Teaching fractions at ordinary level: A case study of mathematics secondary school teachers in Zimbabwe

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The aim of the study was to investigate the teaching of fractions by Ordinary level mathematics teachers at Radcliff high school in Zimbabwe. The main objective of the study was to establish how teachers teach the concept of fraction and to find out why they teach in the manner they do. A case study research design was used and purposive sampling was implemented from the whole population of mathematics teachers at the school to select the sample for the study. The sample consisted of three ‘O’ level mathematics teachers. Documentary analysis, lesson observations and interviews were used to collect data to answer the research questions. The collected data was qualitatively interpreted and analyzed. The results of the study revealed that teachers use traditional methods which are anchored on practice of problem tasks, exemplification (teaching by giving examples), drill and teaching of rules and algorithms in the teaching of fractions. Post data interpretation led to the findings that teachers promote procedural understanding of the concept of fraction. The way they teach is heavily influenced by their beliefs on teaching. They believe that giving worked examples and having students to follow rule based procedures will enhance the students’ problem solving capabilities. As justification of their strategy, teachers cited shortage of time to prepare for the lessons, examination driven curriculum and limited resources. Basing on these findings, the researchers recommend staff development workshops and seminars to equip teachers with skills which will enable them to employ child centred teaching strategies that may result in the conceptual understanding of fractions.

Key words: Conceptual understanding, procedural knowledge, instrumentally, relationally, teaching strategy.

INTRODUCTION

Many people use fractions in one way or another in everyday life. For example, when a person wakes up and allocates the time to spend on each item of the daily chores, the time spent on one of the items is part of the twenty-four hours in a day. One may further come into contact with fractions on checking time. One quarter of an hour can be read as quarter past that is;

\[
\frac{15}{60} = \frac{4}{3}
\]

There are many other forms in which the concept of fractions is applicable to everyday activities. In the majority of cases many people including students do this without realizing that they are coming into contact with fractions (Burns, 1992).

The foregoing discussion implies that fractions are central to peoples’ everyday lives (Burns, 1992). People need to have a deep understanding of fractions for them to be able to apply them in their day-to-day activities. Researchers have established that many students have considerable difficulty in understanding fractions (Behr et al., 1992). The sources of the students’ problems of understanding fractions may be categorized into two major related groups. These are the formal teaching strategies and informal learning of fractions concept as argued by Behr and others (1992) that the obstacles students encounter in understanding fractions are either

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inherent to the nature of fractions or are due to the instructional approaches employed by the teachers.

Informal learning starts at an early stage before pupils go to school and continues during formal learning. This implies that prior knowledge on a particular concept like fractions is so vital in the learning process. Students’ prior knowledge is a combination of different life experiences they have encountered in their day to day lives that require them to use knowledge of fractions. If students fail to realize their informal encounter with fractions in their daily activities, the formal learning of such concept remains abstract and irrelevant.

Students’ learning depends on the teaching strategy. When students are taught concepts by rules, they are not afforded the opportunity to construct a conceptual foundation for understanding. Rules can facilitate conceptual understanding if they are used relationally, but experience has shown that most teachers use them instrumentally and as a result students fail to understand the concepts. If rules are handled relationally, they facilitate conceptualization than when they are used instrumentally. Such instructional strategies usually divorce the classroom concept from the real life situation and make the concept difficult to comprehend. Real life situations entail what students already know and what they encounter in their day to day activities. Since the teachers’ teaching strategies are influenced by the teachers’ understanding about the students’ prior knowledge, content and how students build productively upon prior knowledge (Thompson and Saldanha, 2003) teaching by rules makes the classroom learning of concepts irrelevant and abstract. A major concern of researchers who focus on the concept of fractions is that students’ understanding is often too procedural in nature and limited as they rely on their taught rules (Carpenter et al., 1993).

According to Sowder and others (1998) the progression of the concept of fraction from low to high level school curriculum requires a transition from additive reasoning to multiplicative reasoning. This implies that it is appropriate for teachers to use the additive reasoning approaches when teaching fractions at lower levels (primary level and junior level) but needs to shift to the multiplicative approaches at higher levels. A cursory look at the secondary school mathematics curriculum in Zimbabwe shows that multiplicative relations underpin almost all number-related fractions, called the multiplicative conceptual field.

