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Effects of using dynamic mathematics software on pre-service mathematics teachers' spatial visualization skills: The case of spatial analytic geometry

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The purpose of this study was to investigate the effects of using dynamic geometry software on pre-service mathematics teachers' spatial visualization skills and to determine whether spatial visualization skills can be a predictor of success in learning analytic geometry of space. The study used a quasi-experimental design with a control group. There were 46 students in the experimental group and 48 students in the control group. The Purdue Spatial Visualization Test (PSVT) was applied as both pre- and post-test to measure the students' spatial visualization skills. Furthermore, the Prior Knowledge Test (PKT) was administered at the beginning of the study to measure their prior knowledge, and the Academic Achievement Test (AAT) was used to evaluate their ability in analytic geometry of space. The results of the study showed that using dynamic geometry software was effective for improving students' spatial visualization skills. It was also found that spatial visualization skills are a significant predictor of success in spatial analytic geometry.

Key words: Spatial visualization, dynamic geometry software, spatial analytic geometry, success.

INTRODUCTION

Geometry is the branch of mathematics that deals with point, line, plane, space, spatial figures and the relationships among these. Baki (2008) summarizes the general objectives of geometry education as recognizing the properties of geometric shapes in plane and space, exploring the relationships among them, defining the locus, explaining their transformations and proving geometrical propositions. These are the key objectives of both two-dimensional and three-dimensional geometry. In fact, within these general objectives, the main purpose is to understand and explain the physical world around us. However, the physical world cannot be explained by two-dimensional geometry alone, because everything around us has a three-dimensional geometric shape. Therefore, it is considerably important to study three-dimensional geometry to achieve this aim.

The field of geometry consists of several sub-branches. One of these branches is analytic geometry, which unites algebra and geometric concepts. Descartes, who is known as the founder of analytic geometry, argued that the concept of coordinates in geometry, which are based on the idea of a point, can be represented by numbers.

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Since the time of Descartes, analysis proper and geometry have continued to develop side by side (Young, 1909). Today, various sub-branches of analytic geometry are also recognized.

Analytic geometry dealing with both the algebraic and geometric forms of geometric shapes in coordinate systems can be conceptually studied according to two main categories: planar and spatial analytic geometry (Ertekin, 2014). While the analytical examination of point, line and planar shapes is discussed in terms of planar analytic geometry, spatial analytic geometry deals with point, line, plane and three-dimensional shapes in space. The main topics in spatial analytic geometry include coordinate systems, vectors, planes and lines in space, and surfaces. Understanding all of these complex concepts requires well-developed three-dimensional thinking skills.

Namely, three-dimensional thinking is required in order to visualize three-dimensional objects in the mind and perform mathematical operations on them. As such, this skill is especially important for the teaching of three-dimensional geometry, because understanding and interpreting the two-dimensional views of three dimensional objects found in textbooks necessitates three-dimensional thinking ability. In this respect, Ertekin (2014) points out that three-dimensional thinking has special significance in spatial reasoning, which involves the location and movement of objects and ourselves, either mentally or physically, in space (National Research Council, 2006). In other words, visualizing the movement of objects and manipulating them mentally requires well-developed spatial ability.

The literature on spatial ability has introduced various terms for this skill, including spatial sense, spatial insight, and spatial reasoning; but there is little consensus on the definitions of these (Bishop, 1980; Pellegrino et al., 1984; Grande, 1990; Bennie and Smit, 1999). Moreover, the definition of spatial ability itself varies according to different scholars. Linn and Petersen (1985), for instance, defined spatial ability in terms of the mental processes used in perceiving, storing, recalling, creating, arranging and relating spatial images. On the other hand, Lohman (1993) defined spatial ability as the capacity to generate, retain, retrieve, and transform well-structured visual images. Smith (2009), furthermore, in stressing that there are numerous definitions for spatial ability, characterized it as being able to view, conceive, and manipulate objects or ideas within the mind's eye.

