A non-econometric analysis with algebraic models to forecast the numbers of newly hired and retirement of public primary school teachers in Taiwan

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In recent years, the "street teachers" issue has caused social concern in Taiwan. This study estimates the retirement of and needs for newly hired and public primary school teachers in 2010 using an algebraic model from the paper by Hussar (1999). This recursive methodology predicts the number of newly hired public primary school teachers due to teachers leaving the work force, and due to the predicted enrollment with age-specific continuation rates of teachers. The age-specific continuation rates were calculated from the Yearbook of Teacher Education Statistics, Ministry of Education to predict how many teachers will continue teaching from one year to the next. We assumed the age distribution of new teachers would be the same as the 2009 distribution. Depending on the assumptions, we estimate there will be 41 to 1491 newly hired public primary school teachers by 2010 and about 1662 teachers will retire.

Key words: Primary school teachers, newly hired teachers, retirement teachers, algebraic model, age-specific continuation rate.

INTRODUCTION

Following major education reform in 1994, teacher education has changed from a closed-door policy to a mechanism of open competition. Although the results shows there is a positive effect of democratic pluralism, it has also caused some issues, including "street teachers" problems (Li, 2007). "Street teachers" are unemployed teachers who hold a teacher certificate but have not yet been employed in a permanent teaching position. According to the Yearbook of Teachers Education Statistics, the Republic of China in 2009, the cumulated number of teachers prepared and certified under Teacher Education Act with specializing in primary school education is 62,733. The number of in-service teachers (with permanent position) with specialty in primary school education is 29,684, which is 47.32% out of certified teachers in this specialty. This means more than half are "street teachers" who could not get a permanent teaching position. This issue has caused social concern.

In addition, the birth rate of Taiwan is decreasing year by year and last year was among the world’s lowest. According to estimates by the CIA World Factbook, the crude birth rate of 8.3 newborns per 1,000 people last year puts Taiwan above only Germany, Hong Kong, Italy, and Japan (Ralph, 2010). The Ministry of Education pointed out; for the next seven years, Taiwan will reduce 99,615 classes and if there is 1.5 teachers per class in primary school, that will reduce the number of teachers by about 15,000 (Chang, 2005). Moreover, Taiwan has faced an economic downturn in recent years that have made teaching positions more attractive due to their perceived stability. These factors will result in imbalanced
supply and demand and worsen the issue of "street teachers".

The retirement of public primary school teachers is divided into voluntary retirement and compulsory retirement. According to Article 4 to 6 under the Civil Service Retirement Act (Ministry of Civil Service, 2010); the conditions for voluntary retirement are the teacher teaching more than 5 years and over 60 years of age, or 25 years of service. The conditions for compulsory retirement are teachers with more than 5 years of service and over 65 years old, or over 5 years of service and not competent for teaching due to mental or physical disadvantages. Therefore, the expected retirement ages are 50 to 54. In 2004, due to the Ministry of Education increasing the teachers' retirement funds; the number of retired teachers increased to 9,033 but dropped to 5,522 in the following year. According to the Ministry of Education Statistics, the estimate of primary school teachers eligible for retirement is around 2000 within next 5 years (Lin, 2007). Therefore, the demand for retirement has been reduced. However, the number of retired teachers depends on the financial capacity of local government. This will affect the retirement system for teachers as less financial capacity means fewer job will be available.

The official statistics on teacher education given by the Ministry of Education are normally published in August of the following year. Therefore, this study aims to estimate the needs for newly hired and retirement of public primary school teachers in 2010 using an algebraic model, \( T_2 = C_2 + A_2 \) from the paper by Hussar (1999); where \( T_2 \) is the total number of teachers in year 2; \( A_2 \) is the number of newly hired teachers in year 2; \( C_2 \) is the number of continuing teachers in year 2. This recursive methodology is used to predict the number of newly hired public primary school teachers because of teachers leaving the work force, and because of the predicted enrollment. Also, this methodology is used to examine the impact that the existing age distribution of teachers will have on the composition of the teacher force.

The rest of the article is structured as follows. First, the brief research goals, notations, and assumptions used in this study are given. Second, the utilized models, formulas, and data sources are explained, followed by the results and findings. The last section concludes with a summary and discussion of this study.

**Aims of the study**

The purposes of this study are to predict the number of newly hired public primary school teachers in Taiwan as follows:

i) To estimate the need of new public primary school teachers in 2010 according to two scenarios (constant pupil/teacher ratio and constant number of teacher) with two continuation rates (2008 and 2009).