Clark and Kamii (1996) wrote about the importance of distinguishing between additive and multiplicative reasoning. Sowder and others (1998) use an investment scenario to illustrate the crucial difference between additive and multiplicative thinking as follows: one person invests $2 and gets back $8, while the other person invests $6 and gets back $12. Typically additive thinkers simply calculate the difference between the investment and the profit, and conclude that the deals are the same because both people make a $6 profit. Multiplicative thinkers, on the other hand, can appreciate that the first investment quadrupled, whereas the second investment only doubled, so the first investment is a "better deal."

Though research has increasingly proved that multiplicative approaches to teaching of fractions (Clark and Kamii, 1996) have numerous advantages to learning of the concept, there are also several studies revealing that in most cases students resort to additive approaches to interpret and solve fraction problems (Hart, 1978). The additive approaches that are sometimes referred to as the building up strategy, at times lead to a correct theoretical solution that lacks practical relevance. This often happens when the problem situation is complex and demands multiplicative reasoning.

For example the national assessment of educational progress assessed the fractional reasoning skills of 8th and 12th grades using the following problem: In 1980 the population of Town A and Town B was 5000 and 6000 respectively. In 1990 the population was 8000 and 9000 respectively. Compare the 1980 and 1990 population of towns A and B. Most students reasoned additively and concluded that both town A and B grew at the same rate. They solved the problem by finding the difference. Using multiplicative reasoning to solve the above problem will yield the following results: Town A’s 1990 population is \( \frac{8}{5} \) of its 1980 population resulting in a 160% growth rate and Town B’s population is \( \frac{9}{6} \) of its 1980 population thus giving a 150% growth rate. As a result students can conclude that Town A had a higher growth rate than Town B and resources will be allocated equitably.

Pirie and Kieren (1994) point out that fraction learning involves constructing an ever more elaborate, complex, broad and sophisticated fraction world and developing the capacity to function in more complex and sophisticated ways within it. This implies that students’ understandings of the concept are reflected in their ability to solve real life situations. This can be attained if teachers adopt multiplicative approaches to teaching fractions concept. However, multiplicative reasoning is difficult for students and often requires formal instruction and multiplicative reasoning is the entry point to the world of more complex fractions (Sowder et al., 1998). The teaching strategies aimed at developing multiplicative reasoning in the context of this study are known as multiplicative approaches.

The problems students encounter with understanding of fractions is common and recurrent in most nations the world over (Behr et al., 1993). In Zimbabwe, research is scarce on the subject of fractions and multiplicative reasoning. Our observation as mathematics teachers have been summarized by Land (1963) who observed that school mathematics work is characterized by unimaginative teaching. If the emerging multiplicative
approaches to teaching fractions create opportunities for students to competently solve fraction problems then the goal of making school mathematics relevant to the students will be attained. However one wonders whether teachers in Zimbabwe are aware of this approach. If they are aware, how far are they creating classroom opportunities for students to develop multiplicative reasoning skills? In an effort to provide answers to the questions raised and contribute to empirical evidence on how fractions are taught in Zimbabwe the researcher conducted a case study at Radcliff High school guided by the following research questions. How do teachers teach fractions and why do teachers teach the fractions the way they do? Answers to the above research questions will give insights into an understanding of the present teaching of fractions in the Zimbabwean context.

Conceptual framework

The assumptions in this study reflect a social constructivist perspective of teaching and learning, and a corresponding belief that assessment should be an integral and constructive part of the learning process. This perspective acknowledges that: Learning involves the active construction of knowledge through personal experience and influenced by prior knowledge as well as student and teacher attitudes and approaches towards learning. Learning does not occur in isolation and is not fixed, but rather it is socially negotiated and expressed through language that focuses on explanation and clarification. Learning is enhanced through collaboration with more knowledgeable others through a scaffolding process where learners progress from assisted to independent performance and assessment is an integral part of the learning process and should be consistent with learning principles (Clarke, 1997; Yackel et al., 1992).

However, the principles of social constructivism do not dictate specific teaching methods (Stake, 1995). Therefore, it could be argued that classroom teachers seeking to merge their beliefs about learning with their pedagogical practices could benefit from the abstraction of shared practices that reflect the principles of social constructivist learning and teaching. The essences of these principles suggest that students need to make sense of the information they are constructing through socially interactive processes so that understanding becomes the goal of learning.

