academicJournals

Vol. 10(3), pp. 338-350, 10 February, 2015
DOI:10.5897/ERR2014.2017
Article Number: 49FB50F50226
ISSN 1990-3839

http://www.academicjournals.org/ERR

Educational Research and Reviews

Full Length Research Paper

Students' perceptions and development of conceptual understanding regarding trigonometry and trigonometric function

Omer Faruk CETİN

Erzincan University Education Faculty, Turkey.

Received 25 November, 2014; Accepted 29 January, 2015

This study aims to analyse university level mathematics education students' perceptions on conceptual understanding of trigonometry and trigonometric functions and their content development of these concepts. A case study was conducted with 90 freshman students of Elementary Mathematics Department. The data were gathered via a scale; they included ten questions related to perceptions of concept knowledge and a knowledge test with five open ended questions related to the mathematical content that students use. Descriptive and content analyses were applied within the frame of the research goal. Findings showed that though students had a high level of perception about the fundamental concepts of trigonometry and trigonometric functions, specifically angle-angle measurement and arc-arc measurement, they were not successful in understanding their conceptual development. But, they were aware of this issue. In addition, though the students had the visual images of these concepts, their mathematical content usage in the development of conceptual understanding of these concepts were far from the original issues of content development of these concepts. Based on the findings, some suggestions were given about angle, arc and their measurements to lessen the problems hindering the teaching of trigonometry and trigonometric functions in Mathematics education.

Key words: Angle measurement, arc measurement, conceptual understanding, content development.

INTRODUCTION

This study came after an eighth grade student's question in a mathematics course, "Teacher, you say the sine of an angle equals to division of the length of an opposite leg to length of the hypotenuse and these lengths cannot be negative. However, you also say that the sine of a 270 degree angle is "-1". Isn't there a contradiction here?" As Mathematics is an international language, it is very natural to come across similar problems in teaching mathematics across nations. According to the relevant literature (Aydın, 1998; Brown, 2005; Boyacıoğlu et al., 1996; Çetin, 2011, Durmuş, 2004; Fi, 2003; Kültür et al., 2008; Tatar et al., 2008; Thompson et al., 2007; Weber, 2005), problems in teaching trigonometry are the relationships between angles and sides in a triangle and teaching trigonometric functions, which is all about defining the measurement of a directional angle to real

E-mail: fcetin80@hotmail.com

number set in high schools and universities.

Studies on these problems show that the problems are derived from a number of factors such as abstractness of topics (Durmuş, 2004), lack of students' motivation (Durmuş, 2004), students' misunderstanding of fundamental concepts (Güntekin and Akgün, 2011; Steckroth, 2007), and students' failure in connecting the relationships between concepts (Çetin and Dane 2004; Thompson, 2008). From these problems, basic concepts stated in some studies which are related to students' misunderstanding of fundamental concepts are as follows: the concept of angle defined as an intersection of two rays having common beginning point (Çetin, 2011; Mithcelmore, 1998; Mitchelmore and White, 2000), angle measurement (Clements and Burns, 2000; Moore, 2009), concept of radian (Akkoç, 2008; Fi, 2003; Orhun, 2004; Topçu et al., 2006; Steckroth, 2007); arc concept in circles and the concept of arc measurement (Moore, 2012). Students' prior learning experiences about trigonometry and associations with new ones can make them to think that trigonometry is a difficult course to understand (Thompson, 2008). Pre-learning experience is a prerequisite for meta-learning experiences, but this prerequisite sometimes cannot be met properly by all students. An instance is the issue of trigonometry and trigonometric function. While having an idea of what an angle means is a necessity for defining trigonometric functions, knowing the concept of angle well does not mean that a student who is good at that topic will define trigonometric functions easily. Connected with these issues, it was found in the study of Çetin and Dane (2004) that university level students defined the geometric concepts which are connected to each other such as angle, angle in circles and angle measurement as if they are independent topics from each other. Then, letting students know these concepts by developing a conceptual understanding of trigonometric functions appropriately to the content constructs seems the best way in teaching them (Moore, 2013). In addition, strong relationships between these concepts should be considered in the conceptual development process (Moore, 2013; Thompson, 2008). These relationships can be summarised as follows.

Trigonometry and trigonometric functions and their relationships with fundamental conceptions

Figure 1, prepared by the author with the help of definitions of concepts, eases the definition process of the relationships among concepts of trigonometry and trigonometric functions and angle. The figure gives details about teaching order of these concepts for a concrete conceptual development.

The figure shows that the angle should be measured in radians in order to define trigonometric functions. Therefore, we need to measure the arc in terms of radian in a circle. Real number axis should be used for the arc measurement in radian. The arc and arc measurement in circles concepts listed among the aforementioned fundamental concepts is related to the concept of angle measurement and is also a prerequisite for the measurement. There are very few studies in the literature on arc, arc measurement angle and angle measurement concepts (Moore, 2013). This fact might be one of the reasons why solutions to the problems in teaching trigonometry and trigonometric functions have been delayed. In trigonometry, any concept is strongly related to another one taught before and after the present concept and there is a natural sequential order in teaching these concepts (Altun, 1998; Çetin, 2011; Dikici and İşleyen, 2004). Also there is less emphasis in the literature on the use of real number axis in terms of radian measurement. This study also deals with this issue, with emphasis. University students will definitely and frequently use conceptual development in constructing trigonometry and trigonometric functions in some courses in their curricula. Therefore, determining students' preparedness about these concepts and checking their prior experiences in learning them will be helpful for shaping further teaching activities and creating long lasting solutions to deficiencies in learning. In addition, there are some studies focusing on the fundamental concepts of trigonometry and trigonometric functions and emphasising the importance of the studies dealing with conceptual relationships between these concepts for relevant literature (Kutluca and Baki, 2009; Kültür et al., 2008).

Considering the above issues, this study aims to research freshman elementary mathematics education students' perceptions which they formed previously about the conceptual understanding and development of angle-angle measurement and arc-arc measurement concepts. For the fulfilment of the aim, the following research questions were researched:

1. What do students think about their knowledge levels with regard to fundamental concepts of trigonometry and trigonometric functions?
2. What are the mathematical contents which students use in the development of conceptual understanding of trigonometry and fundamental concepts of trigonometric functions?

