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Teaching activity-based taxicab geometry

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This study aimed on the process of teaching taxicab geometry, a non-Euclidean geometry that is easy to understand and similar to Euclidean geometry with its axiomatic structure. In this regard, several teaching activities were designed such as measuring taxicab distance, defining a taxicab circle, finding a geometric locus in taxicab geometry, and real life problems. These activities were carried out for five weeks after introducing students to taxicab geometry. A total of 40 pre-service teachers participated in the study. However the findings included the sample answers that demonstrated the overall situation among the students who solved the problems accurately, provided half of the correct answers, and made logical errors. The study used the teaching experiment model. The study data were obtained from the activities carried out with students. The data were analyzed using descriptive analysis technique. The study found that the participants' interest was aroused by being introduced to a new geometry and to a different interpretation of familiar mathematical concepts with taxicab geometry. The participants also recognized the importance of the taxi distance between two points in defining a taxi circle, discovering π, and finding a geometric location. Finally, the participants learnt these concepts once more in Euclidean geometry and had the chance to realize their weaknesses and to improve them.

Key words: Taxicab geometry, Euclidean distance, taxicab distance, teaching experiment.

INTRODUCTION

According to the distance function defined in Euclidean geometry, we have to move along a straight line. In order to move from one point to another in the real world, however, these points may not always be on a straight line. Sometimes we can move turning right or left to streets. In other words, we cannot arrive at our destination flying like a bird. Buildings prevent us first of all. We need a new distance function in this case. The distance function defined in taxicab geometry, which is a non-Euclidean geometry, offers help with this. This geometry, which is very similar to Euclidean geometry, was defined by Minkowski (1864-1909) and then developed by Krause (1975).

Taxicab geometry is widely used in urban geography. While Euclidean geometry seems to be a good model of the natural world, taxicab geometry is a better model than the artificial urban world built by human being. The coordinate plane of this geometry is the same as the coordinate plane of Euclidean geometry. We can imagine ourselves driving a taxicab along imaginary horizontal and vertical lines in the streets of a perfectly designed city. An important feature of working with this geometry is that it is easy to understand. Its axiomatic structure is similar to Euclidean geometry but it is different from it at the same time because of just one axiom. This is the reason why this geometry is chosen first for teaching non-Euclidean geometries. Therefore, this geometry offers university students original research problems to improve their conceptual skills along with its innovations (Krause, 1975).

There are different ways to calculate distances in Cartesian coordinate plane. The first one is defined in

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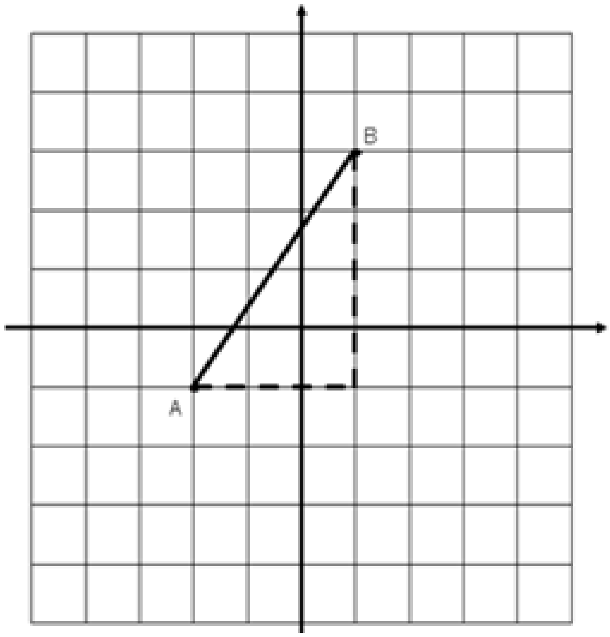

Figure 1. Dashed line is the taxicab distance – straight line is the Euclidean distance.

Euclidean plane as the distance between $A(x_1,y_1)$ and $B(x_2,y_2)$ points

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The other is taxicab distance. The Taxicab distance between A point and B point is

$$|AB|_T = |x_2 - x_1| + |y_2 - y_1|$$

In other words, the shortest distance from A point to B point is equal to the sum of lengths of line segments drawn from A and B points, parallel to the coordinate axes but perpendicular to each other. In Figure 1, the dashed line shows the taxicab distance between A and B points and the straight line shows the Euclidean distance between A and B points.

The aim of this study was to observe how the participants explored the relationships between basic concepts such as circle and geometric location in Euclidean and taxicab geometries, how they explored the similarities and differences between these geometries and recognized the differences presented by taxicab distance function.

METHOD

This study used the teaching experiment method, which is a qualitative research method. Teaching experiments are carried out to create environments where internal and external stimuli can be controlled to investigate cases that are not easy to explore in typical classroom environments. On the other hand, some teaching experiments are aimed at observing purely natural environments while some others are designed to observe similar cases (Anthony, 2000).

The teaching experiment method is used to collect first-hand data about the ways of mathematical learning and thinking. It is difficult for researchers to identify mathematical thinking ways, mathematical procedures and errors without the experiences teaching provides (Steffe and Thompson, 2000). Mathematics teaching is an experimental science. The teaching experiment is a method that makes it possible for researchers and teachers to observe the nature of mathematical thinking and development of ways of mathematical thinking (Czarnocha and Maj, 2008). Apart from observing the classroom environment and monitoring learning process, the teaching experiment method improves teaching-learning activity at the same time (Czarnocha and Prabhu, 2006).

Acting as both a researcher and a teacher, the researcher prepared the teaching-learning activities and, in the role of a researcher teacher, she observed the development of students' (pre-service teachers') learning and thinking skills.

The study sample

The study enrolled a total of 40 first year students studying prospective mathematics teacher. All of the students were in the same class.

Research process

The study was carried out in six weeks: one week for introduction to taxicab geometry and 5 weeks for implementing the activities. In the first week, the researcher made a brief introduction to non-Euclidean geometries and taxicab geometry and explained why this geometry is a non-Euclidean geometry and its uses in our daily lives. In the following weeks, several activities based on taxicab geometry were carried out in classroom. Table 1 shows the weekly objectives of these activities.

Table 1. Weekly objectives.

Activity no	Objective
Activity 1	To explore distance in taxicab geometry
Activity 2	To explore taxicab circle
Activity 3	To explore π in taxicab geometry
Activity 4	To determine geometric locations in taxicab geometry
Activity 5	To use taxicab geometry in real life

Each of the objectives contains sub-problems. The activities were carried out individually during the class hours each week. After each lesson, the researcher checked the students' comprehension, provided additional explanations and guided the students with the problems when necessary. The following are the activities and their sub-problems.

Activity 1

Aim: To explore distance in taxicab geometry
Sub-problems
a) Exploring taxicab distance

b) Comparing Euclidean distance with taxicab distance
c) Discovering the conditions in which distances in the two geometries are equal

Activity 2

Aim: To explore taxi circle
Sub-problems
a) Recognizing that four given points are four taxi units from a given A point
b) (b1) Finding certain points located at a distance of four taxi units from a given A point and locating them in the coordinate system
(b2) Finding certain points located at a Euclidean distance of four units from a given A point and locating them in the coordinate system
c) (c1) Finding the set of all points located at a distance of four taxi units from a given A point and locating them in the coordinate system
(c2) Finding the set of all points located at a Euclidean distance of four units from a given A point and locating them in the coordinate system
d) (d1) Finding P values giving the equation $|PA|_E = 4$ when A (-3,-2)
(d2) Finding P values giving the equation $|PA|_T = 4$ when A (-3,-2)

Activity 3

Aim: To explore π in taxicab geometry
Sub-problems
a) Finding π in Euclidean geometry
b) Finding π taxicab geometry

Activity 4

Aim: To determine geometric locus in taxicab geometry
Sub-problems
a) Finding the equation of the taxicab circle centered at the point A (1,-2) of radius 3 and drawing its graph
b) Finding the geometric location of the points giving the equation $\{P||PB|_T = 2\}$ when B (-3,-1) and drawing its graph
c) for the given points A (-3,-2) and B (1,0)
(c1) Finding $|AB|_T$
(c2) Finding the geometric location of the set of points $\{P||PA|_T = 2 \; and \; |PB|_T = 4\}$ and drawing its graph
(c3) Finding the geometric location of the set of points $\{P||PA|_T = 3 \; and \; |PB|_T = 3\}$ and drawing its graph
(c4) Finding the geometric location of the set of points $\{P||PA|_T + |PB|_T = |AB|_T\}$ and drawing its graph
d) For the given points A (-3,-2) and B (1, 0) points
(d1) Finding $|AB|_E$
(d2) Finding the geometric locus of the set of points $\{P||PA|_E = 2 \; and \; |PB|_E = 4\}$ and drawing its graph
(d3) Finding the geometric locus of the set of points $\{P||PA|_E = 3 \; ve \; |PB|_E = 3\}$ and drawing its graph
(d4) Finding the geometric locus of the set of points $\{P||PA|_E + |PB|_E = |AB|_E\}$ and drawing its graph
(d5) Comparing and interpreting (d4) in question (b) and (d4) in question (c).
e) For the given points A (-3,-2) and B (1,2)
(e1) Finding the geometric locus of the points located at an equal taxi distance from the points $\{P||PA|_T = |PB|_T\}$A and B and drawing its graph
(e2) Finding the geometric locus of the points located at an equal Euclidean distance from the points $\{P||PA|_E \; |PB|_E\}$ and drawing its graph

