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A research on future mathematics teachers' instructional explanations: The case of Algebra

Mustafa Guler* and Derya Celik

Department of Elementary School Education, Fatih Faculty of Education, Karadeniz Technical University, Turkey.

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In this study, explanations of future mathematics teachers about algebra were analysed according to the levels of understanding used by Kinach (2002). The participants for the study were 101 teacher candidates attending the final semester of a teacher training program. For data collection, a form containing four scenario-type items were administered to the participants. In addition, interviews were carried out on six participants selected through purposeful sampling technique. The findings of the study showed that the participants mostly provided content-level explanations based on procedural understanding. The second frequent type of explanation was at the concept level. The problem solving-epistemic level explanations were found to be used less. At the level of content, the explanations were mostly related to rules while at the level of concept, the explanations were based on inductive reasoning.

Key words: Future mathematics teachers, pedagogical content knowledge, instructional explanations, levels of understanding.

INTRODUCTION

One of the desired characteristics of educational programs is that they should reflect changes and innovations. It is one of the reasons for the ongoing revisions in educational programs. On the other hand, teachers are the key to the success of these programs. It is certain that teachers shape their teaching practice based on their professional knowledge.

Nowadays, the assumption of "who knows teaches well" is not valid anymore and instead, it is argued that teachers should also have the knowledge of teaching (Baki, 2012; Ball et al., 2008; Shulman, 1986). Although the knowledge of teaching has been defined in different ways, there is an agreement over somebasic principles (Grossman, 1990; Hill et al., 2005; Ma, 1999). In the context of mathematics teaching, the knowledge of teaching includes the knowledge of processes underlying mathematical operations, awareness of the needs of learners and the knowledge of pedagogical issues such as strategies, teaching methods and techniques (Baki, 2012).

It is certain that the knowledge of teaching is an essential keystone for the teaching profession. On the other hand, the definition of this knowledge base is necessary to identify strategies, methods and techniques while training future teachers. Besides, it gives us an opportunity to focus on specificbases to evaluate

*Corresponding author. E-mail: mustafaguler@ktu.edu.tr.

Table 1. Definitions of understanding levels.

Level	Definition
Content level	Knowing related terms in the subject-matter, using algoritms properly and having procedural skills
Concept level	Having information and experience about concepts and ideas
Problem solving level	Using general or specific strategies and heuristic schemes
Epistemic level	Using proper verification methods
Inquiry level	Searching for new information

possessed knowledge types ofteachers or candidates. Therefore, we may make inferences on the quality of in-service or pre-service teacher training activities. There are various national and international projects to evaluate the knowledge of teaching such as Teacher Education and Development Study in Mathematics (TEDS-M) and Mathematics Teaching in the 21st Century (MT21).

Within the scope of such large-scaled projects involving hundreds of teachers or teacher candidates, mathematics teaching knowledge which captures content knowledge and pedagogical content knowledge was measured in whole subject areas (combination of numbers, algebra, geometry and data). Another recent development was to evaluate the knowledge of teaching in a specific learning domain (Doerr, 2004; Ferrini-Mundy et al., 2003; Li, 2007). This tendency is quite explicit in some rationales. General frameworks give researchers a snapshot of the general picture, however, the picture is seen to always be low resolution and weak on reflecting details. In this study,future mathematics teachers' knowledge of teaching was analysed in the context of algebra, which is one of the significant parts of mathematics.

THEORETICAL FRAMEWORK

In the study, the framework for levels of understanding by Perkins and Simmons (1988) was employed. It was developed to classify understandings in a specific discipline and was used by Kinach (2002) to reveal the quality of instructional explanations and the depth of the subject-matter knowledge. This model argues that in each discipline, understanding is composed of five different levels: content level, concept level, problem solving level, epistemic level and inquiry level (Table 1).

Kinach (2002) related these levels to instrumental and relational understandings in mathematics developed by Skemp (1978). Kinach (2002) argued that content level equals instrumental understanding while the levels of concept, problem solving and epistemic refer to relational understanding. Kinach (2002) further argued that relational understanding does not include inquiry-level understanding. From this point of view, one can make inferences about the depth of understanding by considering the levels without content level.