METHOD

Research design

This work aims to deal, in detail, with students' conceptual development process with regard to relevant topics. A case study research design used for analyzing an event or issue deeply (McMillan, 2000) was adopted for this work.

Participants

Participants comprised 90 freshman university students enrolled

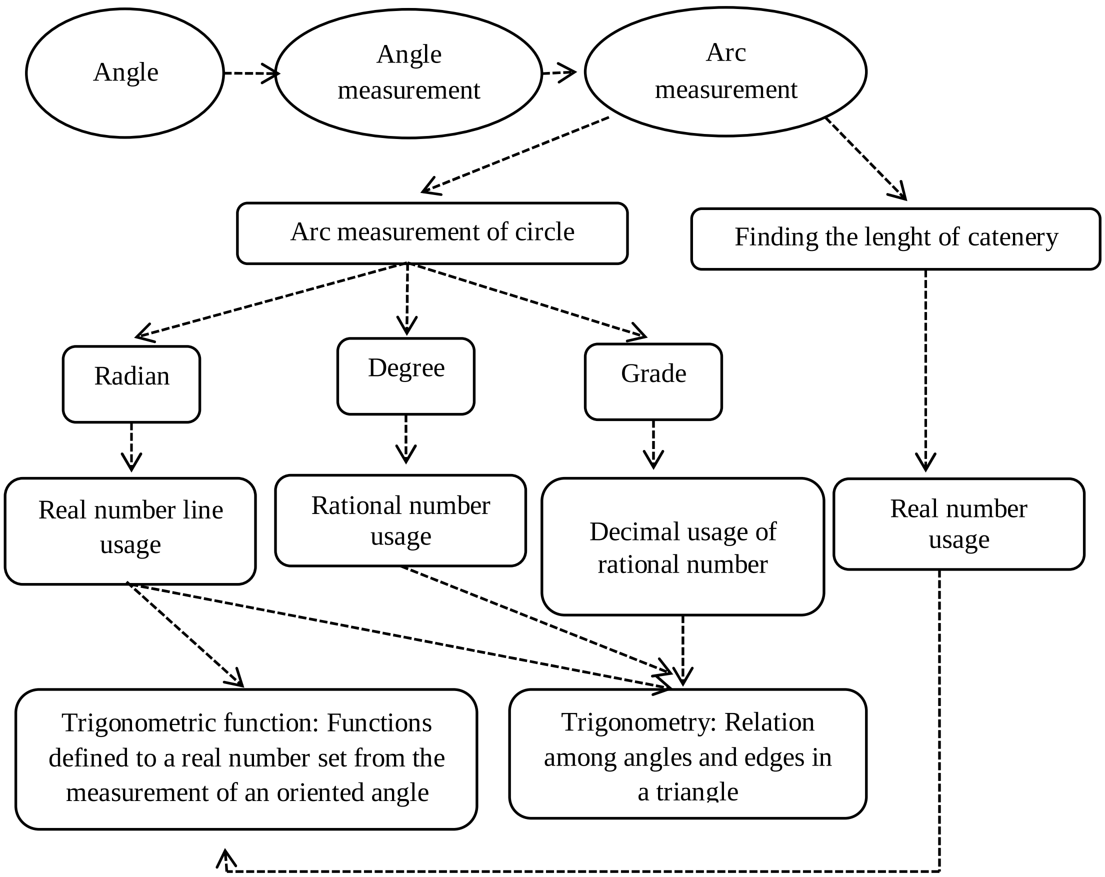

Figure 1. Relationship between the concepts of trigonometry and trigonometric functions.

into Turkish University's Department of Elementary Mathematics Education in the first term of the academic year, 2012-2013. The students did not take any formal course at the department on trigonometry and trigonometric functions as they are first graders and the courses in the curriculum with titles such as Geometry, Analysis I and Analysis II are in the next semesters. As the study is mainly about the students' preparedness on these issues and their prior knowledge, this issue is important for the study. Any demographic details about the participants were not considered for the study. As a qualitative research design was adopted in this study, sample size was not considered and all university students (N: 90) in the related grades of the department participated in the study.

Data collection instruments

The data were collected with two instruments: a Perception of Concept Knowledge Scale (PCKS) and a Concept Knowledge Expression Form (CKEF).

PCKS is a five point Likert scale including ten items about angle-angle measurement and arc-arc measurement concepts. The scale was originally developed by Çetin (2011); some items were removed and new ones were added according to the experts' opinions. The rating of the scale ranged from 1 (Strongly Agree) to 5 (Strongly Disagree). Cronbach alpha of the scale is 0.90. Some of the sample items are as follows: *I know what an angle is, I know what an arc is, I know the difference between angle and arc, I know the relationship between angle and arc, I know what angle measurement means, I can find the measure of a given angle.*

CKEF has five open ended questions about the mathematical content used by the students in the concept development of angle-angle measurement and arc-arc measurement concepts. The questions were prepared by the field experts and the questions are as follows: *Please, define number line and draw it. Please define angle and draw it and then explain how you can measure the angle you drew. Please define an arc and draw it, and then explain how you can measure the arc you drew. Please write the differences and relationships between angle and arc. Please explain why the arc measured counter clockwise is negative*.

Data collection and analysis

PCKS and CKEF were given to the students in a course period and answered within fifty minutes. During data collection processes, the students' influence on one another for answering was lessened, and they were informed about the importance of the study. Then the data were transferred to the computer, and descriptive and content analyses (Yıldırım and Şimşek, 2008) were conducted.

During the analyses of the PCKS, new categories from the Likert range such as precise perception of the conceptual development (Strongly agree and agree), little perception for the conceptual development (Indecisive) and no perception for the conceptual development (Strongly disagree and disagree) were formed and findings were presented in tables with frequencies and percentages. The CKEF was analyzed in two stages. In the first stage, based on the categories, "drawing/not drawing", "defining/not defining", "explaining/not explaining", the data were subjected to

Table 1. Frequencies and percentages of students' perceptions about trigonometry and the fundamental concepts of trigonometric functions.

