

Full Length Research Paper

Students' thinking process in solving combination problems considered from assimilation and accommodation framework

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Received 11 April, 2016; Accepted 15 August, 2016

This study aims to explain the thinking process of students in solving combination problems considered from assimilation and accommodation frameworks. This research used a case study approach by classifying students into three categories of capabilities namely high, medium and low capabilities. From each of the ability categories, one student was chosen as a research subject. The results of this research showed that the student in high ability category, in understanding, planning, implementing and checking the problem solving used assimilation thinking process and they also used accommodation thinking process in solving a problem. The student in the category of medium ability, in understanding, planning, implementing and rechecking problem solving used assimilation thinking process. The student in the category of low ability, in understanding, planning and implementing problem solving used incomplete assimilation thinking process, and in rechecking the results of problem solving he was not able to do the thinking process of assimilation and accommodation.

Key words: Thinking process, combination, assimilation, accommodation.

INTRODUCTION

Combination material is a difficult material for students and is a prerequisite material for studying opportunity and statistics (Garfiel and Ahlgren, 1988). Combination is a part of Discrete Mathematics which has many usefulness in everyday life and is closely related to the real context (Abrahamson, 2008). Permutation and combination is one of the materials in Discrete Mathematics course (Susanna, 2004). However, in reality the students of Department of Mathematics, Universitas

Negeri Malang still have mistakes while solving permutation and combination problems (Sukoriyanto et al., 2016).

Students do thinking process when solving problems (Frenke and Kazemi, 2001). In the thinking process, there is a process between incoming information and scheme (cognitive structure) in a person's brain (Subanji and Supratman, 2015). Experience or new information received will be adapted through the

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process of assimilation or accommodation (Simatwa, 2010).

Cognitive process that occurs when a person integrates perceptions, concepts or new experiences into an existing scheme in his mind is called assimilation (Subanji and Supratman, 2015). While accommodation is the process of integrating the new stimulus through the creation of a new scheme or change the old scheme to adjust to the problems encountered (Subanji and Supratman, 2015). According to Hoppes and Segal (2010), accommodation can happen in two ways, namely to modify the existing scheme to match the stimulus given or to form a new scheme that matches the given stimuli.

To track the type and the mistake location done by the students in solving combination problems, the lecturers need to know the thinking process of students in solving combination problems. By knowing the location and type of mistakes of the students in solving combination problems through tracking thinking process, the lecturer can design an appropriate learning which is suitable with students' thinking process. Students require high thinking level in solving the problem. The steps of problem solving according to Polya this consists of four steps:

1. Understand the problem
2. Devise a plan
3. Carry out the problem, and
4. Look back (In'am, 2014).

Students are expected to develop their problem-solving abilities to face challenges by adopting various perspectives (Tai and Lin, 2015). To give the students experience in using the knowledge and skills possessed can be done through a process of problem solving (Can, 2015).

Students perform process series of thinking in solving problems (Saragih and Napitupulu, 2015). Similarly, when the students solve the combination problem they need to perform series of thinking process. If the students' thinking process in solving combination problem does not get the attention from the lecturer, there may be an impact on students' learning outcomes. Therefore, the disclosure of students' thinking processes in solving combination problems needs to be done to know the students' thinking structure. Based on the problem stated earlier, the problem in this research is "how is the thinking process of students in solving combination problems considered from assimilation and accommodation frameworks?"

METHODOLOGY

The research subjects were 22 students of 3rd semester,

Department of Mathematics, FMIPA, Universitas Negeri Malang. The subjects were those who took Discrete Mathematics subject, whose ages ranged from 19 to 21 years. Furthermore, the subjects were asked to work on the problems associated with combination. The results of the student work were assessed by using score from 0 to 100. The abilities of students were classified into three categories based on the results of test scores. They were in low-ability category if the test scores ranged from 0 to 55; the students were in medium ability category if the test scores ranged from 55 to 80; and the students were in high ability category if the test scores were between 80 and 100. Furthermore, the researcher selected one student from each category to become the research subjects. The selection of research subjects (low, medium, high) was done with the reason that the thinking process of all ability categories could be searched.