Kinach (2002) used this classification to analyze the instructional explanations of classroom and future mathematics teachers about procedures involving integers within the context of subject-matter knowledge. She concluded that the majority of the participants focused on mathematical procedures and had instrumental pedagogical content knowledge. Toluk-Uçar (2011) analyzed the subject-matter knowledge and instructional explanations of classroom and teacher candidates about fractions using the levels of understanding. It was found that the majority of both groups had an understanding of mathematics at the instrumental level and that their explanation was mostly at the same level. Baki (2013) analysed the content knowledge and instructional explanations of primary teacher candidates about division in natural numbers. The study concluded that the majority of the participants did division incorrectly and that the division-algorithm related explanation of those participants who did division correctly was insufficient. Less than half of the participants provided proper instructional explanations in which division algorithm was properly given. All these studies focused on the learning domain of numbers.

This study deals with the future mathematics teachers' understanding about algebra based on the *levels of understanding*. In the study, instructional explanation is analyzed using three levels, namely;

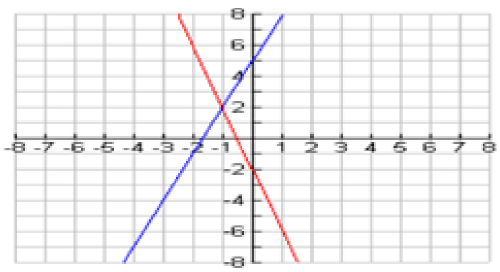
1. Content level,
2. Concept level and
3. Problem solving -epistemic levels.

Although the levels of concept, problem solving and epistemic are all related to relational understanding, the differences between these levels are significant in terms of instructional explanations. Therefore, the levels of problem solving and epistemic are regarded as independent from the level of concept. Given that the participants were not expected to provide explanations about teaching practice, the level of inquiry was excluded.

Aim of the study

The aim of this study is to analyse the quality of

Table 2. Items used in the data collection form.

Items	Variable
1.	While studying exponential numbers you recognized that some students wrote the value of 0 for 2^0. How would you explain them that the correct equation; $2^0 = 1$?
2.	Reyhan teacher wrote the following equation on the blackboard and asked students to solve it. **-x<7** Kübra used -1 to divide the both parts of this equation and found the solution as x>-7. Another student asked that why the equation changes its direction when a negative number is used. What would be your answer to this question?
3.	In some textbooks, coordinate system is used to solve equations. For instance, two linear equations are given on coordinate system. Here {-1} is the apse of the intersection point of the graphics of y= 3 x + 5 and y= -4 x – 2 and is the solution for the equation of 3 x + 5 = -4 x – 2.  Yilmaz thinks that this method works only when there is unknown variable x on both sides and doesn't work when there is a constant on one side. (for instance, this method is invalid for 2x + 7 = 9) Is he right? What would be your answer to Yilmaz? (Adapted from Li, 2007)
4.	Melike teacher provides the following equation and wanted her students to solve it. 2x - 4y = 8 -x + 2y = -4 One of the students solve it as follows: 2x – 4y = 8 + 2(-x + 2y) = 2.(-4) 0 = 0 so the solution is IR. Is this solution correct? If it is not, what would you do to avoid your students such incorrect solutions?

instructional explanations given by mathematics student teachers on algebra.

METHODOLOGY

Participants

The participants of the study were 101 student teachers attending the fourth grade (final semester) of a teacher training program. Specifically, they were attending middle school mathematics education division of this program.

Data collection

The data of the study were collected through a test in which four open-ended scenario-type items were asked(MoNE, 2013). In developing scenarios, the learning domain of algebra at the level of middle school[1]educationwas taken into consideration. Therefore, scenarios concerning basic algebra patterns, equation and inequality) were developed by considering studies in the literature (e.g. Grossman, 1996; Li, 2007).The items were reviewed and answered by five PhD students in mathematics education. Based on these reviews, the items were reorganized and presented (asked) to 30 student teachers who were independent from the participants of the main study. Following the pilot study, the items were finalized as shown in Table 2.

In each item, the participants were given a problem-solving situation and asked how they could explain it to their students (Table 2). The goal was to reveal their potential explanations in case they are faced with the given situation in a real classroom setting and to explore the quality of these instructional explanations.The other data collection tool was interviews; six student teachers were selected to represent different types of level and were asked to explain their solutions to the given items.