Perception of Concept Knowledge Scale Items	Precise Perception						Very Little Perception		No Perception					
	Strongly agree		Agree		Total		Indecisive		Disagree		Strongly disagree		Total	
	n	%	n	%	n	%	n	%	n	%	n	%	n	%
1. I can define what a number line means.	38	42	42	47	80	89	8	9	2	2	-	-	2	2
2. I can define what an angle means.	28	31	47	52	75	83	13	14	1	1	1	1	2	2
3. I know what an angle measurement means.	19	21	38	42	57	63	27	30	2	2	4	5	6	7
4. I can find the measure of a given angle.	21	23	33	37	54	60	25	28	7	8	4	5	11	13
5. I can define an arc.	11	12	29	32	40	44	31	35	13	14	6	7	19	21
6. I know what an arc measurement means.	10	11	12	13	22	24	39	44	21	23	8	9	29	32
7. I can find the measure of a given arc.	10	11	28	31	38	42	29	32	14	16	9	10	23	26
8. I know the difference between angle and arc.	12	13	11	12	23	25	32	36	24	27	11	12	35	39
9. I know the relationship between angle and arc.	10	11	14	15	24	26	33	37	22	25	11	12	33	37
10. I know why the measure of an arc measured counter-clockwise is negative.	12	13	15	17	27	30	26	29	15	17	22	24	37	41

descriptive analysis. In the second stage, the relevant common points of the students' concept, *who can make a definition, drawing and explanation* were determined with the content analysis. Therefore, a code list was prepared and expert's ideas were taken. Later, answers were read again based on the code list and certain codes were brought together and lastly students' excerpts were determined for the presentation of findings. In presenting students' excerpts, a number for each student' paper was given and coded as S1, S2, S3, etc. Lastly, the students' codes and their categories with their frequencies and percentages were presented in tables.

FINDINGS

Research findings are presented in tables.

Findings of the first research question

Findings gathered from the PCKS on students' knowledge levels with regard to concepts of trigonometry and fundamental trigonometric functions are presented in Table 1 and Figure 2.

According to Table 1, the highest and lowest percentages of the items of each main category are as follows: The highest precise perception percentage (89) in the development of conceptual understanding of "number line" and the lowest percentage (24) in "arc measurement", the highest very little perception percentage (44) in the development of conceptual understanding of knowing an arc measurement and the lowest percentage (9) in defining a number line, the highest no perception percentage (41) in the development of conceptual understanding of knowing why an arc measured counter-clockwise is negative and the lowest percentages (2) in defining a number line and an angle. In addition, Figure 2 shows the students' perceptions about the relationships between concepts such as arc-arc measurement and angle-arc measurement as well as their construction of the single concepts.

In the figure, the relationships between the scale items with regard to categories, precise, little and no perception are given. Though the percentages were close to each other in each category after the item number 5 to 10, there were big differences in the percentages of items 1 to 4.

Findings of the second research question

Findings of the second research question about

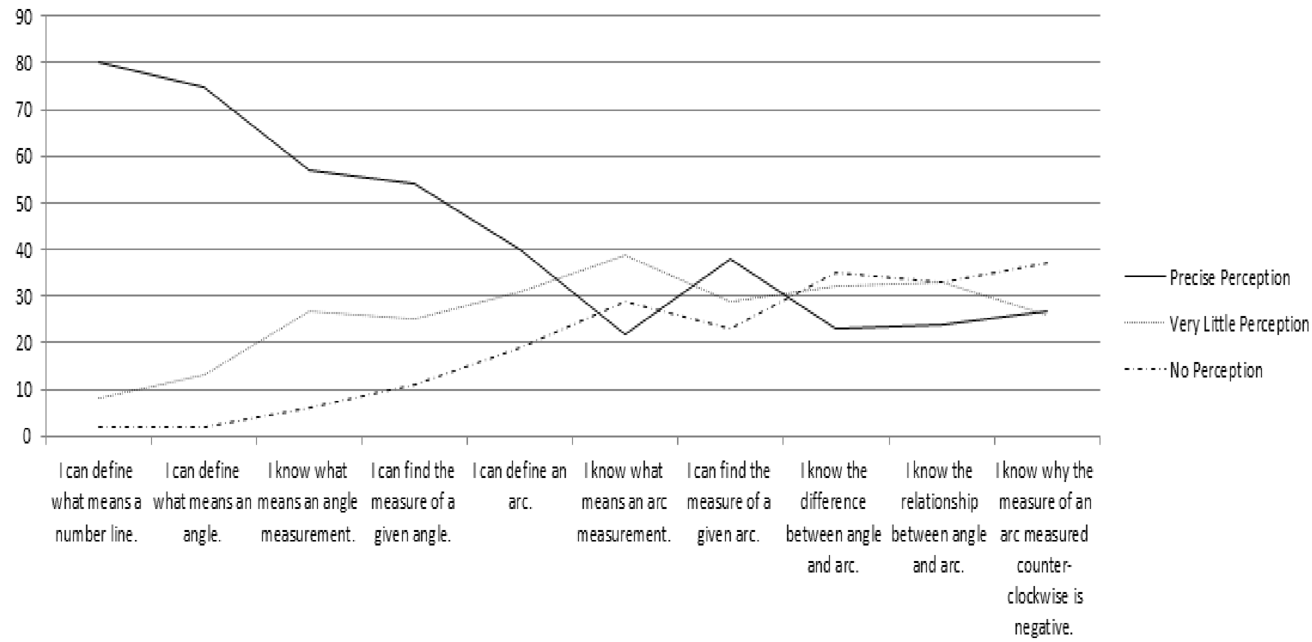

Figure 2. Relationship between percentages of students' perceptions in each category.

Table 2. Frequencies and percentages of the question on defining a number line and the common mathematical content.

PCKS Item: I can define what a number line means.	**Precise perception**		**Very Little perception**		**No perception**		**Total**	
	80		**8**		**2**		**N:90**	
	n	%	n	%	n	%	n	%
CKEF Item: Please, define number line and draw it.								
Those who did not draw.	1	1	1	13	-	-	2	2
Those who drew.	79	99	7	87	2	100	88	98
Total	80	100	8	100	2	100	90	100
Those who did not define.	24	30	4	50	1	50	29	32
Those who defined.	56	70	4	50	1	50	61	68
Total	80	100	8	100	2	100	90	100
The mathematical content used in the development of conceptual understanding of number line								
Has the words such as "real number, line, ordering/taking part/placement"	20	36	1	25	-	-	21	34
Has the words such as "negative infinity, positive infinity, starting point"	15	27	-	-	-	-	15	25
Has the expression "Line with numbers on it"	10	18	-	-	-		10	16
Other	11	19	3	75	1	100	15	25
Total	56	100	4	100	1	100	61	100

mathematical contents which students use in the development of conceptual understanding of trigonometry and fundamental concepts of trigonometric functions were presented according to the open-ended questions of CKEF.