The aforementioned ideas discussed regarding students as active participants in knowledge construction provided underpinnings for understanding the way teachers teach fraction concepts. The pedagogical implications of constructivism are that teachers should act as facilitators who provide appropriate activities and support for students to personally construct meanings, rather than receive them ready made from the teacher (von Glaserfield, 1995). Teachers need to recognise that the students’ actions, ideas, errors indicate their current state of understanding, and from the students’ perspective these are sensible and logical (Wood, 1995). Further, teachers need to design learning activities that engender “periods of conflict, confusion and surprise over long periods of time during social interaction” (Wood, 1995: 337). The related classroom environment should encourage creativity, problem solving, exploration and sharing of ideas.

The understanding of a concept can be classified as either conceptual or procedural. The orientation of any mathematics teaching strategy can be towards competing conceptual and procedural knowledge or interacted between elements of conceptual and procedural knowledge. Rittle et al. (2002) recommend that research in mathematics education focuses on the conceptual knowledge orientation of teaching strategies. The knowledge attained is an outcome of some reasoning approaches should be used in solving a mathematical problem. This raises many questions in the teaching of fractions in Zimbabwe as to whether the teaching of fractions promotes procedural (additive reasoning) or conceptual (multiplicative reasoning).

Knowledge of rules or procedural knowledge will not promote conceptual understanding of fractions. When teaching a mathematics topic such as fractions, the challenge then is whether the teaching is promoting procedural understanding or conceptual understanding. Yin (1994) has observed that quite often students have exhibited more procedural understanding than conceptual understanding. In short students should acquire a deep understanding of fractions and be able to use them competently in problem solving.

The national assessment of educational progress reports that fractions are “exceedingly difficult for children to master” (NAEP, 2001). Additionally, students are frequently unable to remember prior experiences with fractions from lower grade levels. However, students who use multiplicative reasoning in most cases can differentiate between situations that require additive or multiplicative transformations, Clark and Kamii (1966) and most importantly find the grasp of fraction concepts relatively easy. Several studies conducted elsewhere have revealed important findings about the reasoning approaches and development of conceptual knowledge in the teaching and learning of fractions. According to Smith (2002) there are two broad phases of development in solving fraction problems. These categories are, first, the student is to learn what fractions are and then learn how to perform arithmetic operations on them.

The key is to grasp the idea that fractions name the relationship between the collection of parts and the whole, not the size of the whole or its parts (Smith, 2002). Smith suggests that students need practice with partitioning of wholes into many different sized pieces in order to bring understanding of partitioning. The understanding described here by Smith is based on
additive reasoning. The argument presented by Smith here is procedural knowledge and is a link to gaining conceptual knowledge.

Research suggests that students who develop conceptual understanding early perform best on procedural knowledge later (Fraser, 2000). Students with good conceptual understanding are able to perform successfully on near-transfer tasks and to develop procedures and skills they have not been taught. Students without conceptual understanding are able to acquire procedural knowledge when the skill is taught, but research suggests that students with low levels of conceptual understanding need more practice in order to acquire procedural knowledge. A balance is needed between the time students spend practising routine procedures and the time which they devote to inventing and discovering new ideas. Teachers need not choose between these activities; indeed, they must not make a choice if students are to develop the mathematical power they need.

Another important aspect is the teachers’ content knowledge which is vital and complementary to pedagogical knowledge in the teaching of fractions as expressed by the national council of mathematics teachers (2000) that the learners' understanding of mathematics, their ability to use the knowledge to solve problems and their confidence in and disposition towards mathematics are all shaped by the teaching they encounter in school. The extent to which students acquire multiplication reasoning skills and gain conceptual knowledge of fractions is greatly influenced by what happens in the classrooms. Consequently, what happens in the classroom is influenced by what teachers know about the subject and teaching. Therefore successful development of multiplicative reasoning skills in students is actually centered on the teacher's knowledge of fractions and multiplicative reasoning.

According to the NCTM (2000), teachers should understand deeply the mathematics they teach so that they will be able to draw on that knowledge with flexibility in their teaching. Ma (1999) (as cited in NCTM, 2000) argues that it is important for teachers to understand big ideas of mathematics. In that case teachers will be able to represent the ideas as a coherent and connected enterprise. They should be able to select tasks that will enable students to do problems on fractions and reflect on the problems. Ball and others (2001) are of the opinion that teachers must understand concepts and procedures themselves in order to select and construct fruitful tasks and activities for their students as well as flexibly interpret and appraise their ideas. These tasks on fraction concepts that are chosen will help students to understand fractions and build bridges between what the students know and the new information being taught. In line with this Hill, Schilling and Ball (2004), have developed a model called mathematics knowledge of teaching (M.K.T). The model has three knowledge domains most central to mathematics teaching that include common knowledge of mathematics, specialized knowledge of content and knowledge of students and their ways of thinking.