Given this disparity in the definitions of spatial ability, no standardized classification of its components has been agreed upon; however, researchers from a range of disciplines have defined several elements that make up this skill. According to McGee (1979), for instance, there are two main factors: spatial visualization and spatial orientation. In this respect, spatial visualization refers to the ability to mentally manipulate, rotate, twist or invert a pictorially presented stimulus; while spatial orientation consists of the comprehension of the arrangement of elements within a visual stimulus pattern and the aptitude to keep track of the changing orientation in which a spatial configuration may be presented (p. 893). On the other hand, Linn and Petersen (1985) examined spatial ability according to three main factors: spatial perception, mental rotation and spatial visualization. In their view, spatial perception comprises an individual's ability to determine spatial relationships with respect to the orientation of his or her own body in spite of distracting information. Mental rotation, moreover, consists of the capacity to rapidly and accurately rotate an object on a plane or in space; while spatial visualization involves complicated and multi-step manipulations of spatial information. Thus, spatial visualization tasks may include the use of spatial perception and mental rotation skills.

Maier (1996)'s description of spatial ability goes further, defining five distinct elements: spatial perception, visualization, mental rotation, spatial relations and spatial orientation. The skill of spatial perception is required to determine the location of the horizontal or the vertical in spite of distracting information, while visualization refers to the ability to visualize a configuration in which there is movement or displacement among its parts. Furthermore, Maier's view of mental rotation involves rotating a two- or three-dimensional figure rapidly and accurately in the mind; and the term spatial relations refers to the ability to comprehend the spatial configuration of objects or parts of objects and their relation to each other. Finally, spatial orientation means the ability to orient oneself physically or mentally in space. Among all of these differing interpretations, the element of spatial ability that they have in common is spatial visualization; this skill involves mental visualization of the manipulations of objects by means of the translation, rotation, opening and reversing of an object.

Spatial visualization skills are critical for many different disciplines such as science, technology, engineering and mathematics (Uttal and Cohen, 2012). Well-developed spatial visualization skills are needed for to be successful in those domains. Numerous researches showed that spatial visualization skills are related with success in mathematics and geometry (Panaoura et al., 2007; Rohde, 2007; Meyer et al., 2010; Turgut and Yılmaz, 2012). Moreover, some researches stated that spatial visualization is more related to geometry more than mathematics (Battista, 1990; Grande, 1990; Karaman, 2000). Spatial visualization plays an important role in learning geometry because of requiring the interpretation of visual information. Yenilmez and Kakmaci (2015) stated that the forming of a geometrical shape in space in two or three dimensions mentally and look from different points of view, spatial visualization is the most important part of geometrical thinking.

Spatial visualization involves mental visualization of the manipulations of objects by means of the translation, rotation, opening and reversing of an object. These skills are integral to the study of geometry; and in this sense, traditional geometry instruction has experienced a revolution through the use of technology in mathematics education. In traditional geometry teaching, static drawings of three-dimensional objects on two-dimensional paper may be incomplete, thus causing erroneous perceptions; and even if the drawings are perfectly executed, it is impossible to see the configurations of objects from different angles, due to the static nature of a drawing (Kösa, 2011). Several studies have revealed that students have difficulties in interpreting static diagrams representing three-dimensional geometric objects as a result of these limitations (Parzysz, 1988; Bako, 2003; Accassina and Rogaro, 2006; Baki et al., 2008).

The emergence of dynamic geometry software (DGS) has brought about a significant change in the way that geometry subjects are taught. The term DGS refers to specialized geometry software such as Cabri 2D and 3D; Geometer's Sketchpad; GeoGebra; and so on. This software provides an environment in which students can explore geometric relationships and make and test conjectures (Baki et al., 2011). Because of this feature, using three-dimensional DGS in teaching three-dimensional geometry could facilitate dynamic visualization of three-dimensional shapes; and thus, according to De Villiers (1996), DGS has been the most significant innovation in geometry instruction since the time of Euclid.