Activity 5

Aim: To use Taxicab geometry in real life
Sub-problems
a) Mathematics teachers Ayşe and Mehmet are transferred to schools in Ideal City. Ayşe's school is located at A (-3,-1) and Mehmet's school is located at M (3,3). They want their flat to be at an equal distance from both of their schools. Where do you think they should look for a flat?
b) A telephone company wants to place its telephone booths at locations that are 12 blocks from the city center and cover all residents of the city. Place the booths at certain points so that each of them serves everyone within four blocks. Find the minimum number and best locations of the booths to achieve this?

Data analysis

The study data were analyzed using descriptive analysis. In the descriptive analysis, direct quotes were used to reflect the views of the participants effectively by describing the data in a systematic way so that readers could be provided with arranged and interpreted data. The data were then explained and interpreted, cause-effect relations were discussed and some results were obtained (Yıldırım and Şimşek, 2005). The solutions of some of the students were presented and their answers to the problems were interpreted.

FINDINGS

The student samples obtained from the five activities prepared for teaching taxicab geometry are presented as follows. In Activity 1, the students were asked questions about exploring distance in taxicab geometry.

In question (a), the students were asked to find both Euclidean distance and taxi distance between two points. In question (b), they were asked to compare the distance which they found. Figure 2 shows the answers given by one of the students to questions (a) and (b). As can be seen in Figure 2, Özge calculated the distance between the points in both of the geometries and expressed the distance in the graph. Therefore, she got the correct result by using the two different definitions of distance.

In Figure 3, in order to find the conditions in which Euclidean and taxicab distance could be equal, Ayşe equated them with each other. After performing the appropriate steps, she got the solution set. She also stated that $|AB|_T = |AB|_E$ could be achieved if the difference between the first and second components of A point and B point was equal to zero.

Ayşe examined the equation of distance in the two geometries in algebraic terms while Kenan examined this equation in geometric terms (Figure 4). In Figure 4, Kenan came up with a geometric solution to the question. He answered, "In order for taxicab and Euclidean distances to be equal, they have to be parallel and linear to *x* or *y* planes when we combine the coordinates taken."

Regarding questions (a), (b) and (c), almost all of the students easily found both Euclidean distance and taxicab

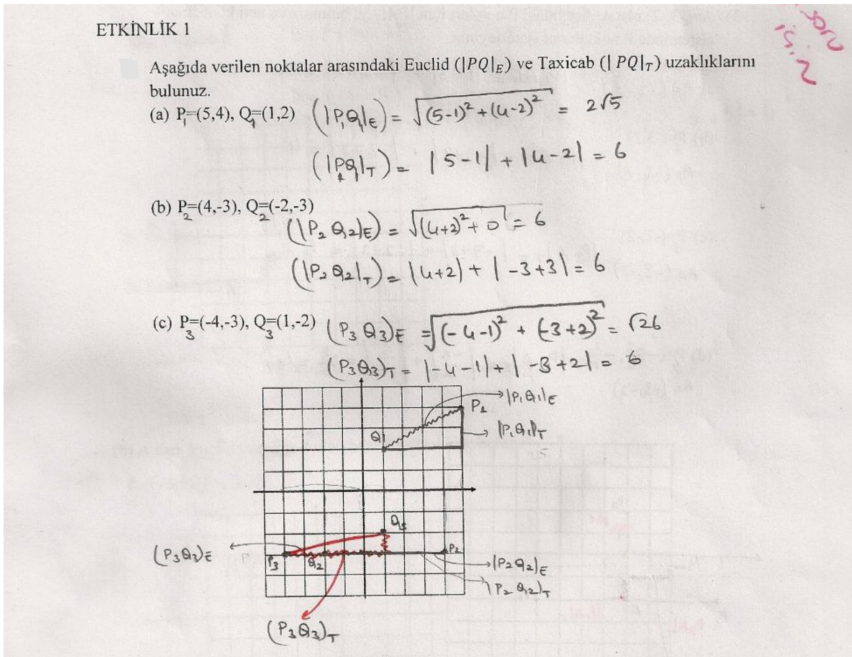

Figure 2. answers given by Özge to questions (a) and (b) in Activity1.

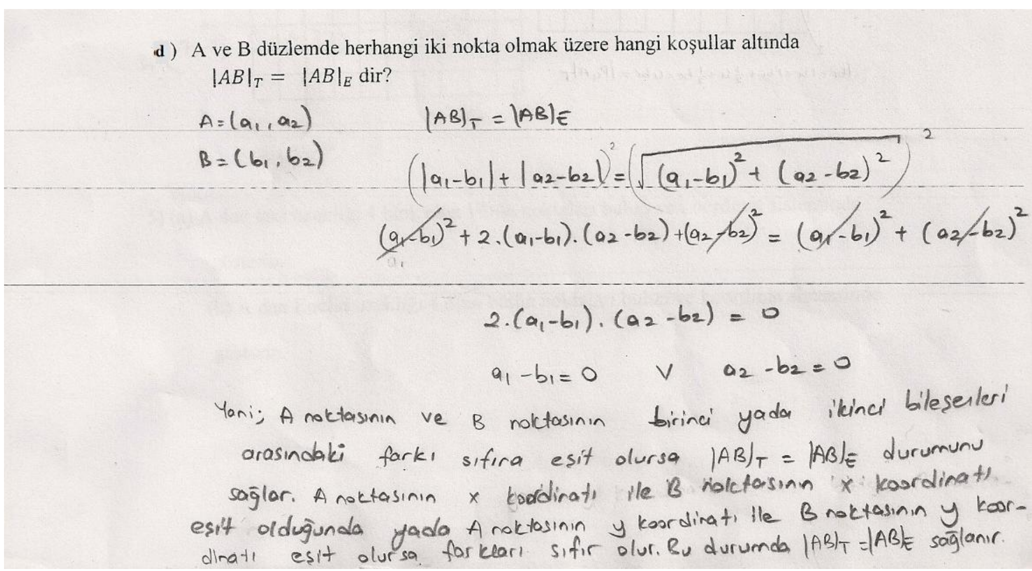

Figure 3. answer given by Ayşe to question (d) in Activity1.

distance between A and B points, expressed their coordinates and the Euclidean and taxicab distances between the two points by highlighting them with color pencils. To question (d), the students gave answers from geometric and algebraic perspectives. Kenan noted, “Both A point and B point have to be located on a line in parallel to x or y coordinates” and he emphasized that when expressing the geometric locus of the points, their axes or coordinates should be same. Therefore, this student made a generalization based on the questions. With an algebraic perspective, Ayşe obtained the equation of the multiplication expression to zero as a result of the algebraic operations by taking the squares of both distances to obtain equal distances and she concluded that the equation was obtained when one of the multipliers was equal to zero.

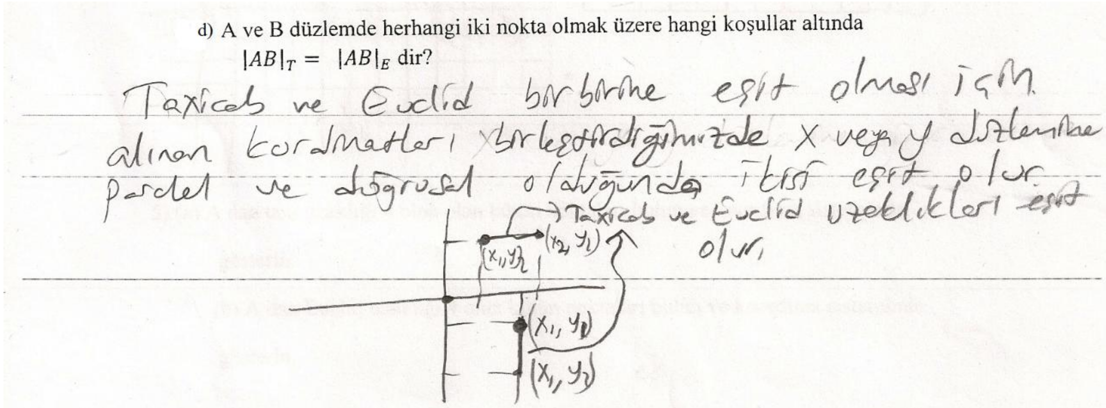

Figure 4. answer given by Kenan to question (d) in Activity1.