METHODOLOGY

The study was carried out at a high school in Mashonaland Central Province, Bindura District. Three teachers were purposively sampled to participate in this study (Yin, 1994; Stake, 1994). Teachers were selected as demanded by the study topic, which looked into the teaching of algebraic fractions. Their selection of teaching strategies, activities and content is highly influential to students' sound understanding of the fractions concept. Only teachers with a minimum qualification of a secondary school teaching certificate in education and at least five years teaching experience were considered for selection. A teaching experience of five and more years was preferred because of the teachers' well-established craft knowledge (NCTM, 2000; Hiebert and Carpenter, 1992; Kent et al., 2002). A minimum qualification of a certificate in education was considered desirable as such teaching qualification ensured that the teacher possessed mathematics knowledge of teaching (Hill et al., 2004).

Instruments

The researcher is a primary instrument for data collection in qualitative research designs (Lincoln and Guba, 1985; Bogdan and Biklen, 1982). This implies that the researchers need to be at the center of data collection activities to subjectively select events to investigate (Lincoln and Guba, 1985; Nyawaranda, 1998). As a primary instrument of data collection the researchers collected data through document analysis, lesson observations and interviews. These three data collection tools were considered to be sufficient multiple data sources to triangulate qualitative data to answer the research questions (Yin, 1994). It also increases the reliability of the data and its collection process (Stake, 1995). The objective of triangulation was achieved when the researcher compared different sources, situations, and methods to find irregularities in the data and recurring patterns (MacMillan and Schumacher, 1997). Cross examination of document analysis notes, lesson observation notes and interview data was the basis for triangulation.

The researchers sought ministerial, school, departmental and individual authority to gain access to the field of study. At each of the MOESAC levels and with each person, the researcher had to gain access to overcome barriers to entrance and provide the legal rights protection of the participant (Lincoln and Guba, 1985). Guaranteeing legal rights of protection made participants free and confident to provide real and rich data (Bogdan and Biklen, 1988; Lincoln and Guba, 1985; Nyawaranda, 1998). The researchers joined the staff as participant observers attached to the department of mathematics. This was because qualitative researchers need to study phenomenon from its natural setting (Nyawaranda, 1998). Peer lesson observation policy with post and pre interview lesson observation was adopted and implemented. All this was done to create a collegial and open working environment as well as to minimize the effects of an intruder in a lesson.

The teacher's professional dossier (Schemes of work, and assessment profiles) and departmental curriculum documents (national mathematics syllabus, school mathematics syllabus and departmental policy) were collected and analyzed. Data was captured by way of field notes. The curriculum documents provided sufficient triangulation on their stipulation of teaching of goals of fractions, content coverage, level of concept complexity and pedagogical approach. The curriculum document analysis provided
the researchers with insights into the national curriculum expectations of teaching fractions in mathematics education at ordinary level. This was followed by teacher professional dossier analysis to determine the extent to which teachers understand and translate the national curriculum expectations and standards of teaching fractions (the teachers' interpretation of the national curriculum).

Each teacher dossier comprised a scheme of work, written work plans and records of marks. However, the analysis of these documents was a continuous process. Information was looked for as and when the researchers needed clarification from the documents. Exercise books were inspected to check on the teacher’s achievement of the assessment and teaching goals as stipulated in their schemes of work and plan of written work. Lesson observation commenced soon after the researchers had completed looking at the teachers’ dossiers. The teachers’ interaction with students in the classroom provided information on the teachers’ implementation of their teaching of fraction plans. The researchers conducted pre and post lesson observation interviews with each participant. The individual interviews were conducted concurrently with lesson observations. Interviews enabled researchers to craft specific questions appropriate to the uniqueness of each participant about the teaching of fractions. The researchers jotted down verbal expressions in response to questions as well as non-verbal clues like feelings and intuitive hunches, and poses. Data gathered from the interviews was then triangulated with data obtained from the observation of lessons. The pre interviews were guided by questions raised and noted from the documents analyzed whilst post interviews were guided by points noted and questions rose during lessons as well as from pre interviews.

RESULTS

To provide answers to the research questions, how teachers teach fractions and why they teach the way they do data for each case is presented. The data collected from the three individual case units was interpreted in an integrated manner.