One of the most notable three-dimensional dynamic geometry software programs is known as GeoGebra. This program offers a combination of both algebraic and graphic representations of geometric structure through 2D or 3D graphics interfaces, as well as an algebra interface. The 2D graphic interface allows for the construction of planar figures, in addition to performing measurements such as angle, length and area associated with these figures. Furthermore, the 3D graphic interface allows to users to construct lines, vectors, planes and solids such as prisms, pyramids and cones on the screen; these can be viewed from different rotations by dragging them on-screen. Moreover, users can perform vector operations such as vector sum, cross product and dot product. The coordinates of a point or vector and the equations of a line, plane or sphere in space can also be represented on the screen (Kösa and Karakuş, 2010); yet the most distinctive feature of GeoGebra that differentiates it from other 3D software programs is the input bar. For example, if the algebraic equation of a 2D or 3D shape is entered in the input bar, the geometric representation of the shape can be seen in the 2D or 3D graphic interfaces. Thus, any algebraic expression's geometric representation can be seen easily. As an additional feature, GeoGebra users can observe how geometric shapes are affected when the parameters of algebraic expressions are changed.

Because of the benefits previously described, three-dimensional software may help to enhance students' ability to understand and interpret three-dimensional dynamic geometric structures in the study of analytic geometry of space. Earlier studies related to three-dimensional dynamic geometry software have generally focused on their use in teaching analytic geometry (Kösa and Karakuş, 2010; Ertekin, 2014; Baltacı et al., 2015) and developing spatial visualization skills (Chirstou et al., 2007; Oldknow and Tetlow, 2008; Güven and Kösa, 2008; Baki et al., 2011). For instance, Kösa and Karakuş used Cabri 3D for teaching line equations in space and found that the program is potentially very useful for teaching spatial analytic geometry. Similarly, Ertekin (2014) investigated whether Cabri 3D was effective for the teaching of special planes in analytic geometry and demonstrated that Cabri 3D is an effective tool for identifying the equations of special planes and their normal vectors and for drawing their graphs. In addition, Baltacı et al. (2015) explored the potential of GeoGebra for teaching analytic geometry concepts and found that using dynamic mathematics software while studying analytic geometry offered convenience for prospective mathematics teachers.

Some researchers have also implemented training programs using 3D dynamic geometry software to improve spatial visualization skills and found that such programs were successful in developing spatial visualization skills (Chirstou et al., 2007; Oldknow and Tetlow, 2008). Furthermore, Kösa and Güven (2008) used Cabri 3D to teach solid geometry and showed that using Cabri 3D developed prospective mathematics teachers' spatial visualization skills. Arıcı and Aslan-Tutak (2015) examined the effect of origami-based geometry instruction on spatial visualization, geometry achievement, and geometric reasoning of tenth-grade students.

The results of their study showed that origami-based instruction had a significant effect on spatial visualization, geometry achievement and geometric reasoning. Karaman and Toğrol (2009) investigated the relationship between gender, spatial visualization, spatial orientation, flexibility or speed of closure abilities and the performances related to the plane geometry subject of the sixth grade students.

Their results demonstrated that the three predictor variables explained the 35 percent of the variance in plane geometry test scores. Abu-Mustafa (2010) examined the relationship between spatial ability and mathematics achievement of sixth grade students. The results showed that there was a positive correlation between mathematics achievement and students' spatial ability. Although there were many studies about spatial visualization and plane geometry, spatial analytic geometry and spatial visualization skills together was less studied. Moreover, there is no evidence of any research intended for revealing whether spatial visualization skills can be a predictor of success in analytic geometry of space.

Which of the below is the equation of the line through the points $(-1,2)$ and $(3,-4)$?

A) $3x+2y=8$ B) $x-y=4$ C) $2x+3y=3$

D) $3x+2y=1$ E) $3x-2y=17$

$A(2,0,0)$, $B(0,2,0)$ and $C(0,0,2)$ are the coordinates of the corner points of ABC triangle. What is the area of the triangle ABC?