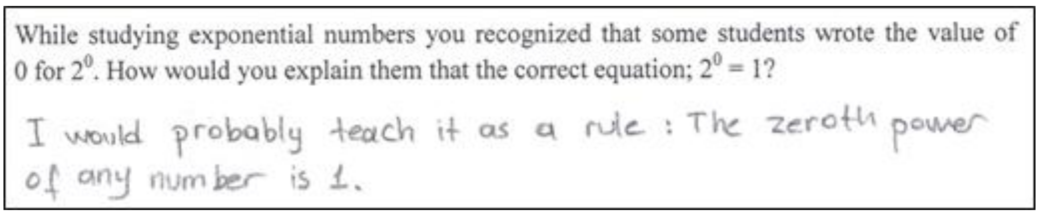
Data analysis

The data were analyzed using the theoretical framework employed by Kinach (2002). As stated earlier, the quality of the answers given by the participants was analyzed in terms of the levels of content, concept and problem solving-epistemic. The level of content refers

[1] In some education systems, middle school level is named lower-secondary or classified as a part of primary education.

Table 3. Distribution of explanations based on the levels of understanding.

Levels	Content level		Concept level		Problem-solving and epistemic levels		Either incorrect or no response	
	Frequency (f)	Percentage (%)	Frequency (f)	Percentage (%)	Frequency (f)	Percentage (%)	Frequency (f)	Percentage (%)
1 item	44	43	27	27	20	20	10	10
2 item	27	27	40	39	16	16	18	18
3 item	40	39	21	21	16	16	24	24
4 item	10	10	7	7	4	4	80	79


While studying exponential numbers you recognized that some students wrote the value of 0 for 2^0. How would you explain them that the correct equation; $2^0 = 1$?

I would probably teach it as a rule : The zeroth power of any number is 1.

Figure 1. Answer of $P_{33}$to the first item

to the skills of the participants on rules, algorithm and procedural skills and covers explanations without any reasoning. Such explanations are regarded as the indication of content level understandings of the participants. Their explanations about conceptual definitions and basic characteristics of concepts were coded as concept level understandings. The problem solving-epistemic level understandings were related to the different representations of concepts, relations and reasoning.

Coding was separately undertaken by both authors. For coding which lacked mutual agreement, additional reviews were made until an agreement was reached. The data obtained from interviews were used to support the quantitative findings and to exemplify relevant cases. “Here, the interviewees were coded as *Px* in reference to the study ethical considerations.” sentence via “The participants were coded as "*Px*" and the researcher "*R*" in reference to the study ethical considerations.”The data presented in the study, both students’ written and oral responses, were translated from Turkish into English, while maintaining the essence of their meanings.

RESULTS

Table 3 indicates the distribution of participants’ explanations based on the levels of understanding used by Kinach (2002). Table 3 showsthat the explanations providedbytheparticipantstothefirst and third scenarios are similar. However, the rate of incorrect answers or no responses to the third scenario is found to be higher than for the first scenario (25 and 10%, respectively). For both scenarios, the most frequent level of understanding is content level, followed by the levels of concept and problem solving-epistemic. With regard to the second scenario, the participants provided mostly concept level explanations (39%), followed by the levels of content and problem solving-epistemic. Concerning the last item, majority of the participants (79%) incorrectly thought that the solution of the problem was right or provided no response. Answers to each item are discussed in detail as follows:

First item

As stated earlier, the explanations given to the first item were mostly at the level of content. It was found that although the participants knew the equation of $2^0 = 1$, they could not provide deeper explanations for it (Figure 1).

Figure 1 shows that the participant provided an explanation that the equation should be taught as a rule. The following statements support this explanation:

P_{34}: …If my students have some incorrect views about this topic, I will tell them that in mathematics some points are made up of acceptance such as the zeroth power of a number equals zero.
P_{65}: …Up until now, I have not questioned why the zeroth power of any number is one. It was just taught to us like that…

Explanations regarding this item which were at the level of concept are generally based on inductive reasoning (for instance, Figure 2).Figure 2 shows that the participants looked for a pattern for positive integer exponents of 2 and reached a generalization based on this pattern. The explanations based on the levels of

While studying exponential numbers you recognized that some students wrote the value of 0 for 2^0. How would you explain them that the correct equation; $2^0 = 1$?