Results from document analysis

Documentary analysis was done so as to get an insight into the teaching of fractions. ‘O’ Level Mathematics syllabus, school syllabus, the departmental policy provides the framework for the teaching and learning of mathematics at Ratcliffe High School. A single copy of each document was in the department.

The school syllabus was compiled from the national syllabus by the syllabus development team. Experienced members with many years in the teaching field were chosen, as their experience is handy in linking topics.

The unit of algebraic fractions in the school mathematics syllabus was divided into sections, which involved addition and subtraction of fractions in algebra, multiplication and division of algebraic fractions, equations which involve fractions, ratios, rates and proportionality. These were further sub-divided into smaller sections. The components of the school mathematics ordinary level syllabus included a list of topics to be covered with time frame, and suggested teaching methods. The document had no aims and objectives. The question that can be asked is “How are they going to come up with objectives for teaching mathematics. Some teachers may have problems in coming up with aims and objectives for teaching a particular topic if they are not clearly outlined in the syllabus document. When the researchers requested for an explanation from the Head of Department over the issue she quickly admitted that the school syllabus team made an oversight.

The departmental policy document contained the following components: goals of teaching mathematics, scheming and planning, lesson delivery, assessment, departmental resources, and staff development workshops.

The policy document was quite skeletal without specificity. This gave teachers leeway to do as they wish. The three participants confirmed that they rarely refer to the policy document as a result their operations is guided in most cases by routine procedures.

The participants

The background information of the three participants of the study: Chido, Kunatsa and Tafara (not their real names) are presented in the Table 1.

Mr Chido

Mr Chido is the most senior and experienced member of the mathematics department. The O-level pass rate documents over the past five years revealed that Chido’s learners obtain good national examinations results, ranging between 60 to 75% each year. Overall, the students had contact time of two hundred and ten minutes of mathematics per week with Chido covering the topics of fractions in algebra which included addition and subtraction of algebraic fractions, multiplication and division of algebraic fractions. Chido prepared his lessons in form of a scheme-cum plan as shown in Figure 1.

The scheme-cum plan is a scheme of work coupled with lesson plans all in one. The day-to-day activities are heavily summarized making it difficult to distinguish work that is to be covered in a lesson. The column headings were quite explicit in the scheme cum plan that made it simple for the researchers to pick teaching methods and teaching and learning materials selected by the teacher. There were no teaching media listed other than the chalk board and work sheets. The outward appearance showed lack of regular reference to the document. Chido confirmed this during interviews as exemplified by the following interview except:

With my experience there is no need to refer to the scheme. I do it to satisfy the requirements of the department....
Table 1. Teachers’ background information.

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Sex</th>
<th>Professional qualification</th>
<th>Position held in school</th>
<th>Experience in years</th>
<th>Level taught</th>
<th>Lessons observed</th>
<th>Periods/week</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chido</td>
<td>M</td>
<td>B.Ed Mathematics</td>
<td>Senior teacher</td>
<td>19</td>
<td>Form 3A, B, C U6 science</td>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>Kunatsa</td>
<td>F</td>
<td>B.Sc. Maths and Statistics</td>
<td>Teacher</td>
<td>12</td>
<td>Form 4A, B, CL6 commercial</td>
<td>5</td>
<td>30</td>
</tr>
<tr>
<td>Tafara</td>
<td>M</td>
<td>B.Sc.Ed Mathematics</td>
<td>Teacher</td>
<td>10</td>
<td>Form 4D, E, F U6 commercial</td>
<td>5</td>
<td>30</td>
</tr>
</tbody>
</table>

Chido underplays the importance of scheming and planning in teaching. Underplaying planning of lessons and relying on experience in our view is evidence of the development of procedural knowledge through rule based teaching. It also implies that the differences in students he meet year in and year out are not taken into consideration as all that matters is the need to master rules so as to pass an examination. There is no link between his teaching and lesson preparation.

The types of activities that dominated Chido’s class was whole class interactive, Topin (1987). The activity of presenting concepts and principles to the class for the first time was characterized as direct teaching. This form of teaching was encountered when a new concept on fractions for instance the simplification of algebraic fractions of the form

\[ \frac{6a^2 + 2b^2}{3ab} = \frac{4a - b}{2b} \]

was introduced or some specific principle or law such as factorizing the denominators so that their lowest common multiple (LCM) can be used as the common denominator was presented. Chido demonstrated how to simplify the following problem on the chalkboard. As the teacher demonstrates, students watched carefully and some were seen taking some notes. They actually admired Mr Chido as he worked the problem to the final solution.