A) $2\sqrt{3}$ B) 4 C) $2\sqrt{6}$ D) 8 E) $4\sqrt{3}$

Figure 1. The examples of PKT and AAT test items

The purpose and research problems of the study

The purpose of this study was to investigate the effects of using the three-dimensional dynamic geometry software program GeoGebra on the spatial visualization skills of pre-service mathematics teachers who were studying analytic geometry of space. Another aim was to reveal whether spatial visualization skills could be a predictor of success in analytic geometry of space. Thus, the following research questions were addressed:

1. Does using the dynamic geometry software program GeoGebra help students to develop their spatial visualization skills?
2. Could spatial visualization skills be a predictor of success in analytic geometry of space?

METHODOLOGY

A quasi-experimental research design was used in this study, which was carried out with one experimental and one control group. Quasi-experimental methods that involve the creation of a comparison group are most often used when it is not possible to randomize individuals or groups into treatment and control groups (White and Sabarwal, 2014).

New teaching strategies can be effective practices in studies that measure one group with a pre-test, implement a treatment manipulation, and then measure the same variable with a post-test (Cohen et al., 2007). The author of this study chose this method in order to determine the effects of using dynamic mathematics software on pre-service mathematics teachers' spatial visualization skills within the context of a course in analytic geometry.

Participants

The participants were 116 pre-service mathematics teachers in the second year of the undergraduate programme in the Department of Elementary Mathematics Education at the Karadeniz Technical University. Each participant was enrolled in Analytic Geometry course in the fall semester of the 2015-2016 academic year. All participants took the Euclidean Geometry course in the first year of their undergraduate program.

The Department of Elementary Mathematics Education at the Karadeniz Technical University has two branches: Class A and Class B. The academic achievement of both classes is equivalent. There are 57 students in Class A and 59 students in Class B. Random assignment was used to determine the experimental and control groups. Thus, Class A is assigned as control group and Class B is assigned as experimental group. However, nine students from control group (Class A) and thirteen students from experimental group (Class B) were eliminated from the sample because they did not take either the pre-test or the post-test. Thus, 46 students were placed in the experimental group, and the control group consisted of 48 students.

Data collection

The data were obtained from three different tests. The first two – the Prior Knowledge Test (PKT) and the Academic Achievement Test (AAT) – were developed by the researcher. The PKT is a multiple choice test that measures students' prior knowledge related to analytic geometry of space. The test consists of 20 multiple-choice items relating to coordinate systems and transformations in the plane, vectors and vector operations in the plane, and equations of line in the plane. The AAT is also a 20-item multiple choice test that includes questions related to analytic geometry of space. The PKT and AAT were administered with 94 students; the reliability of the tests in terms of the Cronbach alpha internal consistency coefficient were 0.86 for the PKT and 0.82 for the AAT. In addition, two colleagues examined the PKT and AAT for content validity and reported that the test items were suitable for measuring the objectives. The first problem provides an example of the PKT test items, and the second one illustrates an example of the AAT test items in Figure 1.

Aside from these two instruments, the Purdue Spatial Visualization Test (PSVT) was used to measure the participants' spatial visualization skills. The test was developed by Guay (1980) and consists of 36 multiple choice items in three sections (Developments, Rotations and Views). The questions in the Developments section require the identification of a closed shape that is given as the open view of a solid. In the Rotations section, students are asked to determine the position of a solid after rotation on a given axis and angling. The questions in the Views section require defining the view of a solid from a given viewpoint. The PSVT, which is commonly used to determine spatial visualization skills, was reported by Guay (1980) to have an internal consistency coefficient (KR-20) of 0.89. On the other hand, Sorby and Baartmans (1996) calculated an internal consistency coefficient of 0.80. Figure 2 provides examples from each section of the test.