$2^3 = 8$
Divide by 2
$2^2 = 4$
Divide by 2
$2^1 = 2$
Divide by 2
$2^0 = 1$

Given that they already know $2^3 = 8$ I would tell them to use the method I wrote, division by 2.

Figure 2. Answer of $P_{56}$to the first item.

Reyhan teacher wrote the following equation on the blackboard and asked students to solve it.

-x<7

Kübra used -1 to divide the both parts of this equation and found the solution as x>-7. Another student asked that why the equation changes its direction when a negative number is used. What would be your answer to this question?

2 is less than 3. We can write it as $2 < 3$. If we divide each of them by -1, $\frac{2}{-1} = -2$ and $\frac{3}{-1} = -3$

Here, $-3 < -2$ or $-2 > -3$. In negative numbers, the value is getting smaller and smaller the farther they are from zero. For this reason, if we divide it by -1, inequality should be inversed.

Figure 3. Answer of $P_{14}$to the second item.

problem solving-epistemic were about algebraic characteristics of these numbers and the participants used these characteristics to account for the correctness of the equation. One fifth of the participants solved the item as follows:

$$2^{a-a} = \frac{2^a}{2^a} = \frac{1}{1} = 1$$

Second item

Explanations of the participants on this item were mostly at the level of concept. However, explanations at the level of problem and epistemic were found to be less. Concept-level explanations used the basic characteristics about the order of numbers and compared numbers based on this characteristic.

As can be seen in Figure 3,two numbers were compared in terms of order and this feature was used to support the correctness of the claim. At this level, student's written response evokes specific heuristic strategy while other student responses consisted of similar answers.For instance, elements chosen from a specific set of numbers and explanations rely on comparing their greatness. The following excerpt is from the interview with a participant who provided a problem solving-epistemic level explanation to the item:

P_5: If it was explained through examples, it would be limited. So I taught it as an equation.
R: How?
P_5: Like in equations in an inequality, we may add a number to both sides. For instance, I added -7 to each side. This resulted to –x-7 <0 . I added x to each side and had -7<x which means x>-7.

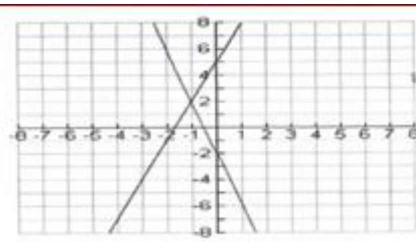
In the earlier mentioned example, the student teacher based his argument on inequality axioms and tried to prove the correctness of the claim. On the other hand, the number of incorrect or no response to this item was

Kübra used -1 to divide the both parts of this equation and found the solution as x>-7. Another student asked that why the equation changes its direction when a negative number is used. What would be your answer to this question?

I would tell them that, at the end of a war between negative and positive numbers (this war can be thought as division or multiplication) negative numbers are always the winners. Thus I would make it easier to remember.

Figure 4. Answer of P_{94} to the second item

In some textbooks, coordinate system is used to solve equations. For instance, two linear equations are given on coordinate system. Here $\{-1\}$ is the apse of the intersection point of the graphics of $y = 3x + 5$ and $y = -4x - 2$ and is the solution for the equation of $3x + 5 = -4x - 2$.



Yilmaz thinks that this method works only when there is unknown variable x on both sides and doesn't work when there is a constant on one side. (for instance, this method is invalid for $2x + 7 = 9$) Is he right? What would be your answer to Yilmaz?

The view of Yilmaz is incorrect. Because using proper algebraic procedures, a constant in one side can be transferred to the other side and it gives us a conventional equation. I would teach it to the student through giving the solution and through drawings.

Figure 5. Answer of P_{32} to the third item.

higher. Such incorrect explanations mostly included improper analogies. Figure 4 shows an example of incorrect explanation for the item. As seen in Figure 4, the participant used an analogy to explain a mathematical statement. However, the explanation has nothing to do with the mathematical content.