Mr Chido: Have you seen how I have done it.... Its simple try to follow the steps as you copy the example in your note books. Okay.

Students copied the example in their notebooks as they struggled to make sense of each and every step in the solution. The demonstration shows that Chido followed an instructive teacher led chalk and talk method in his teaching. The most dominant activity was going over problem tasks, particularly those assigned for homework and those encountered in their tests and examinations.

The ‘going over’ activity could best be described as ‘watch carefully how I do it’. This mode of careful explaining was the dominant activity in his classes. He made use of the chalkboard every time when he explained or demonstrated how to solve a problem task. During the lessons, it appeared that conceptual learning is inhibited through the use of direct/authoritative approach of the teacher.

The learning environment could best be characterized as ‘directive’. Learner activities in class were essentially limited to listening, watching; asking questions and at times providing chorus answers as Chido performed. The marking was not very informative as Mr Chido would concentrate on the final answer without checking on the logical sequencing of the computations. The trend of Chido’s teaching which emerged from the data is summarized into four themes:

1) Teaching of fractions strategies are teacher centered and rule based, oriented towards developing procedural understanding of concepts.
2) Students are engaged in the lessons mainly through question and answer technique,
3) Teacher resorts to use of chalk and board,
4) Assessment of student progress was mainly through written exercises and tests.

From this characterization of Chido’s teaching of fractions the researchers inferred that Chido teaches fractions to promote procedural understanding of concepts and underplays conceptual knowledge acquisition that is attained through the use of multiplicative approaches.

When asked why he stuck to direct teaching as a method of instructions, Chido had this to say:

The concept is too abstract that it only requires the teacher to tell the student what to do. If you do not tell them the rules and procedures to follow you won’t move fast…….Chido believes in the
lecture method that involves telling the student what to do rather than letting them construct their knowledge as they interact with each other in the classroom.

**Mrs Kunatsa**

Kunatsa was observed teaching form 4As. She had an incomplete scheme-cum plan by the time the researchers visited her. An extract for the scheme-cum plan is shown on Figure 2.

The methods selected by the teacher were teacher centred. The listed methods are illustrations and demonstrations. The teacher–pupil activities are not explicit from the scheme-cum plan. It is heavily summarized.

Kunatsa would concentrate on procedures than meaning. She taught rules on multiplication of algebraic
fractions. Beneath is an illustration on how she conducted her lesson:

**Kunatsa:** Today we are going to look at multiplication of fraction by fraction. But before we start the multiplication of fraction-by-fraction, let us all try to work out the following problems.

\[
\frac{m+n}{m} \times \frac{mn}{3m+3n}
\]

\[
\frac{a^2-b^2}{2 ab+a^2} \times \frac{2a^3}{ab-a^2}
\]

Students worked in pairs, and some worked individually and produced the solutions to the problem. Kunatsa would then ask individuals to present their solutions whilst the rest are listening. Kunatsa was more oriented to the use of the rule of multiplying fractions that promotes additive/procedural knowledge. This observation concurs with the research findings from a study by Thompson and Saldanha (2003) on fractions and multiplicative reasoning which found that a large amount of curriculum and instruction has the aim of teaching students multiplication as a process of adding the same number repeatedly. However research literature documents how this approach becomes limiting and problematic (Harel et al., 1994).

Kunatsa also used worksheets with some problem tasks. These problem tasks were taken from textbooks and some from past examination papers. The overall percentage of time spent on different activities is displayed in Figure 3.

During the post lesson interview the Teacher explained why she was giving little time for students to solve problems in class.

**Kunatsa:** It is not like I am not giving the students time but you know I need to finish the syllabus before they begin their public examinations. The students would need ample time to practice examination questions, so they will get that time later when the syllabus is completed.

She used illustrations and students were asked to work problem tasks using the invert and multiply algorithm. No explanation was given to the reason why students have to invert the denominator and change the sign to a multiplication sign. It was evident that students were puzzled and would just follow without understanding.

Kunatsa allocated relatively small amount of time for the active involvement of students. The teacher used verbal explanations and chalk board work. About 30% of the lesson time was allocated to students to work on problem tasks. Kunatsa would say:

‘Do the following numbers 1, 2, 10 and 15 in your exercise books.’