Procedure

At the beginning of the study, the students in both groups took the

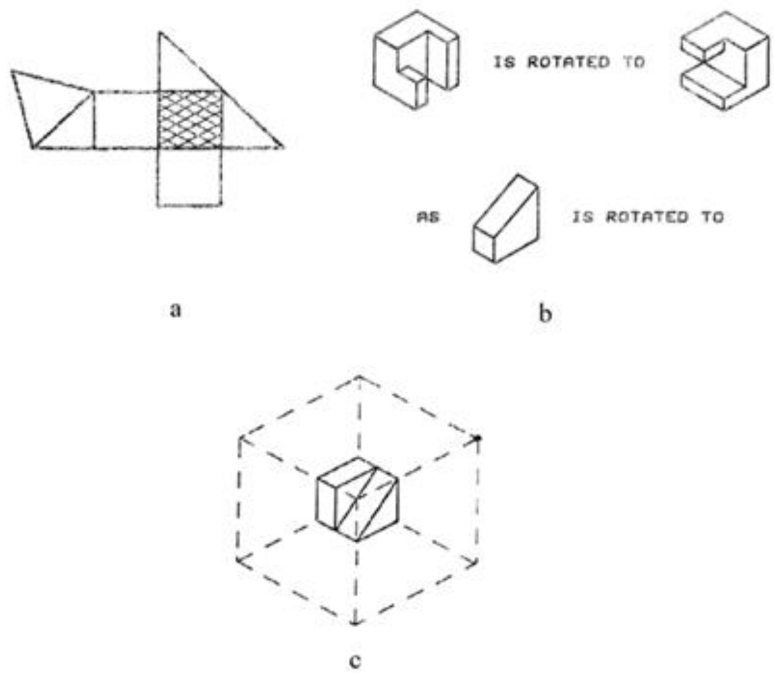

Figure 2. (a) shows a Development section question; (b) shows a Rotation section question; (c) shows a View section question.

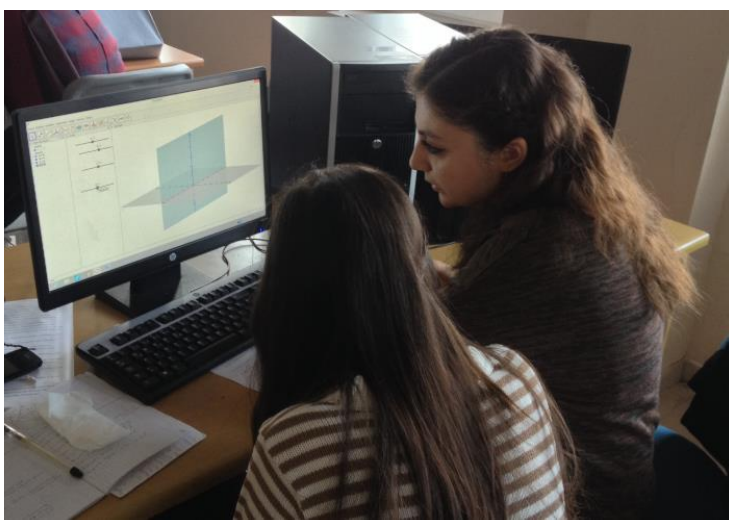

Figure 3. A view of students in the experimental group engaged in learning.

PSVT and the PKT. Then, following the pre-tests, the experimental group received instruction in a computer laboratory. Before the treatment, the students in the experimental group received a three-hour training on using GeoGebra's basic commands and menu functions. During the instructional period, computers were shared by pairs of students. The students in experimental group studied spatial analytic geometry using the GeoGebra program, along with guidance from the researcher, who was also the course instructor. During the treatment, the students constructed some basic geometric forms on their computers and made observations about them (Figure 3).

Meanwhile, the students in the control group took the course in a standard classroom according to a teacher-centered instructional format. The teacher drew figures related to spatial analytic geometry on a blackboard and gave explanations about them, occasionally asking the students questions. The students in the control group solved problems individually.

Throughout the course, which consisted of 3 hours per week over a 10-week period, the same topics in spatial analytic geometry were taught in both the experimental and the control group by the same

Table 1. The content of the course.