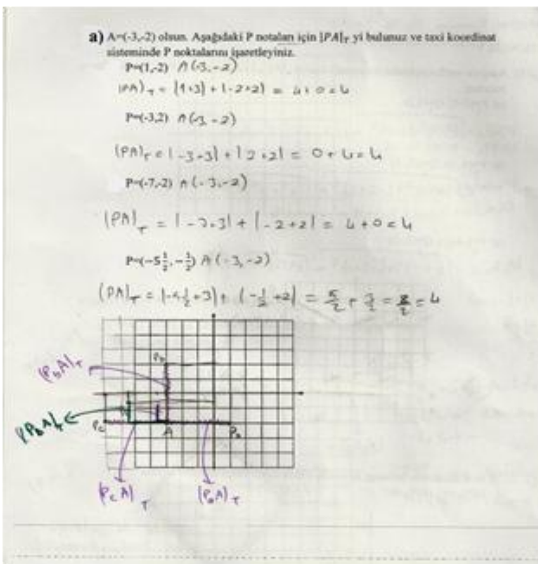

Figure 5. answer given by Rümeysa to question (a) in Activity 2.

Activity 2 was about discovering taxi circle. In question (a), the students were asked to find the taxi distance between a given A point and any given four points with coordinates. Figure 5 shows the answer given by a student to this question.

In Figure 5, Rumeysa found the taxi distances between the A point and the given four points, located the points in the graph and showed the taxi distances in color.

In question (a), all of the students easily determined the points at a distance of four taxi units and four Euclidean units from the given A point and showed them in the coordinate plane.

In (b1) and (b2) in question (b), the students were asked to find certain points at a distance of four taxi units and four Euclidean units respectively from a given A point. Figures 6 and 7 show the solution given by a student to this question in (b1) and (b2). In Figure 6 and, Fatma found some P points at a distance of four taxi units from A point and showed them in the graph. She obtained a geometric shape when she combined the points. In Figure 7, she found particular points at a distance of four Euclidean units and showed them in the coordinate system. She stated that these points displayed a circle centered at the point M(-3,-2) of radius 4.

In (c1) and (c2) of the question (c), the students were asked to find all the points at a distance of four taxi units and four Euclidean units respectively and show them in the coordinate system. Figures 8 and 9 show the solution by one of the students.

In Figure 8, Ahmet wrote the absolute-value equation $4 = |x+3| + |y+2|$ corresponding to all the points at a distance of four taxi units from A point, found the solution set and drew its graph in Figure 9. He stated that the set of points corresponding to the equation which he wrote in the graph was the area which he shaded. Ahmet solved the absolute-value equation correctly and found the solution set but his graph drawing in the coordinate system was not correct. Here, the graph would be correct if the equation had been given with an inequality. This error shows that this student needs to improve his knowledge about basic subjects, especially about graph drawing. As can be seen in Figure 10, this student made the same error when he showed the set of points giving the circle. In Figure 10, Ahmet wrote the equation of the set of the points at a distance of four Euclidean units from A point and wrote, “The Euclidean distance from A point of all the points with all (x,y) coordinates giving this equation are four units”. Apparently, the student made the same error as the one in (c1).

In Figure 10, Ahmet wrote the equation of the set of the points at a distance of four Euclidean units from A point and wrote, “The Euclidean distance from A point of all the points with all (x,y) coordinates giving this equation are four units”. Apparently, the student made the same error as the one in (c1).

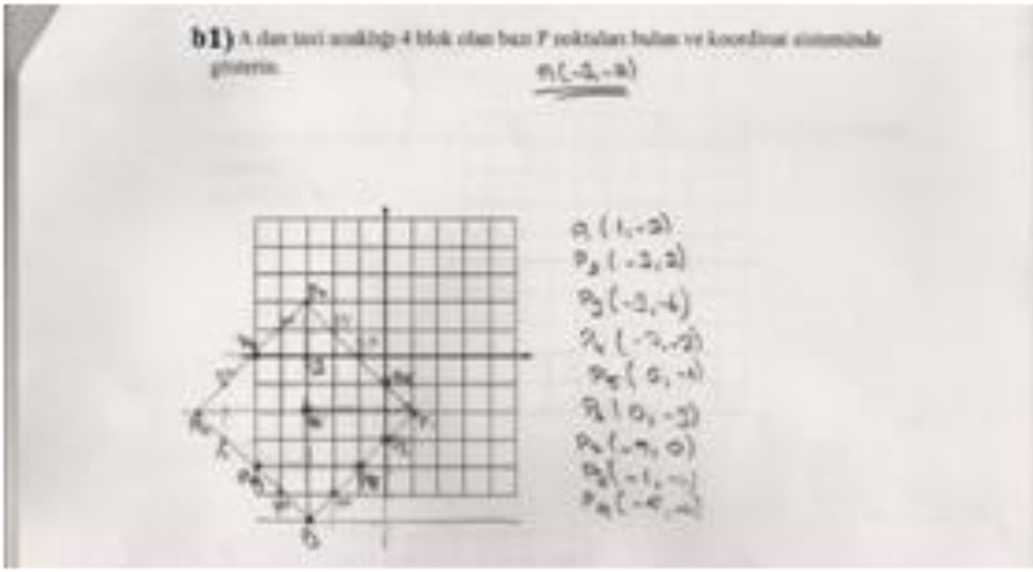

Figure 6. answer given by Fatma to question (b1) in Activity 2.

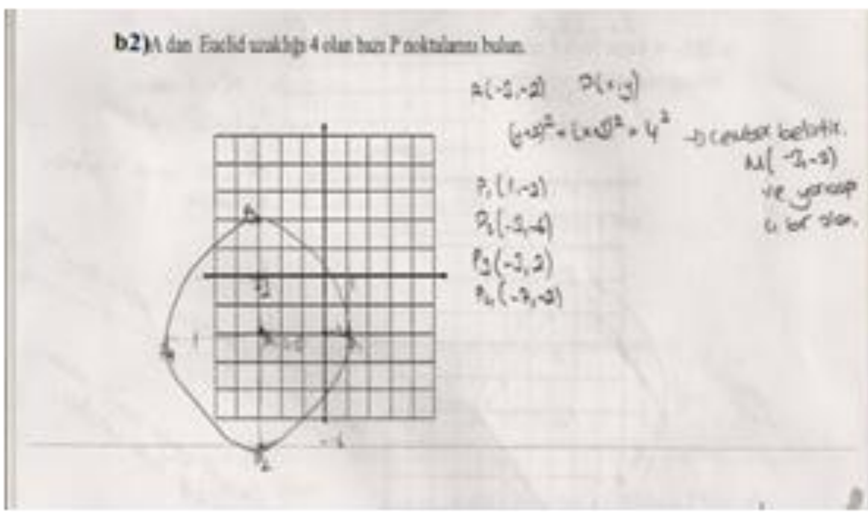

Figure 7. answer given by Fatma to question (b2) in Activity 2.

In (d1) and (d2) in question (d), the students were asked to find the solution set of the points the equation of which were given in both geometries. Figures 11 and 12 and show the answers given by Meral and Fatma to this question.

Meral found $P_{1,}P_2, P_3, P_4$ points giving this equation for (d1) and, at the same time, she stated that these points were the same as the points requested in (d2) and a square was obtained when these points were combined.

Similarly, Fatma wrote the set of points corresponding to the set of points in both taxicab geometry and Euclidean geometry for *P* points giving this equation. She also stated that the set of points in Euclidean geometry indicated a circle while the set of points in taxicab geometry indicated a square and correctly drew the graphs of the two shapes in the same coordinate plane.