Findings of the questions on number line are given in Table 2.

Table 2 shows that the number line was defined by 61 (68%) students. Among these definitions, 15 (25%) of them contained expressions which were not about the number line. When the 46 answers containing expressions about the definition of the real number line were examined, it was observed that "words such as "real number, line, ordering/taking part/placement" were used the most 21(34%) in these definitions whereas the least used expression was "line with numbers on it" 16(16%).

Table 3. Frequencies and percentages for the answers to the question on the concept of angle and common mathematical contents.

PCKS Items: I can define an angle.	**Precise Perception**		**Little Perception**		**No Perception**		**Total**	
	75		**13**		**2**		**90**	
	n	**%**	**n**	**%**	**n**	**%**	**n**	**%**
CKEF Item: Define an angle and draw one.								
Those who did not make a drawing	12	16	2	15	2	100	16	18
Those who made a drawing	63	84	11	85	-	-	74	82
Total	75	100	13	100	2	100	90	100
Those who did not make a definition	25	33	5	38	2	100	32	36
Those who made a definition	50	67	8	62	-	-	58	64
Total	75	100	13	100	2	100	90	100
The common mathematical content that students use in constructing the angle concept								
Measure	15	30	2	25	-	-	17	29
Measurement unit	4	8	-	-	-	-	4	7
Location, region, section, part	11	22	1	12	-	-	12	21
Distance, length	4	8	3	38	-	-	7	12
Degree	4	8	-	-	-	-	4	7
Slope	4	8	-	-	-	-	4	7
Intersection	-	-	2	25	-	-	2	3
Combination of two rays with the same starting point/coincident	3	6	-	-	-	-	3	5
Other	5	10	-	-	-	-	5	9
Total	50	100	8	100	-	-	58	100

The sample quotations taken from participants S19 and S61 who used this expression are as follows:

S19. The bi-directional line with no certain beginning and end is named as a number line.
S61. Number line is a set of linear points.

Statements of S45, S53 and S24 who have used the expressions of “negative infinity, positive infinity, starting point” and “line with numbers on it” are respectively as follows:

S45. It is the line on which numbers are ordered according to their magnitude.
S53. It is the line on which all numbers from + infinity to – infinity are arranged in a certain order
S24. It is a line with numbers on it.

Whereas 29(32%) of the students did not make a number line definition, 88(98%) drew a number line. 56(70%) of the 80 students who had a perception what a number line is made a definition and 79 (99%) also made a drawing.

The findings about the mathematical content which students used in constructing the angle concept are given in Table 3.

As presented in Table 3, 32 (36%) students did not make a definition of an angle and 16 (18%) did not draw. Whereas, 25 (33%) of the 7 students who had a perception about constructing angle concept did not make a definition of an angle; 12 (16%) of them did not make an angle drawing. 8 (62%) of the 13 students who did not have a perception about whether they have constructed the angle concept or not made an angle definition and 11 (85%) of them made a drawing of an angle. 2 students who had a perception on constructing the concept of angle neither made a definition of an angle nor drew one.

Students who had a perception on constructing the angle concept and who did not make a definition of an angle or drew one stated the reason for this using the expressions of, “I know but I cannot define”. The quotations of S63 and S87 are as follows:

S63. I know the concept but I cannot express it.
S87. I know but I cannot define it with my words.

The answers of 5 students who had a perception on constructing the angle concept and who made a definition were not valid. Whereas the number of students who used the “measure” emphasis among the remaining students was highest (29%), the “intersection” emphasis was used the least (3%). The statements of S66, S31, S4, S34, S22, S61 and S49 focusing on measure, measurement unit, “location/region/section/part”, “distance/length”, “degree”, “slope” and “intersection” are as follows:

Table 4. Frequencies and percentages of the answers to the question on the measurement of angle and the mathematical content.

PCKS Item: I can find the measure of a given angle.	**Precise Perception**		**Little Perception**		**No Perception**		**Total**	
	54		**25**		**11**		**90**	
	n	**%**	**n**	**%**	**n**	**%**	**n**	**%**
CKEF Item: How do you measure an angle you draw? Explain								
Those who did not make an explanation about how to find the measure	33	61	17	68	7	64	57	63
Those who made an explanation about how to find the measure	21	39	8	32	4	36	33	37
Total	54	100	25	100	2	100	90	100
The common mathematical content used in structuring the determination of the angle measurement								
Protractor	8	38	2	25	1	25	11	33
Miter	7	33	3	38	1	25	11	33
Compass	4	19	-	-	2	50	6	19
Compass and miter	1	5	-	-	-	-	1	3
Angle meter	-	-	2	12	-	-	2	6
Other	1	5	1	25	-	-	2	6
Total	21	100	8	100	4	100	33	100

S66. It is the measure between two rays.
S31. It is the measurement unit between two infinite lines.
S4. It is the area between two or more lines with coincident starting points.
S34. It is the distance between two rays.
S22. It is the degree that the line makes.
S61. It is the slope of the arc drawn between lines drawn at a point.
S49. It is formed by the intersection of two lines.

Findings about the mathematical content which students used in constructing the angle measure concept are given in Table 4.

Table 4 showed that 57 (63%) students did not make any explanation about how they can find the measure of a given angle. 33 (61%) of the 54 students who had a perception on determining the measure of a given angle did not make any explanation about how they can proceed with finding the measure of the given angle. 8 (32%) of the 25 students who were indecisive about their knowledge of determining the measure of a given angle and 4 (36%) of the 11 students who had a perception of not knowing how to find the measure of a given angle made an explanation. Whereas “Protractor” and “Miter” words were used the most (33%) by students who explained how to determine the measure of a given angle, “angle meter” was the least used (3%) word.

The expressions of S11, S71, S30, S62 and S1 focusing on “Protractor”, “Miter”, “Compass”, “Compass and miter” and “Angle meter” are given below:

S11. We measure by using a protractor.
S71. We measure by using a miter.
S30. We measure by using a compass.
S62. We can determine the measure of an angle by using a compass and miter.
S1. We measure by using an angle meter.

