academicJournals

Vol. 8(17), pp. 1519-1524, 10 September, 2013
DOI: 10.5897/ERR2013.1500
ISSN 1990-3839 © 2013 Academic Journals
http://www.academicjournals.org/ERR

Educational Research and Reviews

Full Length Research Paper

Knowledge levels of pre-service mathematics teachers on the concept of set

Nur Sırmacı* and Fatih Taş

Ataturk University, Turkey.

Accepted 2 August, 2013

This study aims to investigate knowledge levels of pre-service mathematics teachers about set. The study was carried out on a total of 196 students studying at the Department of Mathematics Teaching at Kazım Karabekir Faculty of Education in Atatürk University. Concept testing consisting of 7 open-ended questions aiming to analyze the knowledge of students about the concept of set was used for data collection. Questions were completely related to set. There are two parts in data collection tool. First part is "What is the current knowledge level of pre-service mathematics teachers about set?". Second part is "What is the current knowledge level of pre-service mathematics teachers' on the set-related concepts? The relationship of indicator chart of questions was examined. Whether there is a missing subject in the concept testing was studied by the experts. The answers of the students to open-ended questions in concept testing were evaluated as "Correct", "Partially correct" and "Incorrect". Chi-square analysis was used for statistical analysis. The findings revealed that students did not have adequate level of knowledge about mathematical sets.

Key words: Mathematics, mathematics education, set.

INTRODUCTION

One of the most important needs of students in perfor-Conceptual learning is given a great importance nowadays. So it has its own right place in mathematics education besides operational learning and therefore, while evaluating the learning success of students in both overall education and in mathematics education, it is necessary to scrutinize their both conceptual and operational knowledge (Uğurel and Moralı, 2010). Students start learning with the concepts they have already learned. These concepts are defined as pre-learning beliefs. Among these concepts, those which are contradictory to scientific truths are named misconceptions (Gilberts and Wats, 1983). Zembat (2008) defines misconception as the conception or understanding of any topic in the literature in a contradictory way to that expert has a consensus on. Misconceptions are really dangerous since they overshadow the real concepts related to the same phenomenon. Having no knowledge on any issue is better than having misconceptions on it (Yağbasan et al., 2005).

Therefore in mathematics education, several studies are being conducted to develop the ability to understand mathematics by teachers and educationalists. When considered that mathematics is a course arousing anxiety in students, these studies are crucial in mathematics teaching (Ata and Adıgüzel, 2011). An instruction appropriate to the nature of mathematics should serve the following three aims (Baykul, 2003): (1) Students' understanding of mathematical conceptions, (2) their understanding of mathematical operations, and (3) their relating these conceptions and operations to each other. To Baykul (2003), every conception in mathematics is

*Corresponding author. E-mail: n_sirmaci@yahoo.com

related to each other; each new conception is new relation built on the preceding one. Today, the bases of educational systems are conceptual frameworks. Naturally, there are misunderstandings and misconceptions in mathematics awaiting for education practitioners. These misconceptions can turn into advantages if they are well analyzed by educationalists (Jordaan, 2005; Huillet, 2005). Speedy et al. (1989) who analyzed mathematics and physics education in Australia in 1989 argue that teacher education "should help students understand the main conceptions emphasized in the principles of mathematics education program and thus be aware of the links between the mathematical ideas given at the beginning and high level of mathematical knowledge".