In question (d), while the majority of the students stated that the set of *P* points at a distance of four Euclidean units from a given A point indicated a circle and the set of P points at a distance of four taxi units from a given A point indicated a square. Some of them, however, stated that these sets showed a rhomb. The fact that the students considered the geometric shapes they found in taxicab geometry, which they had just been introduced to, as in Euclidean geometry – for example, definition of a square as a circle in taxicab geometry-might have caused

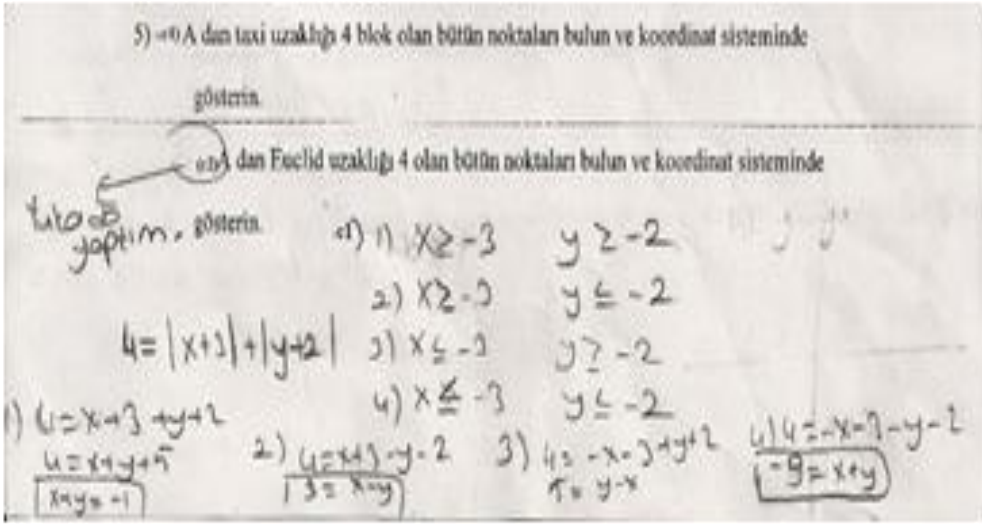

Figure 8. answer given by Ahmet to question (c1) in Activity 2.

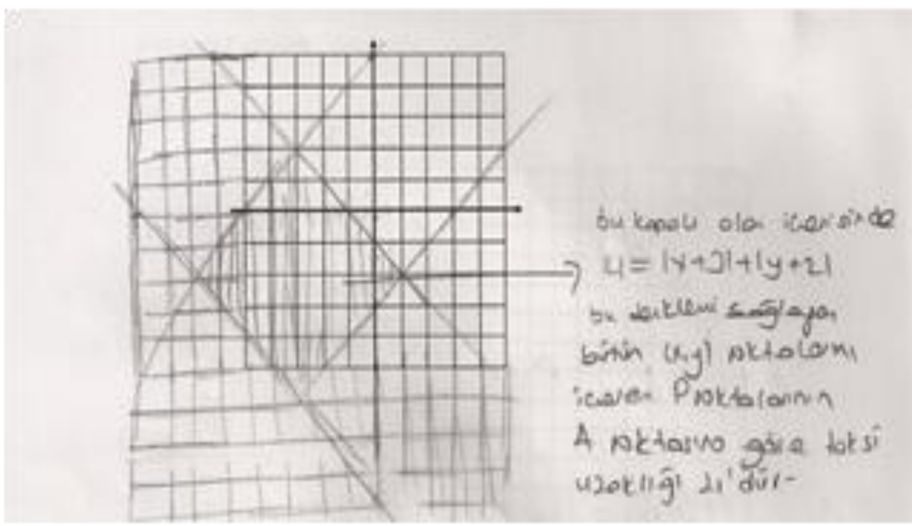

Figure 9. the graph in the coordinate system drawn by Ahmet to question (c1) in Activity 2

them to think that it was not appropriate for the geometric structure they were familiar with.

Activity 3 was about discovering π in taxicab geometry. The first question was "How can we find π in Euclidean geometry?" The second, and primary, question was "How can we find π in taxicab geometry?" Figure 13 shows Çağla's answer to this question. As can be seen in Figure 13, in question (a) Çağla wrote, "If the diameter of the circle is 2r, its circumference 2πr. No matter what the circumferences of the circles are (very large or very small), the ratio of the circles' circumferences to their diameters was always π". In question (b), on the other hand, while she was calculating the circumference, she took the distance not as taxi distance but as Euclidean distance based on the Pythagorean Theorem and then she obtained the circumference by multiplying an edge length that she found by four. Based on the ratio of the circumference to the diameter, she got $\pi = 2\sqrt{2}$ but this result was not correct. If she had calculated the circumference of the circle using taxi distance, she would have correctly got $\pi = 4$.

As shown by this example, when the students were asked how to find π in Euclidean geometry, the majority of them replied, "It is the ratio of the circumference of a circle to its diameter". However, when they were asked how to find π in taxicab geometry, none of the students

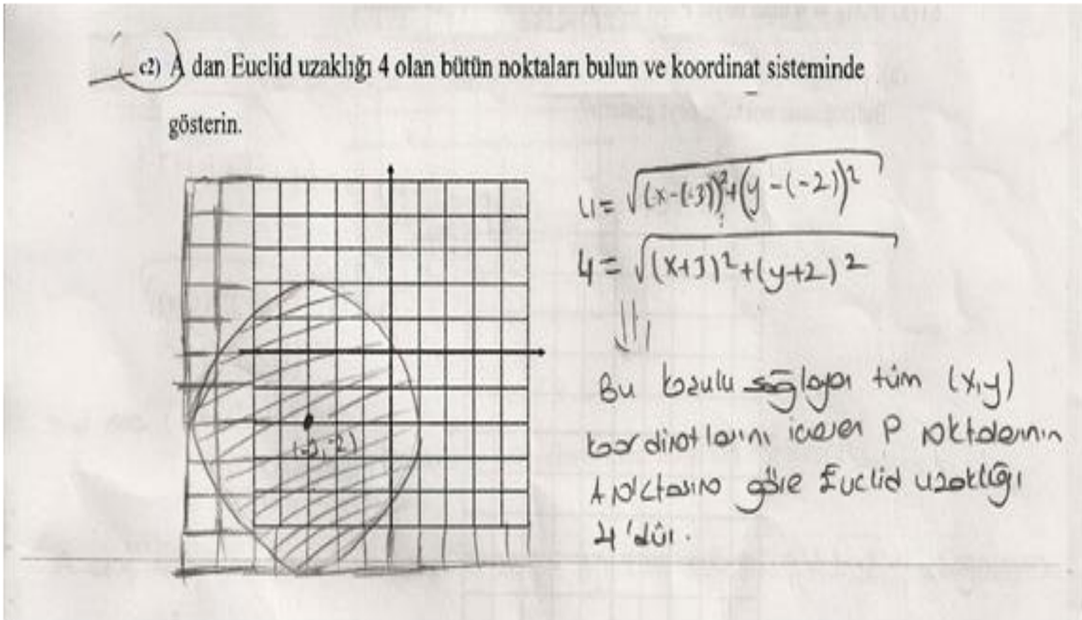

Figure 10. answer given by Ahmet to question (c2) in Activity 2.

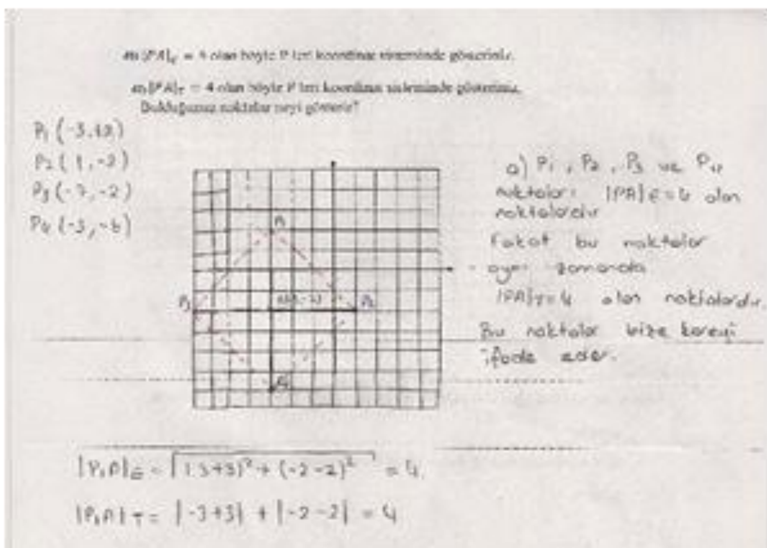

Figure 11. answers given by Meral to questions (d1) and (d2) in Activity 2.

were able to give the correct answer. Most of the students agreed that the ratio of the circumference of a circle to its diameter was π but they could not make a connection of this with taxi distance and, therefore, they made an error.