(ii) To find the best scenario by comparing the actual data and estimation of public primary school teachers that will be needed in 2009.

(iii) To understand the age distribution of the public primary school teachers.

(iv) To estimate the retirement of public primary school teachers.

**Notations**

- \( T_t \): total number of teachers for each year \( t \).
- \( T_{1,a} \): numbers of teachers by individual age \( a \) for each year \( t \).
- \( C_a \): numbers of teachers who taught in the last year who continued teaching in year \( t \).
- \( C_{1,a} \): numbers of continuing teachers of each age \( a \) in year \( t \).
- \( C_{2,a} \): for each age, we have a continuation rate, in year \( t \).
- \( A_{1,a} \): the number of newly hired teachers. (Newly graduated teachers; teachers who had not taught in the last year, but had taught before that; and those who had graduated in prior years, but had never taught before.)
- \( A_{2,a} \): the number of newly hired teachers by age in year \( t \).
- \( R_{1,a} \): is the ratio of the number of newly hired teachers of age \( a \) in year \( t \) to the total number of newly hired teachers in year \( t \). That is, \( R_{1,a} = \frac{A_{1,a}}{A_{2,a}} \).
- \( L_{1,a} \): The total number of individuals of age \( a \) who had taught last year but were not teaching by year \( t \).
- \( P_{2,a} \): The number of people of age \( a \) who had taught last year but who were retired in year \( t \).

**Assumptions**

The assumptions made in this study are:

(i) It is assumed there will be enough supply to meet the demand, which reflects historical precedent.

(ii) For the first scenario, the previous year pupil/teacher ratio is used and is assumed to remain constant to predict the number of public primary school teachers for the next year.

(iii) For the second scenario, it was assumed the number of teachers remained the same as the last year.

(iv) For the third scenario, to predict the number of teachers, the ratio of the previous two years remains constant to the next year.

(v) The continuation rates will be stable over time;

(vi) The age distribution of newly hired teachers will be stable over time.

**MODELS AND DATA**

There are two models in this study; one estimates the number of teachers, by age, who continued teaching from one year to the next through the use of age-specific continuation rates. The other model
estimates the number of teachers retired with age-specific continuation rates.

Data sources

The Newly Hired Teachers Model requires four data items:

1. The number of public primary school teachers by age for a recent year;
2. The total number of public primary school teachers for each year under study including both historical years and forecast years;
3. An estimate of the continuation rate for each age;
4. An estimate of the age distribution of the newly hired teachers.

The past years (2007 to 2009) of the number of public primary school teachers and by age, age-specific retired rates and age-specific continuation rates are calculated from the Yearbook of Teacher Education Statistics, Ministry of Education (Tai, Kuo, Yang, Wei, 2009).

The forecast years of the number of public primary school teachers using the first scenario, which required a constant pupil/teacher ratio and number of enrollments to calculate, are from the Department of Statistics, Ministry of Education (Ministry of Education, Taiwan, 2010).

Formulas

The equation for the number of teachers in year two is:

\[ T_2 = C_2 + A_2 \]

(1) To rearrange the equation for the number of newly hired teachers, \( A_2 \):

\[ A_2 = T_2 - C_2 \]

where the number of teachers who taught in year 1 and who continued teaching in year 2 is:

\[ C_2 = \sum C_{2,a} \]

In year 2 the continuation rate is:

\[ C_{2,a} = C_{2,a}/T_{1,a-1} \]

Number of continuing teachers of each age ‘a’ in year 2:

\[ C_{2,a} = c_{2,a}T_{1,a-1} \]

Therefore, the total number of continuing teachers in year 2, \( C_2 \), can be estimated:

\[ C_2 = \sum C_{2,a} = \sum c_{2,a}T_{1,a-1} \]

The number of newly hired teachers by age, \( A_{2,a} \), can be estimated:

\[ \Sigma A_{2,a} = \Sigma R_{2,a}A_2 \]

In year 2, the ratio of \( R_{2,a} \) is:

\[ R_{2,a} = A_{2,a}/A_2 \]

the number of newly hired teachers by age in year 2:

\[ A_{2,a} \approx R_{E,a}A_2 \]

where \( R_{E,a} \) is the estimate of the ratio of the number of newly hired teachers of age a to the total number of newly hired teachers.