The statements of S73 and S40 were different because they responded to the question “how” instead of “using what” and so their answers are given below:

S73. We measure an angle A by drawing a tangent and connecting the points. We can measure the angles by the method of forming triangles.
S40. I measure from my mind.

Findings about the mathematical content used in constructing the arc concept are presented in Table 5.

Table 5 presented that 29 (32%) of the students made an arc definition and 59(66%) made a drawing of an arc. 21 (53%) of the 40 students who had a perception on constructing the arc concept made a definition of an arc and 10(25%) did not make an arc drawing. 8 (26%) of the 31 students who did not have a perception regarding their construction of the arc concept made an arc definition and 13(42%) made an arc drawing. 2 (11%) of the 19 students who had a perception on not constructing the arc concept made an arc definition and 8(42%) made an arc drawing. 7 arc definitions of the 29 students were not mathematically meaningful. The statement of S61 on this issue is as follows:

S61. The curve used to show the angle is a line.

Whereas the sentence, “it is a part/section of the circle”

Table 5. Frequencies and percentages of the question about defining arch and the common mathematical content.

CKPF Item: I can define an arc.	**Precise Perception**		**Little Perception**		**No Perception**		**Total**	
	40		**31**		**19**		**90**	
	n	**%**	**n**	**%**	**n**	**%**	**n**	**%**
CKEF Item: Define an arc and draw one.								
Those who did not make a drawing	10	25	13	42	8	42	31	34
Those who made a drawing	30	75	18	58	11	58	59	66
Total	40	100	31	100	19	100	90	100
Those who did not make a definition	21	53	23	74	17	89	61	68
Those who made a definition	19	47	8	26	2	11	29	32
Total	40	100	31	100	19	100	90	100
Common mathematical content that students use in constructing the arc concept								
A part/section of the circle	5	35	5	50	-	-	10	34
The distance/length between any two points on the circle	4	29	-	-	-	-	4	14
The length corresponding to an angle on the circle	4	29	3	30	1	20	8	28
Other	1	7	2	20	4	80	7	24
Total	14	100	10	100	5	100	29	100

Table 6. Frequencies and percentages of the question on how to measure and the common mathematical content.

CKPF Item: I can find the measure of a given arc	**Precise Perception**		**Little Perception**		**No Perception**		**Total**	
	38		**29**		**23**		**90**	
	n	**%**	**n**	**%**	**n**	**%**	**n**	**%**
CKEF Item: How do you measure an arc you draw?								
Those who did not make an explanation on how to find it	32	84	25	86	23	100	80	89
Those who made an explanation on how to find it	6	16	4	14	-	-	10	11
Total	38	100	29	100	23	100	90	100
The common mathematical content used in constructing the determination of the measure of an arc								
Miter	2	33	1	25	-	-	3	30
Compass	1	17	-	-	-	-	1	10
By angle measurement	3	50	2	50	-	-	5	50
Other	-	-	1	25	-	-	1	10
Total	6	100	4	100	-	-	10	100

was used the most (34%) among mathematically meaningful arc definitions, the least used sentence (14%) was “It is the distance/length between any two points on the circle”. The quotations of S87, S58 and S1 emphasising “a part/section of the circle”, “the distance/length between any two points on the circle” and “the length corresponding to an angle on the circle” in their arc definitions are as follows:

S87. It is a part taken from the circle.
S58. It is the distance between any two points on the circle.
S1. Arc is the length of the curve corresponding to any angle on unit circle.

Findings about the mathematical content used in constructing the arc measure concept are given in Table 6.

Table 6 indicated that majority of the students (89%) did not make an explanation on how to find the measure of a given arc. 32 (84%) of the 38 students who had a perception on constructing the measuring of an arc did not make any explanation on how to make this measurement. 4 (14%) of the 29 students who did not have a perception about this issue made an explanation. In addition, 23 students having a perception that they did not construct the measurement of an arc did not make any explanation.

Only 10 students made an explanation on how to determine the measure of a given arc and among these,

Table 7. Frequencies and percentages of the question on the differences between angle and arc and the common mathematical content.

CKPF Item: I know the difference between angle and arc.	**Precise Perception**		**Little Perception**		**No Perception**		**Total**	
	23		**32**		**35**		**90**	
	n	%	**n**	%	**n**	%	**n**	%
PCKS Item: What are the differences between angle and arc? Write down.								
Those who did not make an explanation	7	30	23	72	29	83	59	66
Those who made an explanation	16	70	9	28	6	17	31	34
Total	23	100	32	100	35	100	90	100
Common mathematical content used by students to construct the differences between angle-arc								
Angle is measure/degree, whereas arc is length	9	56	2	22	1	17	12	39
Arc is the length corresponding to the angle (on the circle)	2	13	2	22	2	33	6	20
Arc is twice the angle if the angle is not on the centre of the circle	2	13	-	-	2	33	4	13
Angle is expressed on/between lines, whereas arc is expressed as on the circle	-	-	4	44	1	17	5	16
Angle is the combination of two rays, arc is a part of the sets that are equidistant to a fixed point	1	5	-		-	-	1	3
Containing a sentence that has no difference	2	13	1	12	-	-	3	9
Total	16	100	9	100	6	100	31	100

the explanation of the S23 was given below since the student partially responded to the "how" question.

S23. When we think of it on a circle, if the arc divides the circle in two, this arc is 180 degrees. Accordingly, if it is larger than a semi-circle than it is greater than 180 degrees and if it is smaller than a semi-circle than it is smaller than 180 degrees.

Whereas the highest emphasis among the remaining 9 student explanations was on "By measuring the angle" (50%), the least emphasis was on "Miter" (6%). The expressions of S52, S38 and S66 emphasising "Miter", "Compass" and "By measuring the angle" are as follows:

S52. It is measured by using a miter.
S38. It is measured using a compass.
S66. The arc has the same measure with the central angle. The central angle is half the arc.

Findings about the mathematical content used in constructing the angle-arc difference concept are given in Table 7.