Week	Course content
1st week	Coordinate systems in space (Cartesian and Cylindrical coordinate systems)
2nd week	Coordinate systems in space (Spherical and Polar coordinate systems)
3rd week	Vectors in space and basic operations with vectors
4th week	The dot product, the cross product and the volume of a parallelepiped
5th week	Forms of the equation of a line, angle between two lines, distance from a point to a line
6th week	The positions of two lines in relation to each other, the distance between two non-intersecting lines
7th week	Symmetries according to a line (Symmetry of a point according to a line in space, symmetry of a line according to another line in space)
8th week	Equation of a plane, intercept form of the equation of a plane, reduction of the equation of a plane to the normal form, special plane equations
9th week	Angle between two planes, distance to a point from a plane, the positions of a line and a plane in space, the positions of two planes to each other
10th week	Symmetries according to a plane (Symmetry of a point according to a plane, symmetry of a line according to a plane)

Table 2. Descriptive statistics and independent t-test results of the students' PKT scores before the treatment.

	N	Mean	SD	t	p
Experimental	46	73.91	10.26	0.53	0.59
Control	48	72.60	13.28		

teacher. The content of the course is presented in Table 1.

After the treatment, all of the students took the AAT in order to determine whether there was a difference between the achievement levels of the experimental and control group. At the end of the course, the students also took the PSVT.

Data analysis

Before the data analysis, it was examined whether the data are normally distributed. The Kolmogorov-Smirnov normality test showed that the data was in accordance with the normal distribution. Independent sample t-tests were used to determine whether there were differences between the experimental and control group in terms of the PKT and PSVT administered at the beginning of the study. Additionally, analysis of covariance and a paired sample t-test were used to determine whether the intervention had a positive effect on students' spatial visualization skills. After the treatment, an independent t-test was used to determine whether there was a significant difference between the students' academic achievement, and multiple linear regression analysis was used to predict the impact of academic achievement (dependent variable) on spatial analytic geometry using preliminary knowledge and spatial visualization skills (independent variables).

RESULTS

At the beginning of the study, the PKT and PSVT were administered to both groups as a pre-test. Tables 2 and 3 illustrate the descriptive statistics and independent t-test results for the data obtained from the pretests.

As seen in Table 2, the independent t-test results showed no significant difference in the mean scores between the experimental and control group for the PKT ($t=.53$, $p>.05$). This means that the groups were similar in ability at the beginning of the study. According to Table 3, moreover, the independent t-test results indicated no significant difference in the mean scores of each group for the Developments section of the PSVT ($t=.38$, $p>.05$), the Rotation section of the PSVT ($t=-.49$, $p>.05$), the Views section of the PSVT ($t=-.18$, $p>.05$) and the overall PSVT ($t=-.10$, $p<.05$). This shows that there were no statistically significant differences between the spatial visualization skills of the students in either group at the beginning of the study.

After the treatment, the students took the academic achievement test (AAT) related to the course content. Table 4 shows the descriptive statistics and independent t-test results for the data obtained from the AAT.

According to Table 4, the independent t-test results showed no significant difference in the mean scores between the experimental and control groups for the AAT ($t=.75$, $p>.05$). This means that the method applied in the experimental group was not more effective in terms of students' understanding of the course content.

The students in both groups also took the same PSVT test following the treatment. The descriptive statistics for the data obtained from the post-treatment PSVT are presented in Table 5.

As Table 5 demonstrates, the means of both groups' test scores increased after the treatment. In order to determine whether there was a significant difference in the post-test PSVT scores for the experimental and control groups, while controlling for their pre-test scores on the same test, an analysis of covariance (ANCOVA) was performed. Table 6 presents the results of the ANCOVA related to the subsections of PSVT and the overall PSVT.

The ANCOVA results showed that there is was no significant mean difference in the gain scores of the students in experimental and control group with respect to the Developments section of the PSVT ($F[1.93]=0.935$,

Table 3. Descriptive statistics and independent t-test results of the students' PSVT scores before the treatment.

Groups	Experimental Group			Control Group				
Measures	**N**	**Mean**	**SD**	**N**	**Mean**	**SD**	**t**	**p**
Developments	46	7.13	1.61	48	6.97	2.20	0.38	0.705
Rotations		5.80	1.37		5.97	2.01	-.49	0.623
Views		5.63	1.16		5.70	2.60	-.18	0.851
Total PSVT		18.56	2.87		18.66	6.02	-.10	0.917

Table 4. Descriptive statistics and independent t-test results of the students' AAT scores after the treatment.