(2) The number of people of age \( a \) who had taught in year 1 but who were retired in year 2, \( P_{2,a} \)

\[ P_{2,a} = p_{E,a}L_{2,a} \]

Where the total number of individuals of age ‘a’ who had taught in year 1 but were not teaching by year 2:

\[ L_{2,a} = (1 - c_{2,a})T_{1,a-1} \]

Estimate the number of retirees by individual age

\[ P_{2,a} = p_{E,a}L_{2,a} \]

RESULTS AND DISCUSSION

The results and discussion are as follows, for more details on the calculation please refer to Appendix.

Newly hired teachers

Table 1 shows the number of newly hired public primary school teachers needed for 2009 and 2010, according to the continuation rates used and teacher total assumption. For the estimation of 2010, the model projects 41 and 165 newly hired public primary school teachers will be needed using scenario 1 according to the continuation rate of 2008 and 2009, respectively; we estimate there will be 1367 and 1491 needed using scenario 2 according to the continuation rate of 2008 and 2009 respectively.

Comparing the actual data and estimation of public primary school teachers that will be needed in 2009 according to different scenarios, Scenario 1 underestimated by about 738; scenario 2 overestimated by about 924. Therefore, scenario 1 was the closest to the actual data.

Table 2 shows the newly hired public primary school teachers and the graph is shown in Figure 1. Table 2 and Figure 1 show there was a decreasing trend of the number of newly hired public primary school teachers needed for the past three years (2007 to 2009). Therefore, we assumed the estimation of 2010 by scenario 1 was the most accurate.

Changing age distribution of primary school teachers

To compare the results for the alternative scenarios, we looked at the age distributions projected for the three scenarios. Figure 2 shows the estimated number of public primary school teachers for each age group. Since the distributions are so similar, the following analysis will concentrate on the results for scenario 1, the constant pupil/teacher ratio.

Table 3 shows the total number of public primary school teachers by age group, and the predicted value in 2010 is used for scenario 1. The constant pupil/teacher ratio with
Table 1. Number of newly hired public primary school teachers needed for 2009 and 2010, according to the continuation rates used and teacher total assumption.

<table>
<thead>
<tr>
<th>Year</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constant pupil/teacher ratio</td>
<td>Constant no. of teachers</td>
</tr>
<tr>
<td>Actual 2009 data</td>
<td>237</td>
<td>237</td>
</tr>
<tr>
<td>Estimate 2009 according to the continuation rate of 2008</td>
<td>-501</td>
<td>1,161</td>
</tr>
<tr>
<td>Estimate 2010 according to the continuation rate of 2008</td>
<td>41</td>
<td>1,367</td>
</tr>
<tr>
<td>Estimate 2010 according to the continuation rate of 2009</td>
<td>165</td>
<td>1,491</td>
</tr>
</tbody>
</table>

the 2009 continuation rate and the graph is shown in Figure 3. Table 3 and Figure 3 show the predicted and actual age distributions of public primary school teachers are similar. The age distributions of teachers sharply increased from age 22 to 34, and then show a more equal distribution from age 30 to 44. After that, there is an expected decrease in public primary school teachers over time.

Retirement of public primary school teachers

The number of retired public primary school teachers for the years 2007 to 2010 and the graph are shown in Table 4 and Figure 4. It shows a decreasing trend of the number of retired public primary school teachers for the years 2007 to 2010. Using scenario 1 and a constant pupil/teacher ratio with the 2009 continuation rate, about 1,662 teachers will retire in 2010.

Figure 5 shows the age distributions of the number of retired public primary school teachers for the years 2007 to 2010. It showed a large increase in the number of retired public primary school teachers after age 44 until the age of 50 to 54, which achieved the highest point. The age distributions appear similar, except for the age group 45 to 49 where the number of retired public primary school teachers in 2007 was much higher than other years.

CONCLUSIONS

This study aimed to examine a model for predicting the number of newly hired public primary school teachers in Taiwan. The conclusions are as follows:

(i) If the pupil-teacher ratio remains constant, we estimate there will be 41 and 165 newly hired public primary school teachers needed in 2010 according to the continuation rate of 2008 and 2009, respectively. However, scenario 2 results in higher forecasts for the need of newly hired public primary school teachers, which will be 1,367 and 1,491 according to the continuation rates of 2008 and 2009, respectively.

(ii) Comparing the actual data and estimation of public primary school teachers needed in 2009 according to different scenarios, scenario 1 was the most accurate.

(iii) Comparing the estimated number of public primary school teachers of different age distributions according to the two scenarios, the distributions appear very similar.