Answer to each subject (categories of low, medium and high) in solving the combinations problem was identified based on the stage of problem-solving according to Polya. Stages of problem-solving by Polya consist of four steps, namely:

1. Understand the problem
2. Devise a plan
3. Carry out the problem, and
4. Look back.

At every stage of problem-solving according to Polya, students' thinking processes is identified by using a framework of assimilation and accommodation according to Piaget. The main instrument in this research was the researcher himself equipped with problem relating to combination, video tape recorder, and interview sheet. The problem related to the combination was as follows:

In how many ways can you divide 6 students into 3 groups with 2 members of each group?

RESULTS AND DISCUSSION

Based on the results of the students' works in solving combination problem, the division result of the students' ability categories were obtained as in Table 1. Based on the three categories of students' abilities earlier stated, one student from each category was chosen namely subject A for the high ability category, subject B for medium category, and subject C for low category. Analysis of thinking process based on the written data and interview data in solving permutation and combination problems based on the problem solving steps according to Polya that included understanding the problem, devise a plan of problem solving, implementing the plan of problem solving, and looking back at the results of problem solving were as follows:

Subject A that has high ability category

At the stage of understanding the problem, subject A was able to identify that the problems encountered were related to combination. At the time of the interview, subject A could reveal that the problem was combination

Table 1. Categories of students' ability to solve combination problems.

| S/N | Categories of students' ability | Range of scores (x) | | Number of students | Percentage (%) |
|-----|---------------------------------|---------------------|----------|--------------------|----------------|
| 1 | Low | 0 | $x < 55$ | 12 | 54 |
| 2 | Medium | 55 | $x < 80$ | 6 | 27 |
| 3 | High | 80 | 100 | 4 | 19 |

$${}^6C_2 \times {}^4C_2 \times {}^2C_2 = \frac{6!}{4!2!} \times \frac{4!}{2!2!} \times \frac{2!}{0!2!} = 15 \times 6 \times 1 = 90$$

Figure 1. The work of subject A in solving combination problem.

problem. But after being tracked further, subject A doubted that the problems faced was combination problem that should be associated with the other problems. After the student was given the opportunity to think, the student revealed that the problem encountered was the problem associated with the combination of multiplication rule. It showed that in understanding the problem, subject A integrated a new stimulus through the establishment of new schemes to adjust to the problem encountered. Thus, subject oA used accommodation thinking process in understanding the problem (Hoppes and Segal, 2010).

At the stage of arranging plan for problem-solving, subject A was able to relate the problems faced and the problem of combination and multiplication rules. When it was traced through interview, it was found that subject A was able to link the key components of the problems that was the number of students were six, the students were divided into three groups and each group consisted of two students. A subject was able to identify that the division of the group in which each group had two students did not need to pay attention to the order so that the problem used was combination problem. However, subject A had a difficulty in determining the relationship of the division among the first group, the second group and the third group consisting of two people. After being searched further, subject A revealed that the division of the first group, the second group and the third group was interrelated events so it was necessary to apply the multiplication rule. It showed that in making the plan of combination problem solving, subject A modified the existing scheme so it matched the stimulus given. Thus subject A used the accommodation thinking process in preparing a plan for combination problem solving (Reinking and Labbo, 2000).

At the stage of solving combination problem, subject A was able to apply the concept of combination and

multiplication rules. At the stage of determining the members of group 1, subject A was able to finish it by using combination 6C_2 . In determining the members of group two and three, subjects A was also able to finish it by using combination 4C_2 and 2C_2 . However, when determining the final answer, subject A actually was already able to apply the multiplication rule, namely ${}^6C_2 \cdot {}^4C_2 \cdot {}^2C_2 = 90$ but subject A made a mistake when considering that the result should be divided by $3!$. The work of subject A can be seen in Figure 1.

From the interview, it could be seen that in determining the final answer, subject A felt doubtful on the final answer. Subject A tried to translate the existing problem by making an example: when the students were A, B, C, D, E and F, then subject A constructed three groups in which each group consisted of two students. Subject A attempted to think about what happened if one of the group arrangements was AB, CD and EF. The thinking structure of subject A was that AB, CD, and EF did not need to pay attention to the arrangement because AB, CD, EF and CD, AB, EF were the same, so that subject A decided that the correct answer should have been divided by $3!$. It showed that in arranging a plan of combination problem-solving subject A integrated perception, concept or new experience into an existing scheme in his mind when solving the problem. But when it was traced through an interview, subject A constructed a new thinking structure that in determining the final result, it should have been divided by $3!$. Thus subject A in implementing problem solving used assimilation and accommodation thinking process (Reinking and Labbo, 2000).