The basic of mathematics is the theory of sets (Gavalas, 2005). Theory of sets was proposed by Georg Cantor in the 19th century (Fischbein and Baltsan, 1999). According to Cantor (year), set is the concentration of our intuitions or mind in a way that specific and identifiable objects can be understood as a whole. These objects are called the elements of the set and the set is completely determined by its elements (Fraenkel, 1966). Everybody should have a consensus on whether a given object belongs to this collection or not (Skemp, 1993). The most important side of Cantor theory is that while deciding which objects are the elements of a set and which are not we are directed by intuitions. This approach which brought about a different view towards the strategies of thought and, particularly, the conception of infiniteness used up to that time led the birth of a new theory (İpek et al., 2009). According to Dönmez (2002), sets are the products of the efforts to achieve a unity/consensus in the language of mathematics. The conception of set has made great contributions to the progress in the field called modern mathematics and to the development of the disciplines of logic and computer. It has an important role in mathematics education in addition to being the basis of it. Theoreticians of learning advocate that conceptual development is based, to a great extent, on the skill to classify; and skill of classification grounds on the conception of sets (Olkun and Toluk, 2003).

Aim

Set is very important in mathematics teaching. Now that set is fundamental of mathematics. We wonder students' knowledge of set and think that our research may be useful for mathematics education. For this reason, this study aims to investigate knowledge levels of pre-service mathematics teachers about set.

Research problem

What are the knowledge levels of pre-service mathematics teachers about set?

Sub problem

1. What is the current knowledge level of pre-service mathematics teachers about set?
- "What is set?"
- "In how many ways can be sets represented?"
2. What is the current knowledge level of pre-service mathematics teachers' on the set-related concepts?
- "Is the set of integers equivalent to the set of even numbers?"
- "Give information about whether a empty set is finite or not."
- "Is the universal set constant or single?"
- "Can the complement of a set be different?"
- "Give information about whether intervals are sets or not."

METHODOLOGY

Sampling

The sample of the study consisted of 124 female, 72 male junior grade students studying at the Department of Mathematics Teaching at Kazım Karabekir Faculty of Education in Atatürk University. The sampling of the study was random sampling. Random sampling is a sampling technique where we select a group of subjects (a sample) for study from a larger group (a population). Each individual is chosen entirely by chance and each member of the population has a known, but possiblynon equal, chance of being included in the sample (Easton and McColl, 2004).

Research design

Investigation method of this study was survey method.

Data collection tool

The findings were obtained by means of Concept testing. Concept testing was prepared by the researcher. Concept testing containing 7 open-ended questions which aimed to determine the knowledge of students about the concept of set was used for data collection. Questions were completely related to set. There are two parts in data collection tool. First part is "What is the current knowledge level of pre-service mathematics teachers about set?".Questions of this part are "What is set?" and **"**In how many ways can be sets represented?" We believe that these are very important questions, since the definition of set and representation of set are one of the basics of mathematics. Second part is "What is the current knowledge level of pre-service mathematics teachers' on the set-related concepts?". In this part, we aim to investigate students' knowledge about equivalent of sets, empty set, universal set, complement of a set and interval. These concepts are necessary for understanding other subject of the mathematics. The relationship of indicator chart of questions was examined. Whether there is a missing subject in the concept testing was studied by the experts.

Data analysis

The answers of questions were examined by the researcher. The answers of the students to open-ended questions were assessed as “correct”, “partially correct” and “incorrect”. Chi-square analysis was used for statistical analysis.

Chi-square value of the differences between the answers of female and male students was not found to be significant. For the question “What is set?”, it was found that 63.7% of the female students gave correct answers; 0% partially correct and 36.3% incorrect answers (Table 1). On the other hand, 70.8% of the male students gave correct answers; 2.8% partially correct and 26.4% incorrect answers to this question.

A set is a well-defined collection of objects without repetition. The male students who gave partially correct answers to this question just wrote “set should be well defined”. Those who gave incorrect answers claimed that a set was not a collection. On the other hand, those who wrote that it was a collection explained the characteristics of the collection wrong.

Chi-square value of the differences between the answers of female and male students was found to be significant. Sets are represented in three ways. They are list as; common property and Venn diagram methods. Absence of the common property method among the answers is one of the common errors of the students who answered the question partially correct (Table 2). Male students made this mistake more than the female students.