(iv) The predicted and actual age distributions of public primary school teachers were similar. After age 44, there was an expected decrease in the number of public primary school teachers over time.

(v) Using scenario 1 with the 2009 continuation rate, about 1,662 teachers will retire in 2010. The peak age of retirement of public primary school teachers was between 50 and 54.

This paper is not a supply and demand study. However, the continuation rate is a critical factor in the model. It can be influenced by education policymakers and economic factors. Increased salaries or other benefits could be used to help retain teachers in their positions, resulting in an increased continuation rate. In addition, an economic downturn might make teaching positions more attractive due to their perceived stability, which is currently the case in Taiwan. Taiwan tends to have fewer children and retirement rate is slow down but the teacher supply is exceed, thus our predicting models can be the references for: 1) assessing teacher supply and demand, 2) quality control of recruiting students for the universities with teacher education, and 3) making policies of teacher education for the government.

The age distribution of newly hired teachers can also be influenced by supply and demand forces. The number of qualified teachers available can vary according to changing teacher certification requirements. This would affect the age distribution of newly hired teachers, later affecting teacher demand.
Table 3. Total number of public primary school teachers by age group.

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>22-29</th>
<th>30-34</th>
<th>35-39</th>
<th>40-44</th>
<th>45-49</th>
<th>50-54</th>
<th>55-59</th>
<th>60+</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>96,142</td>
<td>14,532</td>
<td>20,408</td>
<td>23,271</td>
<td>19,084</td>
<td>11,415</td>
<td>5,471</td>
<td>1,564</td>
<td>397</td>
</tr>
<tr>
<td>2008</td>
<td>95,552</td>
<td>11,201</td>
<td>19,769</td>
<td>23,085</td>
<td>20,786</td>
<td>12,933</td>
<td>5,666</td>
<td>1,702</td>
<td>410</td>
</tr>
<tr>
<td>2009</td>
<td>94,529</td>
<td>7,974</td>
<td>19,041</td>
<td>22,438</td>
<td>22,337</td>
<td>14,326</td>
<td>6,171</td>
<td>1,787</td>
<td>455</td>
</tr>
<tr>
<td>2010</td>
<td>93,203</td>
<td>7,862</td>
<td>18,774</td>
<td>22,123</td>
<td>22,024</td>
<td>14,125</td>
<td>6,084</td>
<td>1,762</td>
<td>449</td>
</tr>
</tbody>
</table>

Figure 1. Number of newly hired public primary school teachers.

Figure 2. Estimated 2010 age distribution of public primary school teachers for two scenarios using a continuation rate 2009.

Table 3. Total number of public primary school teachers by age group.

\[
A_{99,a} = (A_{98,a} / A_{99}) \times A_{99} \quad \text{Estimate using scenario 1 with the 2009 continuation rate.}
\]
Figure 3. Age distribution of public primary school teachers for the years 2007 to 2010.

Table 4. Number of retired public primary school teachers for the years 2007 to 2010.

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>22-29</th>
<th>30-34</th>
<th>35-39</th>
<th>40-44</th>
<th>45-49</th>
<th>50-54</th>
<th>55-59</th>
<th>60+</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>1930</td>
<td>40</td>
<td>55</td>
<td>40</td>
<td>50</td>
<td>589</td>
<td>732</td>
<td>265</td>
<td>159</td>
</tr>
<tr>
<td>2008</td>
<td>1709</td>
<td>25</td>
<td>42</td>
<td>41</td>
<td>39</td>
<td>377</td>
<td>777</td>
<td>289</td>
<td>119</td>
</tr>
<tr>
<td>2009</td>
<td>1679</td>
<td>13</td>
<td>33</td>
<td>31</td>
<td>36</td>
<td>484</td>
<td>686</td>
<td>272</td>
<td>124</td>
</tr>
<tr>
<td>2010’</td>
<td>1662</td>
<td>11</td>
<td>30</td>
<td>30</td>
<td>37</td>
<td>477</td>
<td>682</td>
<td>272</td>
<td>124</td>
</tr>
</tbody>
</table>

1 Estimate using scenario 1 with the 2009 continuation rate.

Figure 4. Number of retired public primary school teachers for the years 2007 to 2010.
Figure 4. Number of retired public primary school teachers for the years 2007 to 2010.

Figure 5. Number of retired public primary school teachers by age group for the years 2007 to 2010.

