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Relationship between problem-solving style and mathematical literacy

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Currently, mathematics education is focused on ensuring that students can apply the knowledge and skills they learn to everyday life; students are expected to develop their problem-solving abilities to face challenges by adopting various perspectives. When faced with a problem, students may employ different methods or patterns to solve it. If this assertion is true, then how are the various types of problem-solving styles related to mathematical literacy? This survey was conducted to investigate this critical and noteworthy topic. Research data were obtained from the 2012 Programme test for International Student Assessment in Taiwan, taken by 15-year-old students. Latent class analysis (LCA), which is appropriate for identifying otherwise unobservable subgroups within a population, was conducted to determine how students respond to problem-solving scenarios and identify patterns of association in their problem-solving styles. The results of the LCA reveal that the 3-class model attained the best fit to the data. The students identified as independent group members attained the highest mathematical literacy, followed by those identified as resource-dependent group members and those identified as passive-dependent group members.

Key words: Mathematical literacy, latent class analysis, problem-solving style.

INTRODUCTION

Mathematics is a basic and critical foundation subject. From a formal educational perspective, mathematics is taught from first grade through high school. The purpose of learning mathematics may be to enter higher education or prepare for employment. From an informal education perspective, people first learn basic mathematics from family members for application in daily life before entering school; for example, learning the names of numbers and how to count and use money. Mathematics is not only learned in school but applied in activities of daily living; therefore, methods of connecting school mathematics and life has become a priority in designing school mathematics curricula.

In recent decades, problem-solving ability has been regarded as a critical aspect in mathematics education. In Taiwan, the Grade 1–9 Curriculum Guidelines for Mathematics states that the purpose of mathematics curricula is to cultivate students' abilities in commutating,

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abstracting, reasoning, and communicating, and students are expected to build mathematical foundations for their next stage of learning, learn to apply knowledge and skills to solve problems, and develop a healthy disposition toward mathematics (Taiwan Ministry of Education, 2003). The National Council of Teachers of Mathematics (NCTM, 2000) indicated that problem-solving ability is an integral part of all mathematics learning. In everyday life and in the workplace, the ability to resolve problems can yield considerable advantages. However, solving problems is not only a goal of learning mathematics but a major means of learning mathematical concepts. NCTM (2000) emphasized that instructional programs from prekindergarten through Grade 12 should enable students to build new mathematical knowledge through problem solving, thereby enabling them to solve problems that arise in mathematics and other contexts, apply and adapt various strategies to solving problems, and monitor and reflect on the process of mathematical problem solving. Trends in mathematics education indicate that students must develop various problem-solving strategies, and that such strategies require instructional attention if students are to learn them.

The Programme for International Student Assessment (PISA) test is a triennial international survey aimed at evaluating education systems worldwide by testing the skills and knowledge of 15-year-old students. Since 2000, students have been randomly selected from schools worldwide to sit tests in reading, mathematics, and science. The PISA mathematical literacy domain is concerned with evaluating students' capacities to analyze, reason, and communicate ideas effectively when posing, formulating, solving, and interpreting solutions to mathematical problems in various domains and situations. By focusing on real-world problems, the PISA is not limited to situations and problems typically encountered in school classrooms. Based on this type of assessment approach, problem-solving style would be a central role for mathematical literacy. The styles of problem solving suggest that systematic differences exist in individuals' natural or habitual pattern of acquiring and processing information in problem-solving situations. A core concept is that individuals differ in how they handle such problems. If this assertion is true, how are the various problem-solving styles related to the students' mathematical literacy?

Few studies have explored the relationship between problem-solving style and mathematical literacy. Furthermore, student attitudes toward problems may be a critical factor influencing their mathematical literacy. Hence, this topic warrants investigation.

Based on the aforementioned reasons, this study was conducted to identify 15-year-old Taiwanese students' patterns of problem solving and how these patterns are related to mathematical literacy.