Chi-square value of the differences between the answers of female and male students was found to be significant. The participants’ answers to the question “Is the set of integers equivalent to the set of even numbers?” show that 92.7% of the female and 81.9% male students gave incorrect answers to it (Table 3). The male students gave more correct answers compared to the females. In addition, most of the students could not give the required answer. They expressed that the two sets were not equivalent. The set of integers consists of even numbers and odd numbers. The set of even integers is a sub-set of the set of integers; and, these two sets are not equal. On the other hand, they are equivalent to each other. Students’ errors are related to this point. That is, they have the idea “How can a set be equivalent to its sub-set that is not equal to it?”

Chi-square value of the differences between the answers of female and male students was not found to be significant. Of the female students, 37.1% gave correct answers and 62.9% incorrect answers. On the other hand, of the male students, 38.9% gave correct answers and 61.1% incorrect answers. A null set is finite set. When the answers were analyzed, some explanations such as “empty sets are neither finite nor infinite, this result is related to the absence of elements in these sets; since empty sets refer to vagueness, they are not finite; since these sets do not contain elements, it’s not possible to make any interpretations” were obtained (Table 4).

Chi-square value of the differences between the answers of female and male students was not found to be significant. It was found that 27.4% of the female students gave correct answers and 72.6% gave incorrect answers to this question. On the other hand, 34.7% of the male students gave correct answers and 65.3% gave incorrect answers. Universal set is not constant and single. It can vary depending on the issue worked on or the person who tries to solve the problem. For the same problem, different universal sets can be formed. A great many of the students could not answer the question right (Table 5).

Chi-square value of the differences between the answers of female and male students was found to be significant. Data related to the question “Can the complement of a set be different?” is presented in Table 6. Only 9.7% of the female students gave correct answers while 90.3% of them gave incorrect answers. On the other hand, 23.6% of the male students gave correct answers while 76.4% of them gave incorrect answers to the question. It was seen that the male students gave more correct answers than the female students did. As shown in Table 6, most students could not give correct answers.

Chi-square value of the differences between the answers of female and male students was not found to be significant. While 12.9% of the female students gave correct answers; 64.5% gave partially correct answers and 22.6% incorrect answers; 20.8% of the male students gave correct answers; 48.6% gave partially correct answers and 30.6% incorrect answers. Intervals are special sets (Table 7). However, it was found most students did not know this. The opinion of the students who explained that intervals were sets was: “for instance, the interval (2, 7) is a set and it is represented as {3, 4, 5, 6}”. This is one of the commonest errors of the students.

RESULTS AND SUGGESTIONS

The concept of set is one the main elements of mathematics. Thanks to that nature of set, the students who have fully learnt this concept can be successful in other classes. That the concept of set could not be fully defined and explained is quite thought-provoking. The concept of set should not be simply defined as “cluster of well defined and non-duplicated repeated objects”. Clusters should be formed and these clusters should be analyzed in their finest details to reach a judgment whether they are set or not.

According to Baki and Şahin (2004), it is realized, in time, that the examples given during the instruction of sets usually result in students’ misunderstandings. Usually, the fact that it is not possible to form sets in the cases of impossible common properties or impossible coexistences is ignored and, the rules and operations related to sets are directly taught. It is seen that, despites its importance and the problems experienced, the studies on the instruction of sets are not at the required levels (Linchevski and Vinner, 1988). Therefore, the studies on the instruction of sets should be at the required levels.

In addition to its definition, ways of representation of sets have important roles in mathematics as well. For instance, a student requires set representation patterns to be able to explain the definition, value, and image set (range) of any function; and to prove that a set shown by common property method is a relation. Most of the participants experienced difficulties in expressing the way of representation by common property method. Therefore, common property method, one of the representation ways of sets, should also be focused on. Exercises such as giving common property sets and asking students to write them with listing method or to do vice versa should be involved in classes.

The sets whose number of elements can be defined as a natural (counting) number are called finite sets; and the others are called infinite sets. In finite sets, a set is not equivalent to any of its sub-sets, except for its subset equal to itself. In infinite sets, an infinite set can be

Table 1. Answers given to the question “What is set?”