Table 7 showed that majority of the students (66%) did not express the difference between angle and arc. 7 (30%) of the 23 students having a perception that they constructed the difference between angle and arc did not say a difference between angle and arc. On the contrary, 9 (28%) of the 32 students who did not have a perception about the construction of the difference between angle and arc made an explanation about the difference between angle and arc. Similarly, 6 (17%) of the 35 students who thought that they did not construct the difference between angle and arc made an explanation about the difference between angle and arc.

3 of the 31 answers to the question of expressing the difference between angle and arc did not express a difference. On this issue, S35 said as follows:

S35. Angle and arc are the same if drawn from the centre of the circle.

In the rest 28 answers, students emphasised "Angle is a measure/degree, whereas arc is length" the most (39%); they emphasised "Angle is the combination of two rays, arc is a part of the sets that are equidistant to a fixed point" the least (3%). Similarly, the expressions of S61, S57, S53, S27 and S10 who respectively emphasised, "Angle is a measure/degree, whereas arc is length", "Arc is the length corresponding to the angle (on the circle)", "Arc is twice the angle if the angle is not on the centre of the circle", "Angle is expressed on/between lines, whereas arc is expressed as on the circle" and "Angle is the combination of two rays, arc is a part of the sets that are equidistant to a fixed point" are given below:

S61. The angle is a measure. The arc is length.
S57. Arc is the length corresponding to the angle on the circle
S53. Arc is twice the angle if the angle is not on the centre of the circle
S27. Angle is between two lines whereas arc is the measure on the circle.
S10. Angle is the combination of two rays. Arc is a part of

Table 8. Frequencies and percentages of the question about the relationships between angle and arc and the common mathematical content.

PCKS Item: I know the relationship between angle and arc.	Precise Perception		Little Perception		No Perception		Total	
	24		33		33		90	
	n	%	n	%	n	%	n	%
CKEF ITEM: What kind of relationships are there between angle and arc?								
Those who did not make an explanation	8	33	22	67	29	88	59	66
Those who made an explanation	16	67	11	33	4	12	31	34
Total	24	100	33	100	33	100	90	100
Common mathematical content used by students to construct the relationship between angle-arc								
Arc is the length/section that is opposite of an angle	3	19	3	27	-	-	6	19
Arc is the length/distance/part in a circle corresponding to the angle	3	19	1	9	-	-	4	13
If angle is at the centre, it equals to the arc and if it is at the edge, it equals to half of the arch	4	25	-	-	1	25	5	16
Other	2	12	1	9	1	25	4	13
Sentence expressing no relationship	4	25	6	55	2	50	12	39
Total	16	100	11	100	4	100	31	100

the sets that are equidistant to a fixed point.

Findings about students' mathematical content used in constructing the angle-arc relationships are given in Table 8.

As it is seen in Table 8, most of the students (66%) did not express any relationship between angle and arc. 8 students (33%) of 24 who had a perception that they constructed the relationship between angle and arc did not present any relationship. However, 11(33%) of the 33 students who did not have any perception regarding their construction of the relationship between angle and arc and 4(12%) of the 33 students who had a perception that they did not construct the relationship between angle gave an explanation.

12 of the 31 related with expressing the relationship between angle and arc were more geared towards giving the differences. 3 of 19 answers did not have a common emphasis. The answers of the S10 whose answer did not express a relationship and that of the S3 whose answer did not contain a common emphasis are as follows:

S10. Angle is formed by the intersection of two rays. Arc is a section of the sets that are equidistant to a fixed point.
S3. Angle is the measure between the intersection point of two lines whereas arc is the circular length between the tips of the lines.

Of the rest 15 answers, students stating the relationship between angle and arc emphasised "Arc is the length/section that the angle faces" the most (19%), whereas the least emphasis was on "Angle is equal to the arc if at the centre, arc is half of the inscribed" (13%). The statements of S51, S2 and S39 focusing on "Arc is the length/section that the angle faces", "Arc is the length/distance/part on the circle" and "Angle is equal to the arc if at the centre, arc is half of the inscribed angle" are as follows:

S51. Arc is the length of the section that the angle faces
S2. Arc is the section of the circle that the angle faces
S39. Arc has the same degrees with the central angle; it is half of the measure of the inscribed angle

Findings on the students' mathematical content which they used in constructing negativity or positivity concept of angle-arc measurement are given in Table 9.

Table 9 indicated that most of the students (69%) did not make an explanation about the reason why the measure of an arc measured counter-clockwise is negative. 13 students (48%) of 27 who had the perception that they had constructed the reason why the measure of an arc measured counter-clockwise is negative did not make an explanation, 14 students (52%) made an explanation. Whereas 31 students (84%) of 37 who had a perception that they did not construct this issue did not make any explanation, 6 (16%) of them made an explanation. 8 students (31%) of 26 who had very little perception on this issue could make an explanation. 12 answers of 28 about the reason of this issue included expressions having dogmatic information; in other words did not have any mathematical reason. On this issue the students S7 and S29 said as follows:

S7. I think this is because counter-clockwise direction is accepted as positive.
S29. I think this is dogmatic information

Among the other 16 responses, the students emphasised

Table 9. Frequencies and percentages of the question about the reason of negativity of the measure of an arc measured counter-clockwise.

PCKS Item: I know why the measure of an arc measured counter-clockwise is negative.	**Precise Perception**		**Little Perception**		**No Perception**		**Total**	
	27		**26**		**37**		**90**	
	n	%	n	%	n	%	n	%
CKEF Item: Why is the measure of an angle measured counter-clockwise is negative?								
Those who did not make an explanation	13	48	18	69	31	84	62	69
Those who made an explanation	14	52	8	31	6	16	28	31
Total	27	100	26	100	37	100	90	100
Common mathematical content used by students in constructing the negativity of the measure of an arc measured counter-clockwise								
Trigonometry, trigonometric function	2	14	1	12	1	17	4	14
Placement of numbers on the number line	2	14	-		-	-	2	7
Coordinate system on the plane	6	43	2	26	-	-	8	29
Slope	1	7	1	12	-	-	2	7
Other (Dogmatic information e.g. rotation direction of the world etc.)	3	22	4	50	5	83	12	43
Total	14	100	8	100	6	100	28	100

"Coordinate system on the plane" the most (29%) and "Placement of numbers on the number line" and "Slope" (7%) the least. The answers of S58, S77, S20 and S73 emphasising "Trigonometry, trigonometric function", "Placement of numbers on the number line", "Coordinate system on the plane" and "Slope" are given below respectively:

S58. This is because of trigonometric functions.
S77. This is because of numbers continuously decrease as we go to the left on the number line
S20. The real direction is the direction on the coordinate system. Clockwise direction is opposite to that. That is why it is negative.
S73. This is because the arc is negative in the counter-clockwise direction if the slope does not yield a positive result.