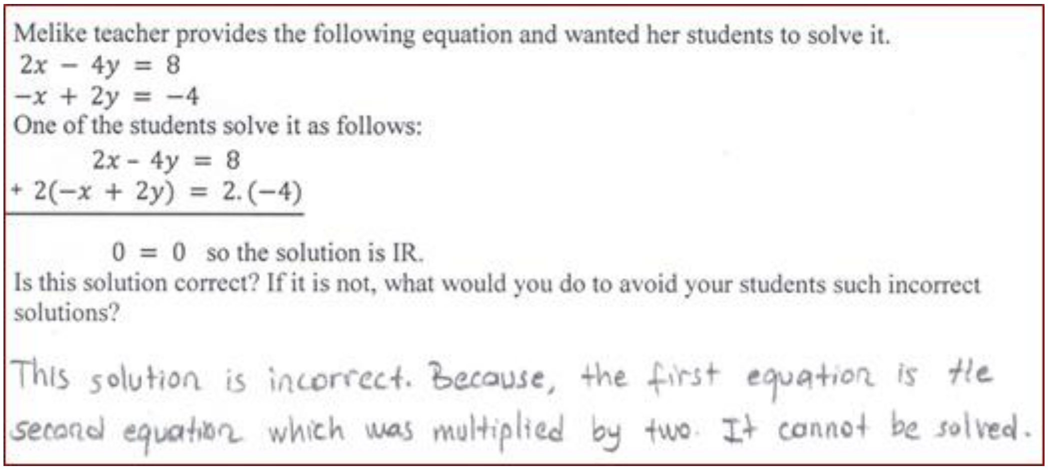
Third item

In the third scenario, a solution was presented by using graphs on a coordinate system to solve an equation. Afterwards, it is asked whether the solution works with another equation or not. The explanations for this item were generally at the level of content (39%). Such explanations mostly argued that the view of the student was incorrect. However, no adequate explanation was given with regard to why it was incorrect. In general,the participants ignored the graphical solution given in the figure and employed conventional equations. They reported that the equations 2x+7=9 and 3x+5=-4x-2 have similar algebraic solutions. Figure 5 gives an example of the explanations for the item.

Figure 5 shows that ST_{32} provided an explanation based on an algebraic solution of the problem. Nearly one-fourth of the participants (24%) gave incorrect or no response to this item. They incorrectly claimed that the view of the student was correct and that the proposed geometric approach was improper for the solution of the item. The following statement shows such incorrect explanations.

R: You stated that the view of the student was correct. Why?

P_{65}: In the first equalities there are two equations. But



Melike teacher provides the following equation and wanted her students to solve it.

$2x - 4y = 8$
$-x + 2y = -4$

One of the students solve it as follows:

$$\begin{array}{r} 2x - 4y = 8 \\ + \; 2(-x + 2y) = 2.(-4) \\ \hline 0 = 0 \end{array}$$

so the solution is IR.

Is this solution correct? If it is not, what would you do to avoid your students such incorrect solutions?

This solution is incorrect. Because, the first equation is the second equation which was multiplied by two. It cannot be solved.

Figure 6. Answer of $P_{58}$to the fourth item.

thereare no two equations here.
R: How?
P_{65}: It is 2x+7=9. It is already an equation. But here, it refers to 3x+5=-4x-2 and there are two unknowns.

ST_{65} had a similar view with the student in the scenario. His argument "*In the first equalities there are two equations. But there are no two equations here"* actually means that in the first statement, $y = 3x + 5$ and $y = -4x - 2$ are two equations which can be drawn on a coordinate system independent of each other. However, the participant thought that the second situation equation $2x + 7 = 9$ can be represented via only one linear line. This explanation implies that the graphical solution cannot be used. This shows that the knowledge of the participants about horizontal and vertical equations is limited. Concept-level explanations included different observations. For instance, some participants stated that the intersection point for the line $9 = 2x + 7$ and $x = 1$ line gave the value of $y = 9$, then the solution would be $x = 1$. Although this solution is correct, it cannot fully account for the situation and therefore, it was regarded as a concept-level explanation. On the other hand, the problem solving and epistemic-level explanations argued that the apse of the intersection point for the lines would be the solution. This explanation fully accounts for the case at hand. 16% of the participants provided this explanation.

Fourth item

The last item which inquires the solution of an equation system with two variables was the most difficult problem for the participants. Majority (80%) of the participants provided either incorrect or no response for this item. It suggests that the participants did not have the necessaryconceptual background to understand the topic at hand. In other words, lack of content knowledge prevented participants from making correct instructional explanations. All explanations in which real numbers were given as the solution for the item were regarded as incorrect. The other type of incorrect explanations was that the view of the student was incorrect and that this problem cannot be solved (Figure 6).

