use of strategies, use of multiple representations, and use of explanations and justifications. Teachers tend to make arithmetic generalization and naive inductions rather than algebraic generalizations. They prefer to use visual models instead of tables, but they did not effectively use them to reach algebraic rule. Mostly they used them for visualisation. Although they did not use algebraic generalization, they wanted students to use algebraic notation with the use of trial and error.

Factors affecting students' implementation of tasks in the classroom

According to the theoretical framework, the factors affecting task implementation are classroom and task conditions, teachers' instructional dispositions, and students' learning disposition. With these factors in mind, the following three main categories appeared are task conditions, classroom habits, and teaching approaches (Table 4).

Table 4. Factors affecting students' implementation of tasks in the classroom.

Categories		K_{13}	K_5	K_6	K_2	K_{15}	K_4
Task conditions	Understanding the nature of the task	√	-	√	-	√	-
	Misunderstanding the nature of the task	-	√	-	√	-	√
	Adapting tasks according to conditions	-	√	√	√	√	-
Classroom habits	Informing about the lesson	√	√	√	-	√	√
	Acting as a whole	-	√	√	-	√	-
	Allowing to work individually	√	-	√	√	-	-
	Prioritizing taking notes	-	√	-	√	-	√
Teaching approaches	Asking supportive questions	-	-	√	-	√	-
	Asking one-way questions	-	√	-	-	√	√
	Using reinforces	√	-	√	-	√	-

As seen in Table 4, we analysed the factors affecting students' implementation of the task within the classroom through the context of the teacher. We take into account understanding the nature of the task correctly, being able to adapt it, the habits that came to be through the teachers' interaction with the class, and teacher approaches during task implementation considering framework. As previously stated, given tasks required the use of both tables and visual models. However, only three of the teachers understood the nature of the task. For example, K_5 used visual models in order to write the pattern numerically:

"How many cubes are there in the first step? In the second, third steps, how many cubes are there, write them underneath the model"

K_5 preferred to directly turn the visual model into a number pattern, rather than make effective use of it. K_5 also opted not to ask some of the questions existing within the task to the students, instead attempting to adapt the task to the situation. K_{15} similarly explained skipping questions during the task as:

"Our figure is here, I only want the number of bars in the fourth step. And I want the rule of the pattern."

K_{15} has preferred to ignore the questions asking the number of bars for the 100 and 1000th steps. As a result of the observations, some of the habits teachers have in the classroom came to attention. K_4 has a preference to inform the students of what they will be doing at the beginning of the lesson:

"…In our lesson today we're going to see patterns. We will learn how to find a general rule. In order to do these we'll start with exponential numbers, what is an exponent, what are the base numbers, how to multiply exponential numbers…"

K_2 however, preferred instead to start by asking what the pattern is without giving any explanation. Along with this, some of the teachers gave information about the tasks and their enactment at the beginning of the lesson. For example, K_6 has expressed the wish to perform the task with the whole class together as follows:

"…Wait, we're not starting yet. We are going to do it together. Let everyone get it. We will go step-by-step. Yes, we're looking at first task"

Contrarily, K_{13} has expressed allowance for individual performance with these words:

"In the second task, we see the term and its number. Let me give you some time […] Think well. I'm coming over to you […] Now everybody read the task thoroughly yourselves…"

K_6 while preferring to walk through the task altogether with the class at the beginning, allowed the students to work on the tasks by themselves after some time. Some of the teachers were sensitive about note taking during the lesson. K_4 has given extra time for the students to take notes:

K_4: Do you understand? Do you know the difference between this and that? You would better to write it down real quick, come on.
Students: Teacher, we are still writing.
K_4: Alright, write it down, I'm waiting.…"

Contrarily, K_{13} has said that the students could not learn thoroughly while writing:

"…Can we look this way please? I do not want you to write, I want you to learn. OK?"

In addition, teachers approached the students with the aid of one-way or supporting questions. One-way questions refer to the questions teachers ask the students within the lesson, yet answer themselves. The following excerpt from K_5 is an example:

"Have we discovered a rule about n*? Yes, we have. How did we discover this rule? We looked at the relations between the numbers. We looked at the increasing and decreasing numbers in this model. Thus, we have found the rule. Can we find a rule through this model? Yes. What was this rule? 5n+3."*

K_{15} however, instead of telling the students they made mistake when they give a wrong answer, used questions to notice:

"So you're saying that in one square four bars are used […] now look, for three squares ten bars have been used. Can we do this with ratios then? Can such a ratio exist for these?"

Another dialogue in which K_{15} uses supportive questions went like this:

Student: It increases by three. For example in the third step there are ten, I used my fingers for each. 13, 16, 19, 22, 25…
K_5: Hmm. You used the finger counting method. Well, if I were to say how many bars do you need for 50 squares, how would you calculate? How many bars do you need for 100^{th} step?