RESULTS, DISCUSSION AND SUGGESTIONS

In this section, results of the research questions related to perceptions about the conceptual development of concepts and mathematical content used in this process were dealt with together.

According to the results, most of the students had a perception of knowing the concept of angle. This finding is in parallel with the findings of Çetin (2011) which state that the students have a good level of knowing the concept of angle. Also, one sixth of the students did not draw angle and two sixth of the students did not define angle; in other words, they could not express mathematical content that they use in developing a conceptual content in constructing the concept of angle. This issue was clearly seen in students' quotations as, "I know it but I cannot define it". In addition, the finding that only three students from 58 could write mathematical contents used in developing conceptual contents in angle is in parallel with the concept of construction of the concept of angle. Though this finding contradicts the result of Deniz and Erdoğan (2012), that states that "students know the angle concept", it is in accordance with the result of Kültür et al (2008), stating that "students have not truly learned the fundamental concepts of trigonometry". Also the findings indicated that the students had the visual image of the concept of angle; in other words they were at the level of *recognition level stage* (Senemoğlu, 2004), but in reality they must have when they should be at the *abstract level stage.* The common emphases of students who have made a definition of the angle are "Measure", "Measurement unit", "Location, region, section, part", "Distance, length", "Degree", "Slope" and "Intersection". Those students also constructed the concept of angle by using very different mathematical contents than the students having the same visual image about angle.

Another finding of the study was about the concept of arch and almost half percentage of the students had a perception that they developed a conceptual understanding on the arc. This finding is in accordance with Çetin (2011)'s result that the students' perceptions about arc are at a good level. In addition, one third of the students had a drawing about the concept of arch and two third of them did not make a definition about the concept. This means that one third of these students failed to express the mathematical expressions they used to construct the concept of arc. Students stated the reason for this issue by saying "I know but I cannot express it". The construction of almost one third of the students who have expressed the mathematical expressions they used in constructing the concept of arc overlaps with the content structure of the arc concept. The construction of one third of students' mathematical expressions in constructing the concept of arch is consistent with the content structure of the arch. When the number of students who did not make a drawing of an arc and the number of students who did not make a drawing of an angle were considered, it can be said that students had much visual image of the angle than the arc.

In addition, though the number of students who defined angle was almost twice the number of students who defined arc, the number of students who defined arch as appropriate to the content structure of the arc concept was three times of the number of students who defined angle as appropriate to the content structure of the angle. This finding also shows that the number of students who are at the abstract level stage on the arc concept is greater than the number of students who are at the abstract level stage on the angle concept. Students' emphasis on "distance and length" in their expressions regarding "the distance between any two points in a circle" and "the length corresponding to an angle in a circle" is much related to the content of arc measure concept than the content of arc. This case gives some clues for understanding the mathematical content used by students in structuring these concepts. This finding also shows that students use the arc measure much than the arc itself. According to these findings, students were also aware that they failed in developing a conceptual understanding in constructing the differences and relationships between angle and arc. The content structuring of the difference and relationship concepts between angle and arc are directly related with the content structuring of the angle and arc concepts. Since the students' construction of angle and arc did not overlap with the content construction of these concepts, the students' construction of the difference and relationship concepts between angle and arc also do not overlap with the content construction of these concepts.

More than half of the students had the perception that they constructed the measurement of a given angle and about half of the students had the perception that they constructed the measurement of a given arc. However, majority of the students could not explain how to measure a given arc and about two thirds could not explain how to measure a given angle. Students who made explanations emphasised "Set-square", "Compass" and "Angle measurement". As the participants are university level students, they can not confuse the question words "with which and how" and these questions were all related to "with which", there may be a lack of content and content construction of these concepts. This finding is consistent with the findings of Deniz and Erdoğan (2012) which is about students experiencing problems in measuring arc and angle, Güntekin and Akgün (2011) which is about very few numbers of students determining the correct measure of a given angle and Steckroth (2007) which is about students confusing the concepts of the length of an arc and the measure of an arc. Another significant finding is that students' responses to the questions about measuring an arch and angle are almost the same content. The fact that the number of students who made an explanation to the question of angle was about three times more than the question of arch indicated that students' conceptual construction about angle measurement is stronger than their construction about arc measurement. There is no common content between the definitions of angle and arc concepts other than the definition that these concepts are both sets with an infinite number of points. However, the "angle measurement" taken as issuing a corresponding number to the infinite set defined as angle is carried out by making use of "arc measurement in the circle". In short, angle measurement is conducted with arc measurement (Moore, 2013). Hence, such a finding shows that development of conceptual understanding of the angle measurement is actually students' construction of arc measurement and they are not aware of this case.

About one third of the students had the perception that they developed a conceptual understanding on why the measure of the arc measured in the clockwise is negative, and only one third of these students gave a reason to this question. This finding shows that students do not have knowledge about this issue and they make inferences based on their former experiences. In that case, it is almost impossible for the students not to experience difficulties in trigonometric functions. In addition, some students gave non-mathematical expressions (even dogmas) such as "the rotation direction of the world" as reasons for this issue. This shows that students accept these expressions as equivalent to mathematical axioms. This issue can create a much harder problem, knowledge conservatism in students which is actually harder than solving the real problem itself. The "number line" concept which can be accepted as the answer to this question and used by the students in measurements was shown only in the "placement of numbers on the number line" expressions of two students which are actually wrong. Majority of the students had the perception that they constructed the number line concept. The finding about almost all students drawing the number line shows that they had the visual image of the concept rather than the concept itself. Also two thirds of the students made a number line definition and very few of them were acceptable ones. This issue also supports above finding. In general, students had visual images of the concepts and visual images are much effective in developing conceptual understanding of the concepts of angle and arc concepts.