Gender	N/%	Correct	Partially Correct	Incorrect	Total
Female	N	79	0	45	124
	%	63.7	0	36.3	100
Male	N	51	2	19	72
	%	70.8	2.8	26.4	100
Total	N	130	2	64	196
	%	66.3	1	32.7	100

DF = 2; χ^2 = 5.161; p>0.05.

Table 2. Answers given to the question “In how many ways can sets be represented?”

Gender	N/%	Correct	Partially Correct	Incorrect	Total
Female	N	92	26	6	124
	%	74.2	21	4.8	100
Male	N	41	28	3	72
	%	56.9	38.9	4.2	100
Total	N	133	54	9	196
	%	67.9	27.6	4.5	100

DF = 2; χ^2 = 7.352; p<0.05.

Table 3. Answers given to the question “Is the set of integers equivalent to the set of even numbers?” Explain.

Gender	N/%	Correct	Incorrect	Total
Female	N	9	115	124
	%	7.3	92.7	100
Male	N	13	59	72
	%	18.1	81.9	100
Total	N	22	174	196
	%	11.2	88.8	100

DF = 1; χ^2 = 5.329; p<0.05.

Table 4. Answers given to the question **“**Give information about whether a empty set is finite or not."

Gender	N/%	Correct	Incorrect	Total
Female	N	46	78	124
	%	37.1	62.9	100
Male	N	28	44	72
	%	38.9	61.1	100
Total	N	74	122	196
	%	37.8	62.2	100

DF = 1; χ^2 = 0.062; p>0.05.

Table 5. Answers given to the question **"**Is the universal set constant or single?

Gender	N/%	Correct	Incorrect	Total
Female	N	34	90	124
	%	27.4	72.6	100
Male	N	25	47	72
	%	34.7	65.3	100
Total	N	59	137	196
	%	30.1	69.9	100

DF = 1; χ^2 = 1.155; p>0.05.

Table 6. Answers given to the question **"**Can the complement of a set be different?"

Gender	N/%	Correct	Incorrect	Total
Female	N	12	112	124
	%	9.7	90.3	100
Male	N	17	55	72
	%	23.6	76.4	100
Total	N	29	167	196
	%	14.8	85.2	100

D.F.= 1; χ^2 = 7,015; p<0.05.

Table 7. Answers given to the question "Give information about whether intervals are sets or not."

Gender	N/%	Correct	Partially Correct	Incorrect	Total
Female	N	16	80	28	124
	%	12.9	64.5	22.6	100
Male	N	15	35	22	72
	%	20.8	48.6	30.6	100
Total	N	31	115	50	196
	%	15.8	58.7	25.5	100

DF = 2; χ^2 = 4.911; p>0.05.

equivalent to one of its sub-sets which is not equal to it, as long as there can be an exact surjection and trans-formation. However, this is true of only infinite sets. Almost all the students gave wrong answers to the question about this subject. The reason of why they answered the question wrong stems, as it was mentioned above, from lack of knowledge. Instruction should be designed in a way to dispel this lack.

When the wrong answers were analyzed, it was seen that since the participants did not know what a finite set was they made some wrong interpretations on whether a empty set was finite or not. They were fully taught the finite and infinite sets again. In this way, they would be able to identify whether a given set was finite or not.

The participants were observed not to be successful as a result of that, during the instruction of the definition of universal set, the statement “A set which contains all the units that operations can be done on is called universal set”, they were not fully taught that a universal set was not single or constant and examples and problems that would exemplify this fact were not involved in the instructional process. The rule that the universal set is not single or constant should be taught by dispelling the troubles mentioned above.

The complement of a set is expressed according to the universal set. The complements of a set vary depending on different universal sets. Since the participants do not know what a universal set refers to, it is inevitable for their lack of knowledge to reflect on here. Universal sets should be taught to the students in a retentive way.