	N	Mean	SD	t	p
Experimental	46	78.04	11.37	0.75	0.45
Control	48	75.93	15.42		

Table 5. Descriptive statistics of students' PSVT scores after the intervention.

Groups	Experimental Group			Control Group		
Measures	**N**	**Mean**	**SD**	**N**	**Mean**	**SD**
Developments	46	7.43	2.07	48	7.16	2.43
Rotations		7.89	1.56		6.14	2.31
Views		7.15	1.42		5.83	2.87
Total PSVT		22.47	3.43		19.14	6.81

Table 6. Covariance analysis results for each section of PSVT for groups.

Measures	F	Df	p	Partial Eta Squared
Developments	0.935	1	0.336	0.010
Rotations	6.048	1	0.016	0.063
Views	9.048	1	0.003	0.091
Total PSVT	3.439	1	0.037	0.047

p=0.336), where the Developments section of the PSVT pre-test was used as a covariate. However, there was a significant mean difference in the gain scores of the students in the experimental group and control group with respect to the Rotations section of the PSVT (F[1.93]=6.048, p=0.016), where the Rotations section of the pre-test was used as covariate. The effect size, which was calculated using partial eta square as 0.063, would be considered important according to the guidelines proposed by Cohen (1988) (small effects = .01; moderate effects =.06; moderate effects; large effects = .14). The ANCOVA results also indicated a significant mean difference in the gain scores of the students in the experimental group and control group with respect to the Views section of the PSVT (F[1.93]=9.048, p=0.003), where the Views section of the pre-test was used as covariate. The effect size, which was calculated using partial eta square, was 0.091; this would again be considered as an important effect. In addition, the ANCOVA results revealed a significant mean difference in the gain scores of the students in the experimental group and control group with respect to the total PSVT (F[1.93]=3.439, p=0.037). The effect size, which was 0.047, would be considered as moderate in this instance.

Given the eta squared values for the experimental group, we can conclude that there was a moderate effect in the Rotations and Views sections, as well as the overall PSVT scores, that were obtained before and after the treatment.

Multiple regression Table 7 shows the results of this analysis was then employed to disclose whether spatial visualization skills and preliminary knowledge related to plane analytic geometry were effective in predicting achievement in spatial analytic geometry.

Table 7. Regression analysis results related to prediction of academic achievement in analytic geometry.

Variable	B	Standard Error	β	t	p	Binary r	Partial r
Constant	19.745	3.885		5.083	0.000		
PKT	1.589	0.174	0.662	9.125	0.000	0.894	0.691
PSVT	0.331	0.083	0.289	3.984	0.000	0.820	0.385

R=0.911, R^2=0.829. p<0.01.

As a result of the regression analysis, it can be said that preliminary knowledge related to plane analytic geometry (PKT) and spatial visualization skill (PSVT) variables had an effect on success in spatial analytic geometry. The preliminary knowledge and spatial visualization skill variables together explain 82% of success in spatial analytic geometry. According to the standardized regression coefficients, the predictor variables' order of importance is PKT (β =0.662) and PSVT (β =0.289) respectively. On examining the significance of the tests' regression coefficients, it can be seen that the predictor variables of PKT and PSVT variables, with a level of p<0.01, are significant predictors of success in spatial analytic geometry.

DISCUSSION AND CONCLUSION

One of the main purposes of this study was to investigate whether using the three-dimensional dynamic geometry software program GeoGebra in teaching analytic geometry of space could promote pre-service mathematics teachers' spatial visualization skills. The descriptive statistics for the PSVT pre- and post-tests showed an increase in both groups. Furthermore, to determine whether there was a significant difference in the PSVT post-test scores for the experimental and control groups, while controlling for their pre-test scores on the same test, an analysis of covariance (ANCOVA) was performed. The ANCOVA results indicated that while there is no significant mean difference in the gain scores of the students in the experimental and control groups with respect to the Developments section of the PSVT, there were significant mean differences in their gain scores with respect to the Rotations and the Views sections, as well as the overall PSVT.