LITERATURE REVIEW

Mathematical literacy

The PISA definition of mathematical literacy is as follows:

An individual's capacity to formulate, employ and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain and predict phenomena. It assists individuals in recognizing the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens. (Organisation for Economic Co-operation and Development [OECD], 2014a, p. 37).

Based on this definition, the PISA assessment approach differs considerably from those adopted by, for example, the Trend International Mathematics and Science Study (TIMSS), which is focused on what students have learned at school. The TIMSS test development process places considerable emphasis on covering the curricula of participating countries and uses a detailed scheme based on traditional curriculum content strands to describe national curricula. However, school mathematics is often offered to students as a strictly compartmentalized science, with a particular overemphasis on computation and formulas (OECD, 1999, p. 14). Although acquiring specific knowledge is critical in school learning, the PISA emphasizes that applying such knowledge in adult life depends crucially on an individual's acquisition of broader concepts and skills; in mathematics, reasoning quantitatively and representing relationships or dependencies are more critical than the ability to answer familiar textbook questions to the ability to deploy mathematical skills in everyday life. The metric for the overall mathematics scale is based on a mean for OECD countries of 500 points and a standard deviation of 100 points that were set in PISA 2003 when the first PISA mathematics scale was first developed.

To date, students representing more than 70 economies have participated in the PISA (OECD, 2015). The PISA tests are designed to assess the extent to which students, nearing the end of compulsory education, can apply their knowledge to real-life situations and are equipped to fully participate in society. The information collected through background questionnaires also provides context that can assist analysts in interpreting the results. The number of studies using PISA data to investigate critical educational issues is increasing. For example, Papanastasiou and Ferdig (2006) analyzed PISA 2003 data and showed that the different types of activities performed on computers are associated with different levels and types of thinking, which in turn are associated with distinct results. Chiu and Xihua (2008) examined how family and motivational factors affect student achievements in mathematics across 41 countries by performing multilevel analyses, the results of which showed that students scored higher when living in more economically developed or egalitarian countries, with two parents, without grandparents, and with fewer siblings (particularly older siblings); when their family has a higher socioeconomic status, more books, more cultural possessions, or uses more diverse forms of cultural communication; or when they have greater interest in mathematics, exerted more effort, exhibited more perseverance, and demonstrated higher self-efficacy or self-concept.

The PISA is unique because the tests are not directly linked to school curricula. Students cannot learn everything in school that they will need to know in adult life. What they must acquire is the prerequisites for successful learning in future life. Such prerequisites are of both cognitive and motivational in nature. Students must develop the ability to organize and regulate their own learning, learn independently and in groups, and overcome difficulties in the learning process, which requires them to be aware of their own cognitive processes and learning strategies and methods; therefore, this study explored the relationship between problem-solving styles and mathematical literacy.

Problem solving

PISA 2012 measured students' capacity to respond to nonroutine situations to fulfill their potential as constructive and reflective citizens. In PISA 2012, problem-solving competency was defined as follows:

An individual's capacity to engage in cognitive processing to understand and resolve problem situations where a method of solution is not immediately obvious. It includes the willingness to engage with such situations in order to achieve one's potential as a constructive and reflective citizen (OECD, 2014b, p.30).

The PISA 2012 test assessed individuals' problem-solving competency. A consistent research finding was that expert problem-solving ability depended on domain-specific knowledge and strategies (e.g. Mayer, 1992; Funke and Frensch, 2007; OECD, 2013, p. 120). To measure the cognitive processes fundamental to problem solving, the PISA 2012 problem-solving assessment avoided the need for expert knowledge as much as possible. This approach distinguished the assessment from problem-solving tasks in the core PISA literacy domains of reading, mathematics, and science, all of which incorporate expert knowledge in these areas.

The central feature of the PISA 2012 problem-solving assessment was that the problem situations were authentic and relatively complex—particularly those that require direct interaction by the student to uncover relevant information. Examples include problems commonly faced when using unfamiliar everyday devices such as remote controls, personal digital devices (e.g., mobile phones), home appliances, and vending machines. Problem-solving skills are necessary to attain more than a basic skill level when handling such situations; however, studies have indicated that more skills, in addition to those involved in traditional reasoning-based problem solving, are required (Klieme, 2004; OECD, 2013, p. 121).