By examining the work done by subject A, it seemed that subject A did not write 'recheck' on the result of problem solving obtained. However, based on the interview subjects A rechecked the problem solving result obtained by matching the command about the problem resolved. It showed that in rechecking the

$${}^6C_2 = \frac{6!}{2!(6-2)!} = \frac{6 \cdot 5 \cdot 4!}{2 \cdot 4!} = 15$$

Ada 15 cara

Figure 2. The work of subject B in solving combination problem.

problem-solving result, subject A simply linked the concept or experience possessed related to the answer obtained from the problem with a simple command on the problem solved. Thus subject A in rechecking the result of problem solving used assimilation thinking process (Simatwa, 2010).

Subject B which has medium ability category

The difficult problem of Subject B at the stage of understanding to decide the problem faced was combination problem. At the time of the interview subject B was confused whether dividing six students into three groups in which each group had 2 students was a problem of combination or permutation.

After being given the chance to think subject B revealed that the problem faced was a combination problem because dividing the group did not need to pay attention to the order. It showed that in understanding the problem, subject B in stimulus integration just went through the establishment of a new scheme to adjust to the problem encountered.

According to Hoopes and Segal (2010) subject B used accommodation thinking process of in understanding the problem. At this stage of arranging problem-solving plan, subject B was only capable of linking the problems faced with the combination problem without linking the problem with the multiplication rule. At the time of the interview subject B had a difficulty to find a relationship between the number of the groups and the members of each group having two students. After being given time to think, subject B said that the problem faced was combination problem 6C_2 . Subject B gave the reason that for dividing six students into three groups in which each group consisted two students was the same as counting 6C_2 . It showed that in making the plan of combination problem solving subject B modified the existing scheme so it matched the stimulus given. Thus, subject B used accommodation thinking process although he was wrong in arranging a plan of combination problem-solving (Hoopes and Segal, 2010).

At this stage of combination solving problem, subject B made a mistake in accordance with the steps taken during the problem solving plan. Subject B only determined a lot of ways to divide the groups by using 6C_2 without noticing that the group should be formed into three groups. The work of subject B can be seen in Figure 2.

Based on interview subject B had a difficulty in linking the problem encountered with the concept or another experience related to the multiplication rule. It showed that in arranging plan of the combination problem-solving subject B attempted to modify the existing scheme in mind and it was forced to match the stimulus given. Thus subject B used accommodation thinking process in implementing problem solving (Hoopes and Segal, 2010).

Based on the work and interview, subject B had already rechecked the result obtained. Rechecking the result of problem solving was done by linking the final result of work obtained by reading back the problem. It showed that in rechecking the result of problem solving subject B only related the concept or the experience owned and related to the answer obtained from a problem with a simple command on the problems solved. Thus, subject B in rechecking the result of problem solving used the assimilation thinking process (Simatwa, 2010).

Subject C who has low ability category

Subject C had difficulty in understanding the problems related to a given combination problem. From the result of examining the student's work and the result of interview, it was obtained a description that subject C considered that the problem of dividing six students into three groups with two members in each group was a problem of permutation with the same elements. Subject C at the time to understand the combination problem given was interference with the understanding of permutation with the same elements. It indicated that subject C when understanding the problem used accommodation thinking process (Hoopes and Segal,

$$6 \text{ siswa} \rightarrow 3 \text{ kelompok} \rightarrow 2 \text{ orang}$$

$$\frac{6!}{3!2!2!} = \frac{6 \times 5 \times 4}{4} = 20$$

Figure 3. The work of subject C in solving problem combination.

2010), although he was wrong and subject C had an interference with the understanding of permutation.