Activity 4 was about determining the geometric locus in taxicab geometry. In question (a), the students were asked to find the equation of a taxi circle whose center and radius were given and to draw its graph. In question (b), they were given the equation of circle and expected to find the set of points giving this equation and to draw its graph. Figure 14 shows the answer of a student who could not give a complete answer to question (a). As can be seen in Figure 14, Gizem wrote the equation of taxi circle using the definition and solved the equation, but she could not draw its graph. This was the same problem as the graph drawing error in Activity 2 question (b). Although Gizem found the complete solution set of the

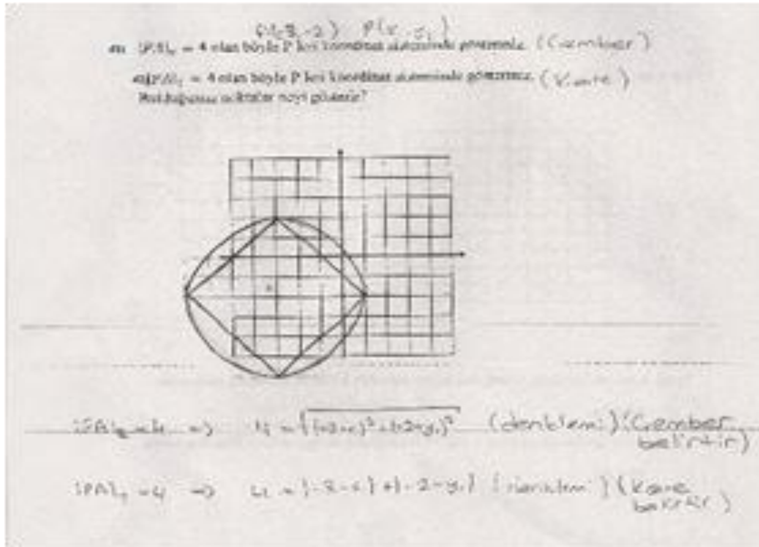

Figure12. answers given by Fatma to questions (d1) and (d2) in Activity2

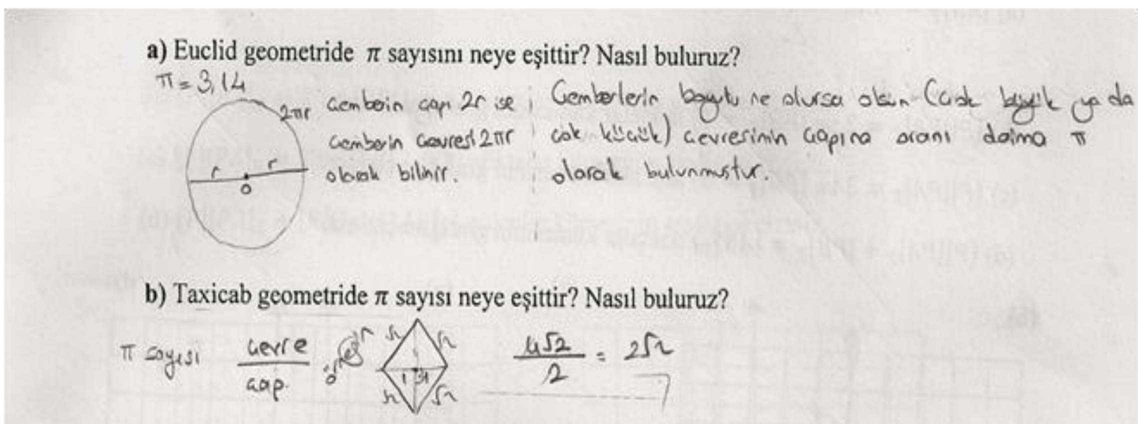

Figure 13. answers given by Çağla to questions (a) and (b) in Activity 3.

equation, she ignored some of the points while drawing the graph with certain intervals and, as a result, she could not form the taxi circle.

Figure 15, on the other hand, shows the complete answer to question (b) given by one of the students. Using the definition of a taxi circle, Kübra wrote the equation, found the complete solution set of the equation and drew its graph (Figure 15).

At the end of these activities about taxi circle, the students realized that the points at an equal distance from a fixed point indicated a taxi circle. They had difficulty in comprehending taxi circle at first as they had never seen any other geometry than Euclidean.

However, in the following activities, they stated that the set of points at a distance of two taxi units from a given B point indicated a taxi circle and the equation was $|PB|_T = 2$ and they did not have any difficulty in finding the solution set of this equation. Here, the students discovered that the distance of any point on a taxi circle from the center was fixed by taking points on the circle.

In question (c), the students were asked to find the geometric locus of the set of points given after the taxi distance between A point and B point was found. Figures 16 and 17 show the answer given by one of the students.

As can be seen in Figures 16 and 17, Merve correctly found the taxi distance between A and B points given in (c1) and drew the graph of the given points in (c2) and (c3). Here, Merve determined the cross-section set of the two taxi circles in the graph and stated that she covered the set of points (-3,0) and (-1,-2) and the remaining points between them while she was writing the solution set of (c2) and that the solution set of (c3) was the set of

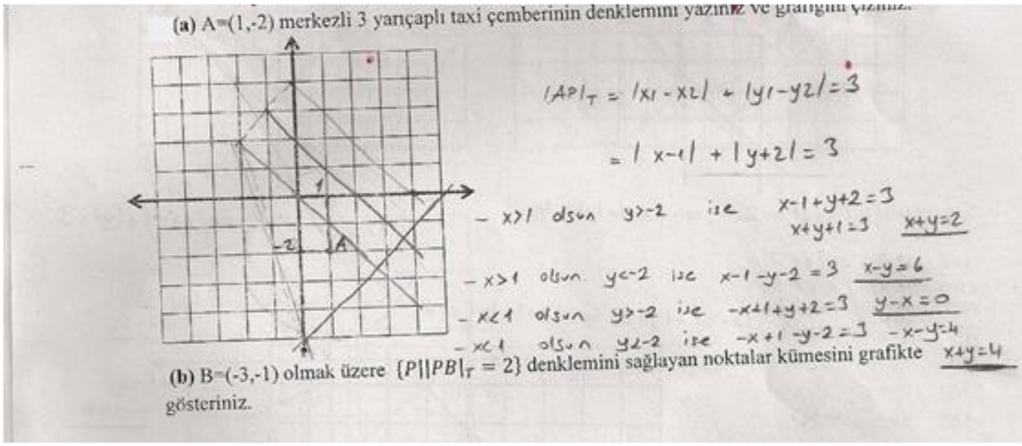

Figure 14. answer given by Gizem to question (a) in Activity 4.

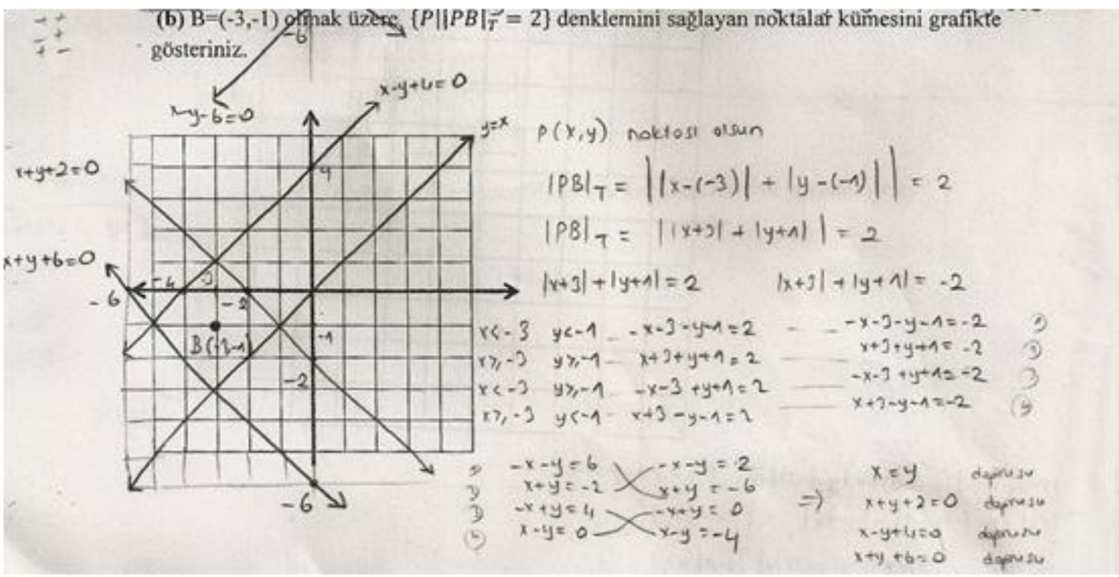

Figure 15. answer given by Kübra to question (b) in Activity 4.

points (-2,0) and (0,-2) and the remaining points between them. She added that, in (c4), P points whose total taxi distances from A and B points were equal to the taxi distance between A and B points were the set of points located in the shaded area.