Intervals are special sets and its elements are real numbers. If students define the interval (2, 7) as a set the elements of which are whole numbers such as {3, 4, 5, 6}, it is clear that they do not know what intervals are. Teaching open, closed, half-open, half-closed, and unbounded intervals effectively will prevent making these and other similar errors.

If we want students to have knowledge about set, teaching with different methods and materials can be used in courses (Cardellini, 2004).

REFERENCES

Ata N, Adıgüzel T (2011). Matematik öğretiminde kavram haritalarının farklı biçimlerinin öğrencilerin kavram haritası yapabilme düzeyi ve akademik başarılarına etkisi. Gaziantep Üniv. Sosyal Bilimler Dergisi. 10(2):803-823.

Baki A, Şahin SM (2004). Bilgisayar destekli kavram haritası yöntemiyle öğretmen adaylarının matematiksel öğrenmelerinin değerlendirilmesi. Turk. Online J. Educ. Technol. 3:2.

Baykul Y (2003). İlköğretimde matematik öğretimi: 6-8. sınıflar için. Ankara: Pegem Yayıncılık.

Cardellini L (2004). Conceiving of concept maps to foster meaningful learning:An interview with Joseph D. Novak. J. Chem. Educ. 81(9): 1303-1308.

Easton VJ, McColl JH (2004). Statistics Glossary 1:1.

Dönmez A (2002). Matematiğin Öyküsü ve Serüveni, “Dünya Matematik Tarihi Ansiklopedisi”, Matematik Sözlüğü.1. İstanbul: Toplumsal Dönüşüm Yayınları.

Fischbein E, Baltsan M (1999). The mathematical concept of set and the collection model. Educ. Stud. Math. 37:1-22.

Fraenkel AA (1966). Set Theory and Logic, Addison-Wesley P. Co.

Gavalas D (2005).Conceptual mathematics: an application to education. Int. J. Math. Educ. Sci. Technol. 36(5):497-0516.

Gilbert JK, Watts DM (1983). Consepts, misconception and alternative conceptions:changing perpectives in science education. Stud. Sci. Educ. 10:61-98.

Huillet D (2005). Mozambican teachers’ professional knowledge about limits of functions. Proceedings of the 29th Conference of the International Group for the Psychol. Math. Educ. 3:69-176.

Ipek A, Albayrak M, Işık C (2009). Sınıf öğretmeni adaylarının küme kavramıyla ilgili algıları. Erzincan Eğitim Fakültesi Dergisi. 11(1).

Jordaan T (2005). Misconceptions of the limit concept in a mathematics course for engineering students.Unpublished Master of Science Dissertation, University of South Africa.

Linchevski L, Vinner S (1988). The naive concept of sets in elementary teachers,Proceedings of the 12th International Conference, Psychol. Math. Educ. 11:471-478.

Olkun S, Toluk Z (2003). İlköğretimde Etkinlik Temelli Matematik Öğretimi. Ankara: Anı Yayıncılık p.221.

Skemp RR (1993). The Psychology of Learning Mathematics.England:Penguin Books.

Speedy G, Annice C, Fensham P (1989). Discipline review of teacher education in science and mathematics. Canberra: Department of Education, Employment and Training.

Uğurel I, Moralı S (2010). Ortaöğretim öğrencilerinin kümeler konusundaki öğrenmelerinin değerlendirilmesi-I.Akademik Bakış Dergisi. 22.

Yağbasan R, Gunes B, Ozdemir DE, Temiz BK, Gulcicek C, Kanlı U, Unsal Y, Tunc T (2005). Konu Alanı Ders Kitabı İnceleme Kılavuzu – Fizik.Ankara: Gazi Kitabevi.

Zembat IÖ (2008). Matematiksel Kavram Yanılgıları ve Çözüm Önerileri. M.F.Özmantar(Ed.),Kavram Yanılgısı Nedir?, Ankara: Pegem Akademi Yayınevi, s. 1-8.