REFERENCES


### APPENDIX

#### Table 1. \( T_1 \)

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual data</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Constant pupil/teacher ratio (^{1})</td>
<td>Constant no. of teacher (^{2})</td>
</tr>
<tr>
<td>2007</td>
<td>96,142</td>
<td>96,142</td>
<td>95,552</td>
</tr>
<tr>
<td>2008</td>
<td>95,552</td>
<td>95,552</td>
<td>95,552</td>
</tr>
<tr>
<td>2009</td>
<td>94,529</td>
<td>93,890</td>
<td>94,529</td>
</tr>
<tr>
<td>2010</td>
<td>93,203</td>
<td>93,203</td>
<td>93,203</td>
</tr>
</tbody>
</table>

\(^{1}\) Projections for 2009 and 2010 were produced by dividing the school enrollment projections from the Department of Statistics, Ministry of Education by the 2008 and 2009 pupil/teacher ratio, respectively. Assume the ratio of public and private school teachers remains constant by using the 2008 and 2009 ratios. 

\(^{2}\) Projections for 2009 and 2010 equal the number for the previous year.

#### Table 2. \( T_{1,a} \)

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>Age group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>22-29</td>
</tr>
<tr>
<td>2007</td>
<td>96,142</td>
<td>14,532</td>
</tr>
<tr>
<td>2008</td>
<td>95,552</td>
<td>11,201</td>
</tr>
<tr>
<td>2009</td>
<td>94,529</td>
<td>7,974</td>
</tr>
<tr>
<td>2010</td>
<td>93,203</td>
<td>7,862</td>
</tr>
</tbody>
</table>

\(A_{99,a} = (A_{98,a}/A_{98}) \times A_{99}\)

#### Table 3. \( T_{1,a-1} \)

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>Age group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>22-28</td>
</tr>
<tr>
<td>2007</td>
<td>96,142</td>
<td>10,688</td>
</tr>
<tr>
<td>2008</td>
<td>95,552</td>
<td>7,639</td>
</tr>
<tr>
<td>2009</td>
<td>94,529</td>
<td>5,066</td>
</tr>
</tbody>
</table>

#### Table 4. Continuation rate \( C_{2,a} \) by age group.

<table>
<thead>
<tr>
<th>Year</th>
<th>Age group</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>22-29</td>
</tr>
<tr>
<td>2007-2008</td>
<td>1.014315</td>
</tr>
<tr>
<td>2008-2009</td>
<td>1.01584</td>
</tr>
</tbody>
</table>

#### Table 5. Estimated number of teachers continuing to teach in 2010 according to Scenario 1.

<table>
<thead>
<tr>
<th>Continuation rate</th>
<th>Total</th>
<th>Age</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>22-29</td>
</tr>
<tr>
<td>2008-2009</td>
<td>93,038</td>
<td>5,146</td>
</tr>
<tr>
<td>2007-2008</td>
<td>93,162</td>
<td>5,139</td>
</tr>
</tbody>
</table>
### Table 6. L2,a

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>22-29</th>
<th>30-34</th>
<th>35-39</th>
<th>40-44</th>
<th>45-49</th>
<th>50-54</th>
<th>55-59</th>
<th>60+</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007</td>
<td>2180</td>
<td>88</td>
<td>116</td>
<td>87</td>
<td>83</td>
<td>611</td>
<td>750</td>
<td>282</td>
<td>163</td>
</tr>
<tr>
<td>2008</td>
<td>2082</td>
<td>145</td>
<td>124</td>
<td>123</td>
<td>82</td>
<td>408</td>
<td>120</td>
<td>791</td>
<td>289</td>
</tr>
<tr>
<td>2009</td>
<td>1789</td>
<td>37</td>
<td>58</td>
<td>56</td>
<td>56</td>
<td>493</td>
<td>691</td>
<td>274</td>
<td>124</td>
</tr>
<tr>
<td>20101</td>
<td>2111</td>
<td>93</td>
<td>103</td>
<td>102</td>
<td>83</td>
<td>447</td>
<td>847</td>
<td>303</td>
<td>133</td>
</tr>
<tr>
<td>20102</td>
<td>1765</td>
<td>31</td>
<td>52</td>
<td>54</td>
<td>57</td>
<td>486</td>
<td>687</td>
<td>274</td>
<td>124</td>
</tr>
</tbody>
</table>

1 Estimate using the 2007-2008 continuation rate. 2 Estimate using the 2008-2009 continuation rate.

![Graph showing age distribution of numbers per year](image-url)