The content-level explanations (10%) recognized that the lines given overlapped, but were not sufficient. The rate of concept-level explanations was lower (7%). Such explanations argued that all ordered double equation systems were not the solution. The rate of the problem solving and epistemic level explanations was also lower (4%). Such explanations included the assumption that in R^2, there were infinite number of solutions and that these solutions were related to the line represented by equations. The following is an example from the interview with a participant who provided a problem solving-epistemic level explanation for the item:

R: You stated that the solution was incorrect. Why?
P_{44}: Because if we use 0 for x and y, it is incorrect.
R: Then what is the correct answer?
P_{44}: For instance, x= 6 and y=1. Or x=0 and y=-2.
R: From this, how many solutions can we find?
P_{44}: We may find infinite solutions.
R: Then why is the solution incorrect?
P_{44}: The infinite solutions do not require that x and y should be real numbers.
R: Why?
P_{44}:Because there are infinite solutions based on the parameters. However, the parameters do not require that x and y should each take real value.

As earlier mentioned ST_{44} identified the student's incorrect answer and gave an example. The participant also provided an adequate explanation forwhy the solution was incorrect by indicating that there were infinite solutions to the problem.

DISCUSSION AND CONCLUSIONS

The current study focused on future mathematics teachers' explanations of the basic concepts in algebra and investigated their quality. For this aim, the levels of understanding used by Kinach (2002) were employed. It was found that the participants mostly provided content-level explanations. The level of problem solving-epistemic was less used in explanations in contrast to the levels of content and concept. Questions that were about the conceptual basis of equations (see 3nd and 4th) were found to be more difficult for the participants to solve. Although problems involving equations can be solved, it becomes harder when the conceptual basis is questioned. Previous studies also indicated that teacher candidates failed to provide the conceptual basis for problem-solving process in which equations were employed (Ferrini-Mundy et al., 2003, 2005; Li, 2007).

Within the scope of this study, some fundamental algebra concepts were examined in each item. By virtue of scenario type questions, mathematical background of the concepts and their properties were used to challenge future teachers' pedagogical content knowledge. Although we directed cases to teacher candidates taken from middle school mathematics education program, many of them had difficulties on basic concepts partaking curriculum. In addition, these scenarios are those which can be frequently seen and used in daily life. The participants mostly provided content-level explanations. Similar findings were found in previous studies by Ball (1990), Bütün (2005), Ma (1999) and Toluk-Uçar (2011). The content-level explanations reflect a procedure-based perspective and emphasize conceptual understanding. The other frequent level of understanding was concept. However, the concept-level explanations given by the participants were not sufficient. Although teacher training programs try to improve problem solving and epistemic level understandings, the findings showed that the number of such explanations was less.

Although, teaching knowledge was limited to PCK, the responses provided were to make inferences about participants' content knowledge. It was mostly indicated that content knowledge is an essential base of mathematics teaching knowledge (Ball et al., 2008; Even, 1993). However, the results showed that participants' lack of content knowledge affected their knowledge of teaching and the explanations they made. For instance, those who had limited knowledge on exponential numbers mostly argued on teaching the zeroth power of a number by memorizing instead of unfolding its mathematical underpinnings.

As stated earlier in this study, future mathematics teachers' explanations about the basic concepts in algebra were analyzed. Their explanations on other topics (such as probability and statistics, geometry, numbers) can be analyzed in relation to different topics, including knowledge of learners and knowledge of teaching. Such studies are crucial in educating qualified future teachers due to their informative role for teacher training programs.

Educational Implications

Pedagogical content knowledge is a special knowledge type which separates a mathematician from a mathematics educator. In other respects, it "goes beyond the knowledge of subject matter per se to the dimension of subject matter for teaching" (Shulman,1986). However, the results show that teacher candidates had difficulties in conceptualizing even basic mathematical statements such as 2^0 and generating explanations about teaching it. The question is not only "What should we teach future teachers?" but also considering the question of "How should we teach to teach mathematical concepts?" Therefore, it is very crucial for teacher training programs to educate future mathematics teachers to acquire knowledge and experience about concepts in school mathematics. At this juncture, such education can be systematically provided in some courses such as special teaching methods or school practice implementations so as to include fundamental mathematical concepts.

Conflict of Interests

The authors have not declared any conflict of interests.

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