It is clear that the students were now able to recognize the equation of taxi circle, draw its graph and even show the points giving the cross-section set of two taxi circles in a graph.

What was aimed by question (d) was to ensure that the students were able to recognize the possible difference when the previous question was considered in Euclidean geometry. Figures 18 and 19 show a sample student answer.

As can be seen in Figure 18, in (d1), Merve found the Euclidean distance between A and B points, drew the Euclidean circles requested in (d2) and (d3) and showed A' and B' cross-section points in graph. However, she could not write these points' coordinates. In (d4), she drew AB line segment and stated that this line segment was the requested set of points. In Figure 19, she stated that the rectangle was formed in (c4) and the line fragment was formed in (d4).

Figures 20 and 21 show a sample correct answer. As can be seen in Figure 20, Çağla gave solutions similar to Merve's answer. However, in Figure 21, she stated that the sum of the distances of P from A and B points was equal to the distance between these two points and (-1,-1) point of P on AB line segment was a single point. In (d5), the students were asked about the relationship

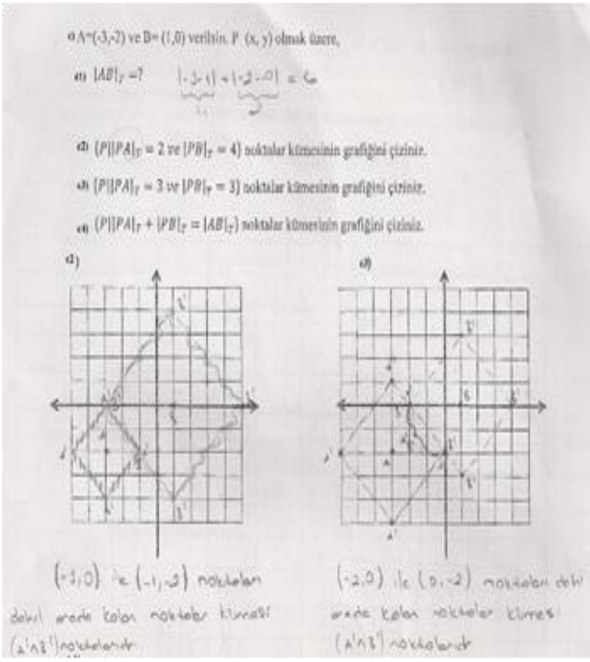

Figure 16. answers given by Merve to questions (c2) and (c3) in Activity 4.

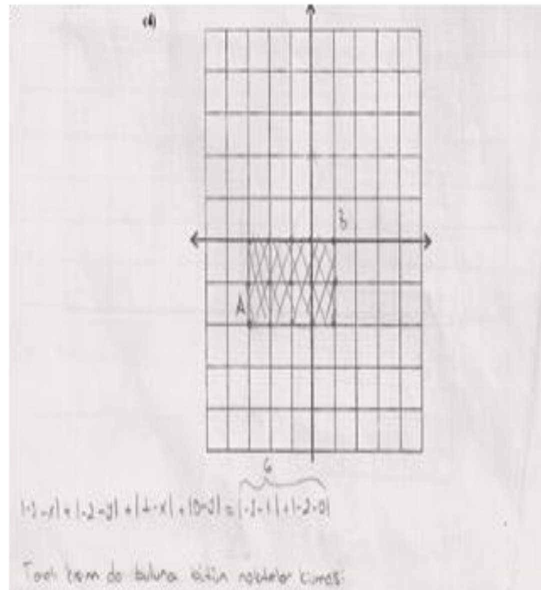

Figure 17. answer given by Merve to question (c4) in Activity 4.

between the result found in (c4) and (d5) in this question. Çağla stated that P set of points corresponded to a single point in (d5) and to a line segment in (c4).

Some of the students stated that the set of points whose sum of distances from two given points in Euclidean geometry was equal to the distance between these points was actually a single point and that this point was the mid-point of AB line segment. On the other hand, they concluded that, in taxicab geometry, the points whose sum of distances from two fixed points were equal formed a rectangular area.

In question (e), the students were asked to find the geometric locus of the points at an equal taxi distance from given A and B points. Figures 22 and 23 show an example of the correct answer to this question.

As can be seen in Figure 22, Ilay wrote the equation of the P set of points at an equal taxi distance from A and B points and gave the solution to it. However, while solving the absolute-value equation, she examined the equation according to x but ignored all the values that could replace y and, as a result, the points turned out to be insufficient for drawing its graph. Therefore, the graph was not drawn completely. On the other hand, when the set of P values at an equal Euclidean distance from A and B points in were requested in Figure 23, İlay wrote the equation and found the solution set. She also drew x+y+1=0 line, which was the solution set, in the coordinate plane.

In taxi plane, the set of points at an equal distance from given A and B points can be expressed as a piecewise function as

$$x = f(y) = \begin{cases} x \leq -3 & y \geq 2 \\ 1 - y & -2 < y < 2 \\ x \geq 1 & y \leq -2 \end{cases}$$

and, in geometric terms, it can also be defined as the combination of $[-3,2] \cup [1,-2]$ interval and the line segment defined above.

In Euclidean geometry, y=ax+b function, the perpendicular bisector of the line segment combining A and B points, is x+y+1=0 line. In this way, the students realized that while perpendicular bisector is a line in Euclidean geometry, it corresponds to the combination of the set of multiple points in taxicab geometry.

Activity 5 dealt with the daily applications of the previous question in real life. Figures 24 and 25 show the answers and solutions given to this question by Damla and Banu respectively, who answered it partly. As can be seen in Figure 24, Damla stated that $|AK|_T = |MK|_T$ should be obtained so that the flat to be rented by Ayşe and Mehmet could be at an equal distance from both A point and M point. Damla wrote the equation correctly but she could neither solve it nor draw the graph. Banu, however, found specific points by trying points at an equal distance from A and B points and drew the graph by combining these points (Figure 25).

The students were supposed to search for the desired location of the flat, a location at an equal distance from both persons' schools, by obtaining the solution set based on a broken line and combination of the two areas. However, some of the students (e.g. Damla and Banu) just wrote the equation but did not give a solution while some others drew just the broken line and took the points

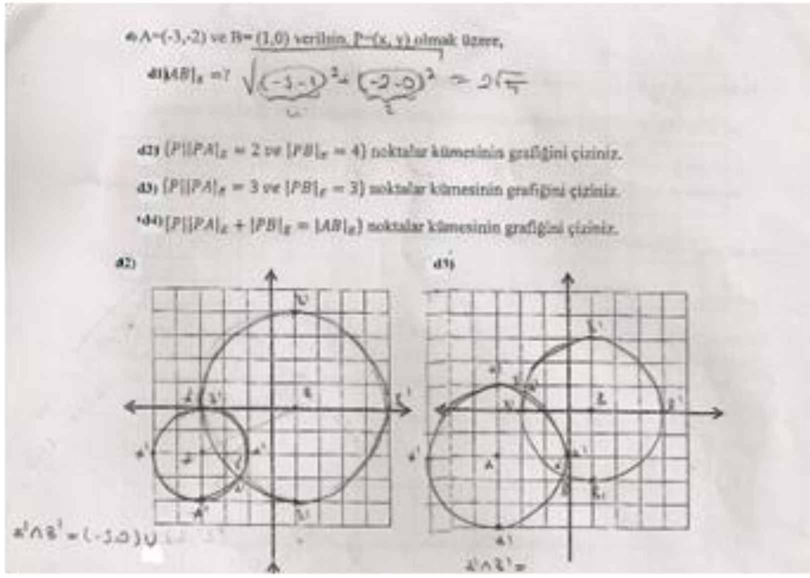

Figure 18. answers given by Merve to questions (d1), (d2) and (d3) in Activity 4**.**

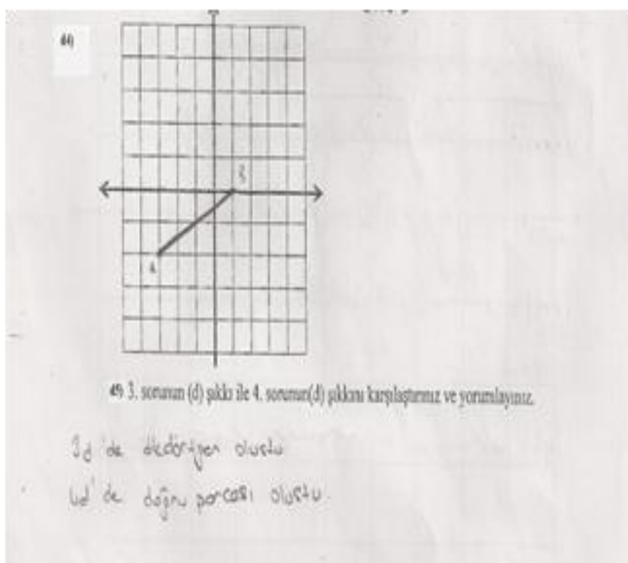

Figure 19. answers given by Merve to questions (d4) and (d5) in Activity 4.

at an equal distance from both of the two points on the broken line.