The problem questions were taken from a work sheet. If students were uncertain about the value to be used or the
algorithm to follow they would simply confer. When Kunatsa was teaching or going over problem tasks, she encouraged the learners to ask questions. However very few students participated in asking or answering questions, without being directly asked by Kunatsa. This was in part due to the presence of five or six learners who dominated interaction to the exclusion of others. When explaining how to solve problems, students would appear lost and would ask those students seated next to them, “where are we?” Kunatsa mostly used direct teaching emphasizing on drill and rule based, students are engaged in the lessons mainly through question and answer technique, teacher resort to use of chalk and board, assessment of students is divorced from lesson planning and delivery. From this characterization of Kunatsa’s teaching of fractions the researchers inferred that she teaches fractions to promote procedural understanding of concepts. She underplays conceptual knowledge acquisition. She cited large class sizes, examination driven curriculum, high teaching load coupled with administrative duties and limited resources as the major drivers of his teaching of fraction.

Mr Tafara

Mr. Tafara was an experienced senior teacher who taught ordinary level Mathematics for 21 years. Mr Tafara was observed teaching a form 4F class. He kept the general guide provided by the school mathematics syllabus. Tafara did not have a scheme cum plan. He extensively relied on the new general mathematics book 4 as a general guide to his teaching. Tafara was doing text book teaching. There were no teaching and learning materials planned for the lessons. He considered the textbooks, chalk and board, as the major teaching materials. During the pre-lesson interview, Tafara mentioned time as a constraint that prohibits him from scheming and looking for teaching materials to teach equations with fraction for conceptual understanding. The topic on solving equations with fractions was dealt with as a separate series of lessons. All the class activities involved Tafara making demonstration and illustration on the chalk board whilst the students listened.

A number of problem tasks would be assigned from worksheets for homework and would be gone over during the next lesson. Following this pattern, like in Chido and Kunatsa’s lessons three major activities dominated in most of his lessons and are described in more detail. The researchers have called these direct teaching with the use of examples, going over problem tasks, and working on problem tasks. The activity of presenting concepts and principles to the class for the first time was characterized as direct teaching. Tafara would proceed by making some demonstration on the chalk board after which students would be asked to solve problems on work cards. The cards had three problems and students were asked to work out the solutions individually or in pairs.

Something unique was that the teacher attempted to use group work methods to provide students with the opportunity to use group discussions as a way of constructing knowledge and assume ownership of the learning process. For example to Solve the following equation:

\[ \frac{4x - 5}{7} = 1 \frac{3}{4} \]

The conversation between the teacher and the group members:

Tafara: Yes let us see how you did this problem. (What is the procedure?)
Student 1: I think we first change the right-hand side to an improper fraction.
Tafara: Yes, how do you covert it?
Student 2: We multiply 4 by 1 then add 3.
Tafara: What is the next step?
Student 1: We then cross multiply and then group the like terms together.
Tafara: Can you demonstrate what you are saying.

Student 2 demonstrated while others were watching. In the discussion, there was evidence of the teacher scaffolding asking leading questions that provoked students to think until they came up with the solutions.

In another lesson where Tafara was teaching word problem that involve fraction equations; an application problem was used as an example as follows:

If Mike can paint his room in ten hours and Susan can paint the same room in seven hours, how long would it take them to paint the room working together?

The idea of representing the part of the task that can be done by a single person in one unit of time is a multiplicative reasoning approach. For example if Mike takes 10 h to paint the room alone, then to paint one unit of the room he will take \( \frac{1}{10} \) h. Similarly, \( \frac{1}{7} \) is the time Susan will take to paint one unit of the room and \( \frac{1}{x} \) is the time they will take if they work together. The multiplicative reasoning idea simplifies the whole problem and it can be solved without any difficulties.

Tafara concluded the lesson by outlining the steps necessary for solving equations with fractions.

Steps for solving fraction equations

1) First find the least common multiple (LCM) of all the denominators in the equation.
2) Multiply all the terms with the LCM (Now the equation doesn't have any fractional terms).
3) Simplify the equation.
4) Re-arrange the equation by bringing all the variables to one side and numbers to one side.
5) Solve the equation for the variable.

Tafara also used questions as an assessment strategy. Tafara’s teaching can be characterized generally as teacher dominated, and teacher directed though in very few times he blended its work activities. Also like Chido and Kunatsa, Tafara cited examination driven curriculum, and limited resources as the major drivers of his teaching of algebraic fractions. On closer interpretation of data, the researchers inferred that the reasons cited are on the surface.