METHOD

Data

The data used in this study were derived from the Taiwan data available from the PISA 2012 website. The PISA is administered triennially to a randomly selected group of 15-year-old students in the subjects of mathematics, reading, and science. At each assessment, one subject is given special focus. The area examined in this study is mathematical literacy because it was the subject that was emphasized in PISA 2012. Three forms (A, B, and C) of student questionnaire were designed to obtain broader and more comprehensive information about factors related to student performance, attitudes, and behaviors, and the functioning of education systems (e.g., demographic variables, previous educational career choices, instruction time, and class size). Therefore, not every student who participated in PISA 2012 responded to the scale of problem-solving experiences. After those who did not respond to the scale were excluded and the students whose data were complete were weighted to ensure that each sampled student appropriately represents the correct number of students in the full PISA population, 193,370 Taiwan students were enrolled in this study.

Problem-solving survey

The student questionnaire was administered after the literacy assessment and required approximately 30 min to complete. Problem-solving experience is one of the aspects in the student questionnaire. The PISA problem-solving experiences scale comprised five units, three situational units for specific situations such as "being unable to send text message from a mobile phone," "planning a trip to the zoo with your brother," and "buying a ticket from a machine that you have never used before," and four items for each unit. The PISA asks students to report whether they would *definitely do this*, *probably do this*, *probably not do this*, or *definitely not do this*.

This study used a latent class analysis (LCA) to explore the student problem-solving styles according to these three situational items. Problem-solving styles are consistent individual differences in the ways people prefer to handle new concepts, manage change, and respond effectively to complex open-ended opportunities and challenges.

Analysis methods

LCA was used to determine whether the criteria attained the best fit

Table 1. Summary of LCA criteria in each class model.

Model	AIC	BIC	Adjusted BIC	Entropy
2-class	43557.21	43714.512	43635.074	0.73
3-class	43274.73	43513.832	43393.085	0.71
4-class	43037.3	43358.209	43196.154	0.58

to a categorical model. To derive the optimal categories or problem-solving styles from the LCA, we viewed the responses definitely do this or probably do this as positive responses (coded as "1"), and the responses definitely not do this or probably not do this as a negative response (coded as "0").

LCA models associate observed categorical variables with latent categorical variables and identify a categorical latent class variable measured by numerous observed response variables. The objective is to categorize people into classes by using the observed items and identify the items that best distinguish between the classes. For LCA models with categorical outcomes, the item parameters correspond to the conditional item probabilities that are class-specific and provide information on the probability of an individual in a class endorsing an item. The class probability parameters specify the relative prevalence (size) of each class (Nylund et al., 2007, p. 539).

This study used Mplus software to perform the LCA and SPSS to perform the descriptive statistics and analysis of variance (ANOVA) for examining the mathematical literacy of students with different problem-solving styles.

RESULTS

Best-fit model of LCA

Currently, researchers apply a combination of criteria to guide decisions pertaining to the number of classes in LCA modeling. Such criteria include the Akaike information criterion (AIC), Bayesian information criterion (BIC), the adjusted BIC and entropy are widely used for model selection. The model with smaller AIC, BIC, and adjusted BIC values and a larger entropy value was the optimal choice.

The results in Table 1 indicate that two classes outperformed a single class and three or four classes was superior to two classes. The AIC, BIC, and adjusted BIC values were smaller for the four-class model, but the entropy was not good enough. The AIC, BIC, and adjusted BIC improved for each additional class, though substantial decreased existed between two and three classes and between three and four classes. The entropy was adequate for the two- and three-class models, but decreased noticeably for the four-class models.

The LCA results revealed that a three-class solution was the optimal fit, and we observed that the figure for the three-class solution was reasonable; therefore, we selected a three-class solution.