According to the second question in this activity, a telephone company wanted to place its telephone booths at locations that were 12 blocks from the city center and cover all residents of the city. Figure 26 shows the answer given by Murat, who placed the booths correctly. As can be seen in Figure 26, Murat determined nine telephone booth locations to serve everyone by placing taxi circles each of which had a four-block diameter in the taxi circle with a 12-block radius from the city center in the coordinate plane.

As a result, the students determined the minimum number of telephone booths in a circular area and helped the telephone company reduce costs.

CONCLUSION

As a result of these activities, the students were

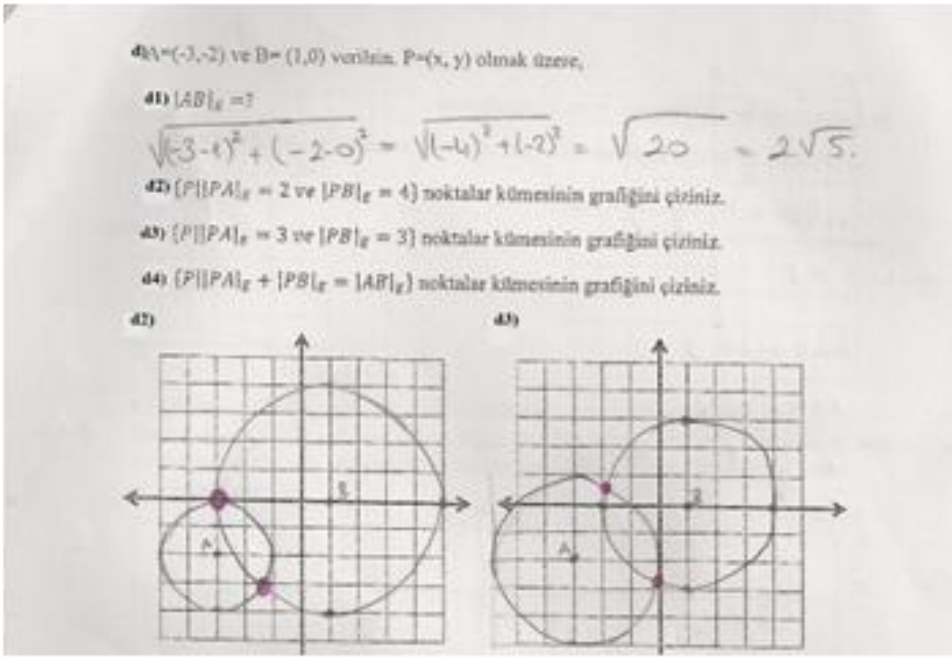

Figure 20. answers given by Çağla to questions (d1), (d2) and (d3) in Activity 4.

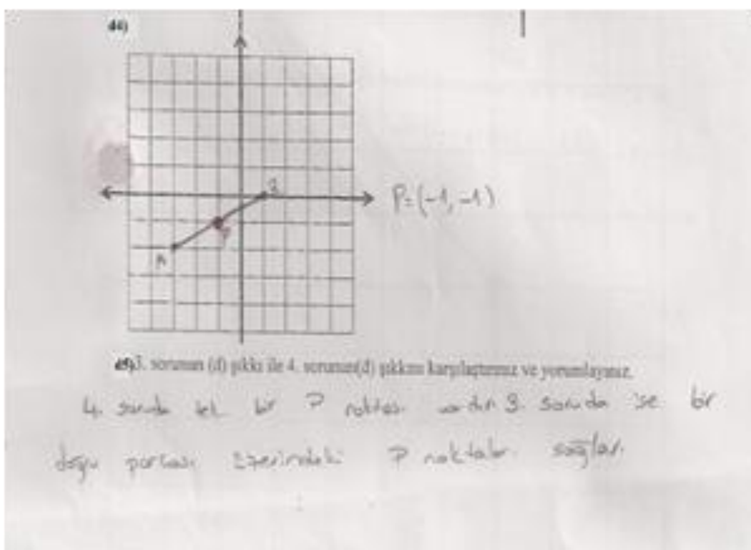

Figure 21. answers given by Çağla to questions (d4) and (d5) in Activity 4.

introduced to a different geometry other than Euclidean and they were given an opportunity to realize that there could be other geometries as well.

In addition, by means of these activities, the students realized their weaknesses regarding their knowledge of the basic concepts in Euclidean geometry. The activities at the beginning were aimed at guiding them into discovering taxicab distance. As a result of the problems they solved, they realized that the real distance used in our real lives was in fact taxicab distance, which was quite an innovation for them. They had difficulty at first because, from primary school until university, they hold the assumption that "there is only one geometry and that is Euclidean geometry". After a few activities, however, they gradually overcame this challenge.

At the end of the activities designed for teaching taxi circle in taxicab geometry, all the students obtained the equation and, by using this equation, they came up with

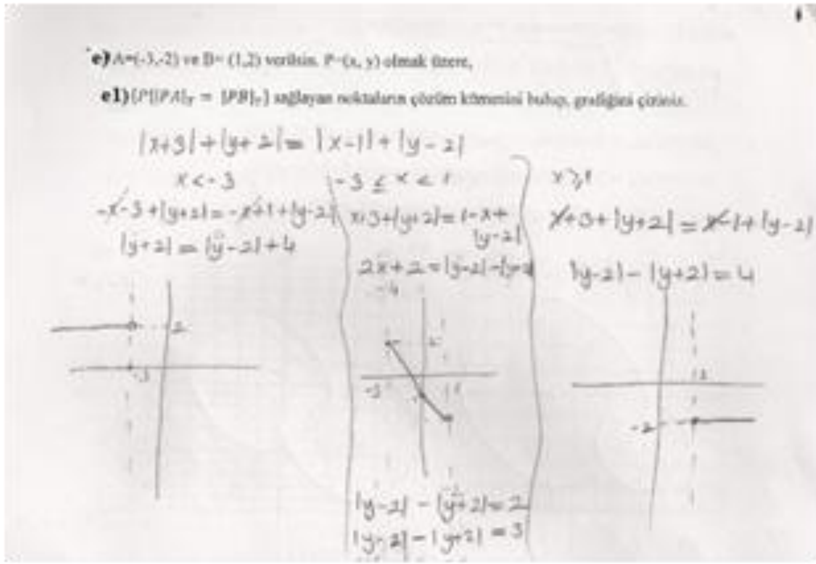

Figure 22. answer given by İlay to question (e1) in Activity 4.

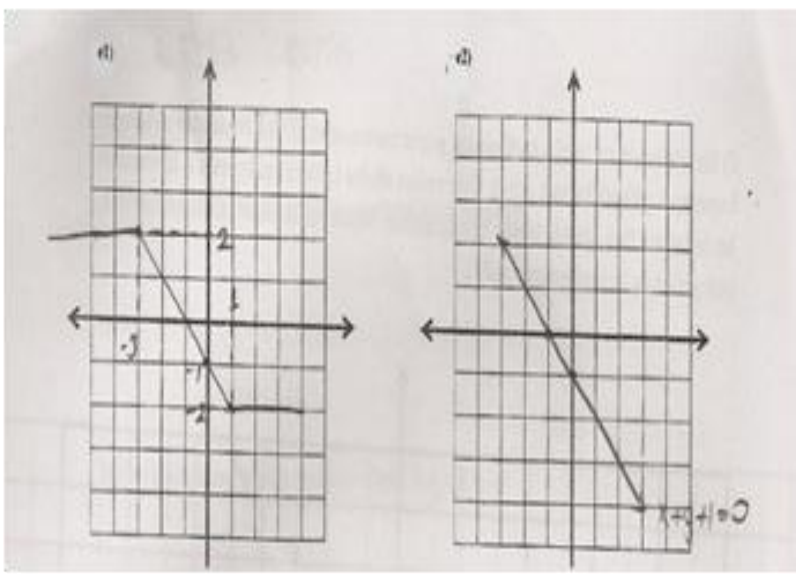

Figure 23. answer given by İlay to question (e2) in Activity 4.