Results from Interviews

The following results came out from post lesson interviews conducted by the researchers. Generally three factors emerged as participants explained the reasons for conducting their lessons the way they did. These factors were time constraints, teachers’ belief and examination oriented culture.

Case units reported time as the major constraint. From the teachers: doing group work or individual work in the classroom could be time consuming. There are fixed amount of syllabus to be covered within limited teaching time. As a result teachers resort to teacher centred methods while the student listens. The teachers acknowledged that student based activities require teachers to spend much time on planning and searching for ideas and resources. Very often teachers do not have sufficient time to plan their lessons because they are tied down by heavy workloads. Teacher centred methods are preferred as they require lesser preparation time.

Teachers believe that giving clear explanations with suitable examples (exemplification) is practical and sufficient to achieve most teaching objectives. It is time consuming to allow students to construct their own knowledge through student based activities. Furthermore they are not confident whether their students have acquired enough knowledge and skills, if the students were allowed to explore by themselves. As a result the teachers feel more comfortable with controlling the teaching and learning pace of students. Also the common belief of ‘practice makes perfect’ was noted during lesson observations as teachers gave students large amount of work to do individually as practice exercises and as homework. The belief of practice makes perfect is prevalent in many teachers.

Examination oriented culture is prevalent in Zimbabwean society. Examination results (public examinations) are used as a yard stick or accountability of school performance. From the education minister to the students’ parents, everyone is anxious about their children’s examination performance. As a result, it is common for school heads to use student performance in examinations as a yard stick to evaluate the teachers’ teaching competency. Consequently, this has strengthened many teachers’ belief that their teaching priority is to ensure that students achieve good results in examinations.

Summary of study

A case study was carried out to investigate the teaching of fractions at Ordinary level at a high school in Bindura District, Zimbabwe. Purposive sampling was implemented in coming up with the sample for the study. The sample consisted of three mathematics teachers drawn from Radcliff High School. Document analysis, lesson observations and individual interviews were the instruments used to collect data to answer the following research questions:

1) How do teachers teach fraction concepts?
2) Why do teachers teach fraction concepts the way they do?

Conceptual framework of the emergent perspective, which deals with formal teaching instigated to enable learning to take place, guided this research study. It involved teaching phenomenon in the classroom like evidence of teaching and learning process within the context of the classroom culture. A technique of constant comparative was implemented to the findings as the process of data collection progressed. Post data analysis interpretation and discussion led to the findings that teachers promote procedural understanding of concepts at the expense of conceptual understanding of knowledge. The traditional methods which are anchored on practice of problem tasks, going over problems and exemplification (teaching by giving examples), drill and teaching of rules and algorithms promote procedural understanding of fraction concepts. However this does not mean students would have developed critical thinking. The mathematics teachers were inclined to the use of traditional whole class teaching strategies and to dominate classroom interaction.

The following reasons for the way fractions are taught emerged from the data. As justification of their strategy in the teaching of fractions teachers cited shortage of time to prepare for the lessons, large class sizes, examination driven curriculum and limited resources. This type of teaching promotes the acquisition of procedural knowledge which underplays the development of multiplicative reasoning skills in students.

Conclusions

The aforementioned illustrative cases described open a
window into real classrooms providing detail of the teachers' experiences of teaching fraction concepts. Teachers in the teaching of fractions employed direct teaching strategy that promotes procedural knowledge and down played conceptual understanding of concepts. The reasons for their teaching that emerged from the study were time constraints, teachers' beliefs and examination oriented culture. There is a wide curriculum gap between the expectation of the syllabus and what actually happens in the classroom as has been revealed by the study.

The following recommendations are proposed to the teaching of fractions at Ordinary level:

1) Staff development workshops and seminars to equip teachers with skills which will enable them to employ child centred teaching strategies that may result in the conceptual understanding of concepts.
2) If teachers make sure they provide a sound definition of a fraction and provide additional time for student exploration with fractions they may find that their students perform better.
3) Teachers should encourage students to generate their own examples as this will help them understand concepts better.
4) It is important for teachers to prove increased time on the topic to give students enough time to conceptualize the concepts.
5) Assessment should be school based on the process of teaching rather than the outcome of teaching, for example, the assessment criteria of the national syllabus should not be centred on public examinations but should incorporate a component of continuous assessment to encourage teachers to use appropriate teaching strategies to meet the curriculum goals of relevant teaching.

REFERENCES