Along with this, K_{13} has used supportive questions, not to get students to notice their mistakes, but to make it easier for them to reach generalizations:

K_{13}: What is happening with each step?
Student: It increases.
K_5: How much does it increase? […] It increases more here. What is happening, how much does it increase by each time?

The participant teachers, along with the explanations they made to correct the mistakes of the students, as seen as earlier stated, they have also used certain reinforces for their correct expressions. For example, K_{13} has used reinforces such as;

"That's it! You are great. […] Thank you, that's exactly the sentence I wanted." during class.

Similarly, K_6 has said
"You're wonderful. You may applaud yourselves."
for their contributions to the tasks.

To sum up, the factors affecting students' implementation of tasks are task conditions, classroom habits, and teaching approaches. Teacher had difficulty in understanding the nature of the task, which affects the implementation at all. Teachers' classroom habits differs as informing about the lesson, acting as a whole, allowing to work individually and prioritizing taking notes. Teachers teaching approaches are mostly based on one-way questions and reinforces.

DISCUSSION AND CONCLUSION

The purpose of this research has been to examine the task implementation of middle school mathematics teachers. We aim to find the factors affecting the task implementation and draw attention to transition between designing and implementing. In this section, we summarize and interpret the findings along with some suggestions in terms of mathematics education.

According to findings, most of the teachers have preferred to make use of multiple representations. Mostly used representations were tables and visual models. However, the explanations of teachers about the representations deprived from meaning were used. Almost all teachers used visual models without focusing on its relationship with the pattern; they made mistakes of counting the shapes, converting the visual patterns into number patterns, and confusing model use with ornamentation. Teachers' lack of subject matter knowledge prevented the implementation of tasks.

Teachers' views were grouped under four subcategories about designing a mathematical task. One of them is that the task must focus on learning outcomes. Participants that were of the idea that the task should be a learning outcome oriented which have stated that tasks that do not focus on learning outcomes tend to scatter the topic and make results difficult to reach. Along with suitability to learning outcomes, another point of focus was tasks structure. Teachers who made this point have emphasized that teachers should implement the tasks after a thriving preparation. Mostly referred subcategories are focus on learning outcome and structuring the task properly. Giving time for the task and alerting students to focus on the tasks is important for teacher behaviours that help students focus on the task (Doyle, 1988).

According to results, we found that twelve of fourteen teachers implement tasks in their math lessons. Some of the teachers were less interested in the content of the task, but more interested in their visual qualities and interestingness, thinking of the tasks as a way to motivate students. The matter most often emphasized by teachers is active student participation, teacher's guidance during the task, and spontaneous interventions during the lesson. Other matters of importance were students and materials state of readiness before the task implementation. Metin and Özmen (2009) studied on the issues that teachers face while designing and implementing mathematical tasks with 5E model. Similar to this study finding they emphasised that effective use of time, classroom management, students' motivation, in short, teacher qualifications are very important for implementation.

About the difficulties faced during implementation of a task, teachers have mentioned that inconvenient classroom conditions create issues. Tasks' implementation ways also affect whether or not they reach the desired result; generally, the participants argued that it does. Teachers' implementation ways varied depending on professional experience; more experienced teachers' ways are consistent with teacher centred approaches.

Based on observations, we saw that most of the teachers found the rule of number patterns through naïve inductions. The tasks given to the teachers contained number patterns that required use of both tables and visual models, yet all of the teachers preferred to generalize with the aid of only visual models. Teachers' understanding of the tasks' nature, their ability to adapt the task, the habits they developed in the classroom and their approach instructing the students were the second dimension of the theoretical framework of this research. As stated earlier, the tasks required the use of both tables and visual models. However, only three of the teachers were able to understand the nature of the task correctly. The tendency was to directly convert the visual model into a number pattern, rather than make appropriate use of the model.

The research findings indicated that teachers must implement tasks following student centred pedagogy. When this does not happen, tasks do not tend to give the intended results. If teacher implement student centred task with a teacher centred manner, the students are unable to get the intended gains from it. This finding brought to attention the importance of developing students in an environment where they are the subjects of their own learning (MEB, 2013). In this case, we suggest preparing tasks in a way that puts the student in their centre; and with cohesion between the designing objective and implementing objective understood by the teacher. This finding puts teachers' importance during task implementation. Likewise, Watson and Mason (2007) stated that student learning is not only affected by students' action on task, but also teachers' interpretation and instruction.

We determined inconsistencies between the answers given in the pre-interview and classroom observations in terms of teaching number pattern generalizations. For example, although the teachers gave answers on teaching number patterns using strategies, we found that they mostly preferred to use trial and error. This finding did not vary by professional experience. All of the teachers with insufficient content knowledge had the same misconception. This finding highlights the importance of having subject matter knowledge.

This study only examines the teacher component of the task implementation process in mathematics classes. Conducting a research, which examines the task implementation with a socio-cultural perspective, paying mind to the student component and classroom norms, is a further research to look into.

Conflict of Interests

The authors have not declared any conflicts of interest.

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