Another important finding of the study was the students' awareness about whether they had the visual images of the concepts or not. However, the visual image of the concept (recognition level) may not be sufficient in grasping that concept and the construction of its content for inter-conceptual associations (abstract level) are also required for conceptual understanding (Senemoğlu, 2004; Thompson, 2008). Arc measurement (Akkoç, 2008; Akkoç and Akbaş Gül, 2010) and angle measurement concepts are among the most difficult topics in Mathematics for students and therefore using number line in the measurement of arch and angle in application can be a good way of solutions to the problems in teaching these concepts.

Conflict of Interests

The author has not declared any conflict of interests.

REFERENCES

Akkoç H (2008). Pre-service mathematics teachers' concept images of radian. Int. J. Math. Educ. Sci. Technol. 39(7):857–878.

Akkoç H, Akbaş Gül N (2010).Analysis of a Teaching Approach Aiming at Eliminating Student Difficulties with Radian. Ankara University J. Faculty Educ. Sci. 43(1):97-129.

Altun M (1998). Matematik Öğretimi [Teaching Mathematics](6. Ed.). Bursa: Alfa.

Aydın N (1998). Liselerde Matematik Derslerinde Zor Öğrenilen Konular, Zor Öğrenilme Nedenleri ve Bunları Öğretme Yöntemleri, VIII. Eğitim Bilimleri Kongresi Bildiriler Kitabı, Cilt 1, 62-67, KTÜ, Trabzon.

Boyacıoğlu H, Erduran A, Alkan H (1996). Permütasyon, Kombinasyon ve Olasılık Öğretiminde Rastlanan Güçlüklerin Giderilmesi, II. Ulusal Eğitim Sempozyumu, Marmara Üniversitesi Atatürk Eğitim Fakültesi, İstanbul.

Brown SA (2005). The trigonometric connection: Students' understanding of sine and cosine.Unpublished Ph.D. Dissertation. Illinois State University: USA.

Clements DH, Burns BA (2000). Students' development of strategies for turn and angle measure. Educ. Stud. Math. 41(1):31-45.

Çetin ÖF (2011). Use of numerical axis in the graphic drawing of tangent and cotangent functions on coordinate plane by means of unit circle. J. Erzincan Univesity Faculty Educ. 13(2):123-141.

Çetin ÖF, Dane A (2004). On level of achievement the geometrical knowledge of the third class students in the class teaching. Kastamonu Educ. J. 12(2):427-436.

Deniz S, Erdoğan EÖ (2012).İlköğretim 7.Sınıflara yönelik geometri Sketchpad ile çember/dairede açı ve yay ölçümü. X. Ulusal Fen Bilimleri ve Matematik Eğitimi Kongresi, 27 - 30 Haziran 2012, Niğde

Dikici R, İşleyen T (2004). The study of learning difficulties in relation and function from the point view of certain variables. Kastamonu Educ. J. 12(1):105–116.

Durmuş S (2004). A diagnostic study to determine learning difficulties in mathematics. Kastamonu Educ. J. 12(1):125-128.

Fi C (2003). Preservice secondary school mathematics teachers' knowledge of trigonometry: Subject matter content knowledge, pedagogical content knowledge and envisioned pedagogy. Unpublished Ph.D. Dissertation. University of Iowa: USA.

Güntekin H, Akgün L (2011).Trigonometrik kavramlarla ilgili öğrencilerin sahip olduğu hatalar ve öğrenme güçlükleri. Çukurova University Faculty Educ. J. 40(1):98-113.

Kutluca T, Baki A (2009). Investigating of views of students, student teachers and teachers about difficult subjects in 10th grade mathematics class. Kastamonu Educ. J. 17(2):609-624.

Kültür MN, Kaplan A, Kaplan N (2010). The evaluations of trigonometry teaching in high school students. Journal of Kâzım Karabekir Educ. Faculty. 17:202-211.

McMillan JH (2000). Educational research: Fundamentals for the consumer (3th ed.). Newyork: Longman.

Mitchelmore MC, White P (2000).Development of angle concepts by progressive abstraction and generalisation. Educ. Stud. Math. 41(3):209-238.

Mithcelmore MC (1998). Young students' concepts of turning and angle. Cognition Instruction. 16(3):265-284.

Moore KC (2009). An investigation into precalculus students' conceptions of angle measure. In Twelfth Annual Special Interest Group of the Mathematical Association of America on Research in Undergraduate Mathematics Education (SIGMAA on RUME) Conference, Raleigh, NC: North Carolina State University.

Moore KC (2012). Coherence, quantitative reasoning, and the trigonometry of students. Quantitative reasoning and mathematical modeling: A driver for STEM integrated education and teaching in context. 75-92.

Moore KC (2013). Making Sense by Measuring Arcs: A Teaching Experiment in Angle Measure. Educ. Stud. Math. 83(2):225-245.

Orhun N (2004). Students' mistakes and misconceptions on teaching of trigonometry. International Conference on New Ideas in Mathematics Education, Palm Cove: Australia.

Senemoğlu N (2004). Gelişim öğrenme ve öğretim (10.Baskı). Ankara: Gazi Kitabevi.

Steckroth JJ (2007). Technology-Enhanced Mathematics Instruction: Effects of Visualization on Student Understanding of Trigonometry. Unpublished PhD Thesis, University of Virginia, Virginia, USA

Tatar, E., Okur, M. ve Tuna, A. (2008). A study to determine learning difficulties in secondary mathematics education. Kastamonu Educ. J. 16(2):507-516.

Thompson PW (2008). Conceptual analysis of mathematical ideas: Some spadework at the foundations of mathematics education. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano & A. Sépulveda (Eds.), Proceedings of the Annual Meeting of the International Group for the Psychol. Math. Educ. 1: 45-64). Morélia, Mexico: PME.

Thompson PW, Carlson MP, Silverman J (2007).The design of tasks in support of teachers' development of coherent mathematical meanings. J. Math. Teacher Educ. 10:415-432.

Topçu T, Kertil M, Akkoç H, Yılmaz K, Önder O (2006) Preservice and in-service mathematics teachers' concept images of radian, Proceedings of the 30th International Conference on the Psychology of Mathematics Education (PME30), Prague, Czech Republic. 5:281-288.

Weber K (2005). Students' understanding of trigonometric functions. Math. Educ. Res. J. 17(3):91-12.

Yıldırım A, Şimşek H (2008). Sosyal bilimlerde nitel araştırma yöntemleri (7 th ed.). Ankara: Seçkin Yayınevi.