the correct graph. On the other hand, although the points giving both the taxi and Euclidean circle equations were only those points on the circle, inside of the circle was incorrectly shaded by few students as shown by the example student answers mentioned Activity 2. This error, which was made in both geometries revealed that the students needed to improve their skills to find geometric locus in Euclidean geometry. The fact that while determining geometric locus in taxicab geometry, some of the students found the solution set of an absolute-value equation. But they took the definition interval of x and took y variable at only certain intervals to find the locus of these points. They transferred them into the coordinate plane indicated that these students were not able obtain the requested graph of the solution set. The students' apparent weakness about geometric locus could have been caused by their previous learning experiences. In the activity asking the students to draw the graph of the equation given in Taxicab geometry, the students stated that the geometric shapes which they obtained when they found the set of points giving the equation and drew its graph could be a square or a

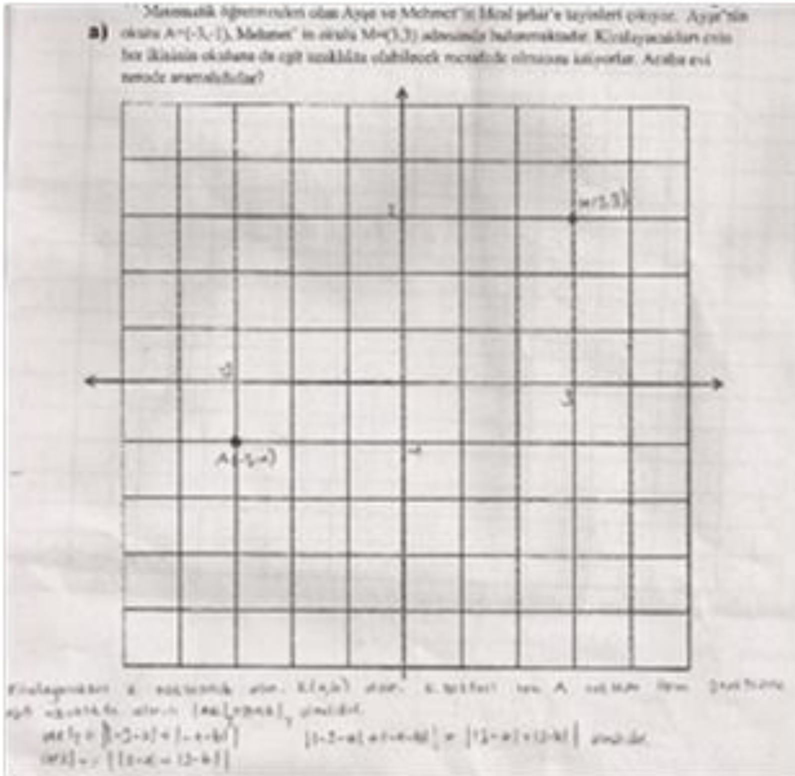

Figure 24. answer given by Damla to question (a) in Activity 5.

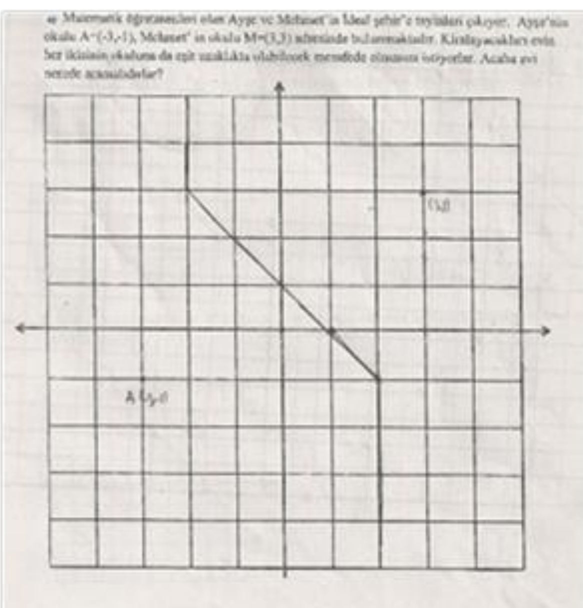

Figure 25. answer given by Banu to question (a) in Activity 5.

rhombus. However, this equation expressed a taxi circle.

The fact that Euclidean geometry had been the only type of geometry they had ever known until that time might have caused them to consider taxicab geometry in the way they normally considered Euclidean geometry. Nevertheless, because they failed to see the relationship between the concept of distance and the definition of a circle, even if it was in Euclidean geometry, they were unable to realize that the definition of circle changed in parallel to the change in distance. At the end of the activity, when they learnt that the square-like geometric shape they obtained in taxicab geometry was actually a taxi circle, they realized that a circle was not always curvilinear but it was tetragonal in this geometry. In the activity asking them to find the number corresponding to π in taxicab geometry, the students tried to obtain it based on the ratio of a circle's circumference to its diameter as in Euclidean geometry. This way was correct, too. On the other hand, while calculating the circle's circumference, few of the students thought in the way as in Euclidean geometry and calculated an edge length of the circle based on the Pythagorean Theorem. The Pythagorean Theorem applies to taxicab geometry, too. However, the theorem is defined in a different way. The main error of the students was taking the distance as Euclidean distance. Most of the students agreed that the ratio of the circumference of a circle to its diameter was π, but they could not make a connection of this with taxi distance and, therefore, they found an incorrect π. Also,

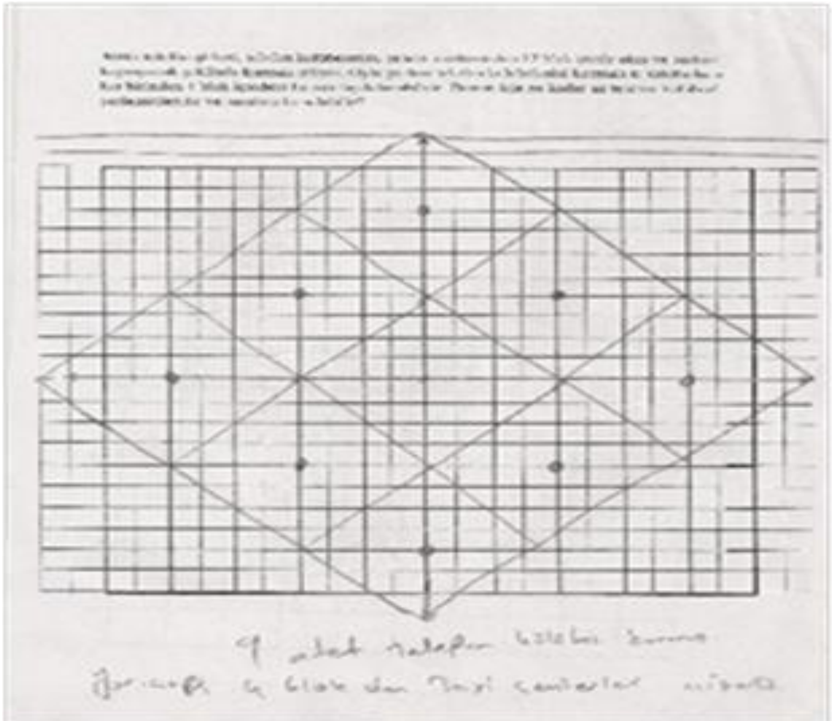

Figure 26. answer given by Murat to question (b) in Activity 5.

they did not answer a question asking them to find π in taxicab geometry. After solving additional problems under the guidance of the researcher, the students used taxicab distance correctly and learnt that in taxicab geometry, $\pi = 4$.

The students found the two obtained taxi circles' locations corresponding to each other (that is, that their cross-section solution set was a line segment), compared the two Euclidean circles' locations corresponding to each other (that is, that they were two points when they intersected with each other) and recognized their differences. The students kept their errors in the earlier activities to a minimum in the last activities. They could now be regarded as successful in geometric locus problems in taxicab geometry. Also, they discovered that, in taxicab geometry, the points whose sum of distances from two fixed points was equal the taxi distance between these points formed a tetragonal area. Conversely, they realized that the set of these points corresponded to a single point in Euclidean geometry.

Finally, in Activity 5, which was intended for linking geometric locus problems with real life, very few of the students were able to solve the problems. The reason for this situation might be the fact that problems related to real life are not commonly seen in Euclidean geometry. On the other hand, when the students solved these problems again under the guidance of the researcher, they realized that there is a connection between geometric locus problems and real life problems. To sum up, by means of the activities carried out, the students had the opportunity to learn about taxicab geometry. Also, they were informed about non-Euclidean geometries by comparing and contrasting the concepts such as distance between two points, circle and geometric locus in taxicab geometry and Euclidean geometry. Relatively simple structure of taxicab geometry in comparison with other non-Euclidean geometries and its similarity to Euclidean geometry increase the importance of taxicab geometry. Therefore, it would be more sensible to introduce students to taxicab geometry when teaching them non-Euclidean geometries.

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