At the stage of arranging plan of combination problem-solving, subject C arranged a plan of problem solving based on the understanding owned by him. Because subject C was interfered at the time to understand the problem that was the combination problem seen as a permutation problem, so when arranging problem solving, subject C related the existing key concept in the problem that the students were six, divided into three groups and each group consisted of two students. It was a permutation problem with the same elements. It showed that in making the plan of combination problem solving subject C tried to modify the scheme existing in his mind and was forced to match the stimulus given.

Thus, subject C used accommodation thinking process in preparing a plan for solving a combination problem (Hoopes and Segal, 2010), although he was interfered with the permutation problem. At this stage of solving a combination problem, subject C was able to solve the problem based on the accomplishment plan that had been made, but the result was still wrong. The work of subject C can be seen in Figure 3.

Based on the result of examining the work and through an interview, it was found that subject C considered that the problem about the six students that would be divided into three groups with 2 members of each group was recurring permutation. Subject C said that $6!$ was gotten from the number of students (six students) while $3!$ was gotten from the number of groups (3 groups), then $2!$ was gotten from the number of members in each group (2 students). Thus, subject C in solving combination problem used the following calculation:

$$\frac{6!}{3!2!2!} = \frac{6 \cdot 5 \cdot 4}{4} = 20$$

It indicated that subject C in solving combination problem was interfered by permutation problem. Subject C also made a mistake in doing calculation operation: $\frac{6 \cdot 5 \cdot 4}{4} = 20$.

It should be 30. At this stage of solving combination problem, subject C tried to modify the scheme existing in his mind and was forced to match the stimulus given. Thus, subject C in solving the combination problem used accommodation thinking process (Hoopes and Segal, 2010), although he was wrong. Based on the result of examining the student's work and interview, it was found that subject C at the stage of rechecking the result of problem solving did it by matching the result of the work to the question which was asked. Subject C said that there was confusion at the time to understand the first problem so that the checking carried out was just based on matching the answer to the question in the problem. Thus, subject C was used in rechecking the result of problem solving used as assimilation thinking process. Based on the description earlier describe, students' thinking process in solving combination problem could be tabulated as shown in Table 2.

The results of this research was supported by the opinion of Simatwa (2010) who says that assimilation is a cognitive process in which a person integrates perceptions, concepts or new experiences into the scheme or pattern that already exists in his mind. Additionally, it was supported by Hoopes and Segal (2010) who says that a person when implementing assimilation thinking process does not need to change the existing scheme. Therefore, when the students do the thinking process of assimilation, the students do not need to change the existing scheme.

The structure of the problems encountered is in accordance with the thinking scheme owned. The accommodation thinking process done by all subjects at the stage of understanding the problems, planning the problem and implementing problem solving is supported by Reinking and Labbo (2000) who says that accommodation transforms the existing information into the new one.

The process of assimilation and accommodation lasted until there is equilibrium conditions (Fajemidagra, 2015). When the student has obtained a combination of problem solving, but not satisfied with the solution, then that person is still going disequilibrium. These conditions will encourage students to hold a reflection of the answers that have been obtained. Conversely, when

Table 2. The students' thinking process seen based on assimilation and accommodation framework.

| S/N | Ability category of students | Troubleshooting steps | | | |
|-----|------------------------------|---------------------------|---------------------|--------------------------------|--------------|
| | | Understanding the problem | Completion planning | Implementing the solution | Rechecking |
| 1 | High | Accommodation | Accommodation | Assimilation and Accommodation | Assimilation |
| 2 | Medium | Accommodation | Accommodation | Accommodation | Assimilation |
| 3 | Low | Accommodation | Accommodation | Accommodation | Assimilation |

the students have been satisfied with the answer, then the process of thinking has already reached equilibrium conditions (Piaget, 1952)

Conclusion

Students with high ability category in understanding the problem, arranging a problem solving plan and implementing problem solving used accommodation thinking process while on the stage of rechecking the result of problem solving, the students with high ability category used assimilation thinking process. Students in medium ability category, in understanding the problem, planning problem-solving, implementing problem solving, used accommodation thinking. Students with low ability category, in understanding a problem, planning problem solving and implementing problem solving used accommodation thinking process even though subject C was interfered by the concept of permutation while in rechecking problem solving stage the student with low ability category used assimilation thinking process although he was wrong.

Conflict of Interests

The authors have not declared any conflicts of interests.

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