Table 2 and Figure 1 present the students' probabilities in each category for the individual indicators. The conditional probability for Class 1 students responded for Item 1 in Situation A is 0.893, implying an 89.3% probability of Class 1 students responding positively to the item "I press every button possible to find out what is wrong."

We can use the probability in each category for individual indicators to assign a meaningful label to each class. The largest class (Class 3, 79%) of students used various resources, whether human or written information. According to the characteristics of these students, Class 3 was termed the "resource-dependent group."

The middle class (Class 1, 12%) favored solving problems independently. These students were more willing to try new things and less afraid of making errors than the other two groups were; therefore, we named Class 1 the "independent group."

The students in the smallest class (Class 2, 9%) were less likely to actively solve problems they encountered and tended to rely on people around them to assist them in solving problems; therefore, this group was named the "passive-dependent group."

Mathematical literacy according to problem-solving style

Particularly noteworthy results of this study were the problem-solving styles derived from the LCA. Table 3 shows the descriptive statistics of mathematical literacy for the three classes of problem solving. The means of three types indicate that the independent group attained the highest level of mathematical literacy (M=596.01), followed by the resource-dependent group, and then the passive-dependent group. One-way ANOVA was then conducted to investigate the relationship between the latent classes and mathematical literacy by conducting a mean difference test across the classes. The results in Table 4 show that the mean difference test across the classes was statistically significant (F=5034.59, p<.01). The measure of association strength η^2 was 4.9%, implying that the problem-solving styles accounted for approximately 5% of the variance in mathematical literacy. The results of a post hoc test indicate that the students who solved problems independently attained significantly higher scores than did those in the resource-

Table 2. Conditional probabilities of each item and latent class probability on the problem-solving scale for 3-class model.

Situation	item	Class 1	Class 2	Class 3
A. Suppose that you have been sending text messages from your mobile phone for several weeks. Today, however, you can't send text messages. You want to try to solve the problem.	1. I press every button possible to find out what is wrong.	0.893	0.408	0.889
	2. I think about what might have caused the problem and what I can do to solve it.	0.994	0.439	0.967
	3. I read the manual.	0.495	0.246	0.759
	4. I ask a friend for help.	0.643	0.661	0.937
B. Suppose that you are planning a trip to the zoo with your brother. You don't know which route to take to get there	1. I read the zoo brochure to see if it says how to get there.	0.891	0.577	0.974
	2. I study a map and work out the best route.	0.806	0.494	0.907
	3. I leave it to my brother to worry about how to get there.	0.296	0.428	0.331
	4. I know roughly where it is, so I suggest we just start driving.	0.535	0.456	0.59
C. Suppose that you arrive at the train station. There is a ticket machine that you have never used before. You want to buy a ticket.	1. I check how similar it is to other ticket machines I have used.	0.911	0.54	0.921
	2. I try out all the buttons to see what happens.	0.481	0.328	0.319
	3. I ask someone for help.	0.275	0.609	0.894
	4. I try to find a ticket office at the station to buy a ticket.	0.668	0.746	0.969
Class Probability		0.12	0.09	0.79

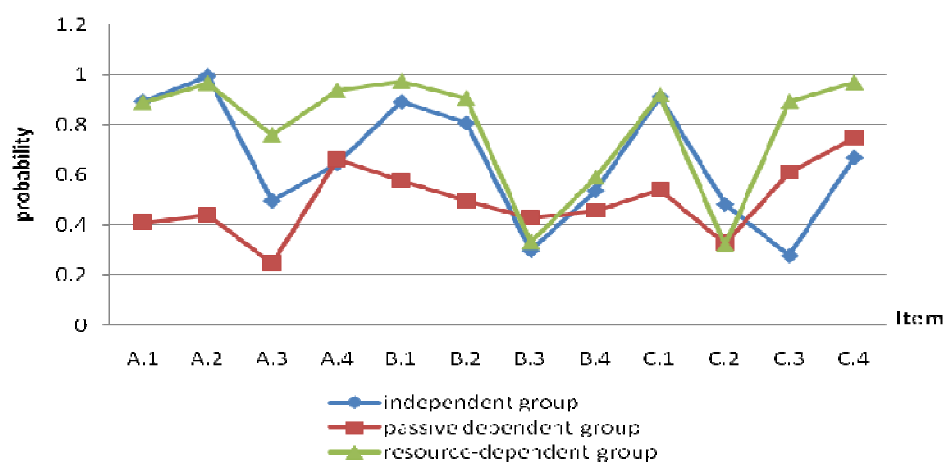

Figure 1. Distribution of conditional probabilities of each item for 3-class model.