While studying spatial analytic geometry in the treatment, the experimental group students performed implementations such as rotating planes in space and observing the geometric structures from the different viewpoints using GeoGebra. The differences in their scores may stem from these implementations. In this sense, a number of previous studies have demonstrated that instruction using computer-based 3D visualizations can assist learners in developing spatial visualization skills (Olkun, 2003; Chirstou et al., 2007; Oldknow and Tetlow, 2008). However, the instruction used in the present study was not directly designed to improve spatial visualization skills.

Some studies showed that spatial visualization skills can be developed by proper instructions by using different types of concrete materials (Arıcı and Aslan-Tutak, 2015; Baki et al., 2011). Folding paper exercises (such as origami) related with geometry or physical manipulatives (such as models of geometric objects) provide students a direct experience to develop their spatial skills by manipulating, rotating, twisting or inverting the objects. Several studies have also shown that spatial visualization skills can be developed using computer software (Sorby and Baartmans, 1996; Olkun, 2003; Christou et al., 2007). However, the software used in these studies was two-dimensional or these studies focused on two-dimensional Euclidean geometry. There are some studies which used three-dimensional software to develop spatial visualization (Accassina and Rogaro, 2006; Kurtulus and Uygan, 2010; Güven and Kösa, 2008). But none of these studies were related with spatial analytic geometry. In the present study, it was found that spatial visualization skills can be improved using 3D dynamic geometry software specifically in the context of studying analytic geometry of space. This is the most important contribution to the literature.

In the relevant literature, a number of other studies have shown that three-dimensional dynamic geometry software can help to develop spatial ability (Oldknow and Tetlow, 2008; Güven and Kösa, 2009; Baki et al., 2011). In this respect, the feature that enables the construction and animation of three-dimensional shapes on a two-dimensional screen could be a key factor in the development of spatial visualization skills. This possibility is supported by the frequent use of implementations such as rotating and imagining views from different perspectives in training programs that are directly designed to improve spatial skills. However, the students in the present study developed spatial visualization skills in the process of studying analytic geometry of space using GeoGebra software. This means that spatial visualization skills can be developed using dynamic geometry software in the context of a geometry course.

Many mathematics educators have asserted that prior knowledge is extremely important to success in learning any topic in mathematics. The importance of prior knowledge cannot be denied, of course; however, the results of this study suggest that success in spatial analytic geometry cannot be explained by this factor alone. Namely, the multi-regression analysis that was implemented in this case showed that both prior knowledge and spatial visualization skills together explained 82% of success in spatial analytic geometry. In other words, in addition to prior knowledge, spatial visualization skills are also a significant predictor for success in spatial analytic geometry. On the other hand, there is no evidence that spatial visualization skills alone can be a predictor for success in this subject.

Several studies have put forth that one of the most important factor in geometry success is spatial ability (Battista, 1990; Karaman, 2000; Panaoura et al., 2007; Turgut and Yılmaz, 2012). However, other factors such as preliminary knowledge, attitudes towards mathematics, teaching methods, etc. are important factors in geometry achievement as well as spatial ability. The present study showed that both prior knowledge and spatial visualization skills together explained the large part of the success in geometry. However, some studies demonstrated that geometrical success is an effective factor on spatial visualization skill (Turgut and Yılmaz, 2012; Yenilmez and Kakmaci, 2015). It means that spatial visualization skill and geometry achievement are the variables affecting each other.

The present study investigated that the effects of using dynamic geometry software on pre-service mathematics teachers' spatial visualization skills in teaching spatial analytic geometry and whether spatial visualization skills can be a predictor on analytic geometry success or not. To explore this matter further, additional studies may be carried out to with respect to other factors that may influence spatial visualization skills and success in spatial analytic geometry.

Conflict of interests

The author has not declared any conflicts of interest.

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