and passive-dependent groups, and that the resource-dependent group solved problems more effectively than did the passive-dependent group.

The results of this study demonstrate that students use different problem-solving styles, and the three styles identified in this study can be considered three types of attitude toward solving problems. Ma and Kishor (1997) conducted a meta-analysis to investigate the positive relationship between attitude toward mathematics and achievement in mathematics. The present study found that problem-solving style plays an important role in mathematical literacy.

Table 3. Descriptive statistics of mathematical literacy for the three classes of problem solving.

Type	N	Mean	SD
independent group	24087	596.01	108.28
passive dependent group	16288	484.12	123.51
resource-dependent group	152995	563.52	111.59

Table 4. ANOVA Results of Mathematical Literacy for Different Problem-Solving Types.

Source	SS	df	MS	F	p	η^2
Between	126869944.16	2	63434972.08	5034.59	0.000	.049
Within	2435871759.59	193367	12597.14			
Total	63393722894.71	193370				

Conclusion

Students cannot learn everything in school that they will need to know in adult life. Therefore, understanding the patterns of problem solving and how they are related to literacy can assist most educators in designing tailored interventions targeting the subgroups of different problem-solving style. In this study, data from the Taiwan PISA 2012 survey and LCA revealed that the three-class model attained the optimal fit to the students' problem-solving styles. The three problem-solving styles were termed "independent," "passive dependent," and "resource dependent." The results of a mean difference test indicate that the average mathematical literacy of the independent group was the highest, followed by the resource-dependent group and passive-dependent group. The mean difference in mathematical literacy scores between the highest and lowest groups was approximately 110, which is nearly one standard deviation for Taiwan. The mean difference in score between the resource- and passive-dependent groups was approximately 80. Overall, the mathematical literacy of the passive-dependent group was considerably lower than that of the two other groups. Hence, students who do not adopt an active problem-solving attitude have poorer mathematical literacy than do those who adopt an active problem-solving attitude. The three groups exhibited distinct attitudes toward problem solving; educators could further understand each group's characteristics according to their conditional probabilities, and educators could design effective instructions to improve student attitudes according to the advantages and disadvantages of the group students are in. Both the resource- and passive-dependent groups tended to seek assistance from others, though the resource-dependent group was more active in solving problems compared with the passive-dependent group. Educators could encourage and assist passive-dependent students in fostering a positive attitude toward solving problems.

The average mathematical literacy score of the independent group was approximately 33 points higher than that of the resource-dependent group. Many similar tendencies existed in the responses of these two groups. However, the independent group tended to think independently, use their own knowledge and skills to solve problems, and was more tolerant of adopting a trial-and-error approach. Thus, this study suggests that educators should encourage resource-dependent learners to engage actively in problem-solving activities and exploit available resources—more importantly, they should encourage these students to attempt to solve problems by themselves.

Identifying the problem-solving styles of students as a basis for providing responsive instruction has never been more critical, with educators increasingly expected to promote their students' mathematics performance effectively. If instruction can help students to adopt effective problem-solving orientation, the students will be able to achieve higher levels of performance and with a more positive attitude towards learning while learning difficult subjects.

This study investigated 15-year-old students from Grades 9 and 10; future research should consider investigating the differences in problem-solving skills between Grade 9 and 10 students or between girls and boys, thereby obtaining a more comprehensive understanding of 15-year-old students' attitudes toward solving problems.

Conflict of Interests

The authors have not declared any conflict of interests.

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