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The difficulties that the undergraduate students face about inner product space

Hülya Burhanzade* and Nilgün Aygör

Department of Mathematics, Faculty of Art and Science, Yildiz Technical University, Davutpasa 34210,Istanbul, Turkey.

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In this qualitative research, we studied difficulties that undergraduate students face while learning the concept of inner product space. Participants were 35 first-year mathematics students from Yildiz Technical University in the 2011 and 2012 academic years. We asked participants to solve 5 inner product space questions. Data were jointly analyzed by researchers. After evaluating the results, we grouped students' answers into "right," "wrong," or "blank." The foremost difficulty that the students encountered while studying inner product spaces is that they could not understand the concept thoroughly and they did not know how to choose the elements of inner product spaces. Additionally, it was seen that because the students simply memorized the concepts instead of learning and understanding them they made conceptual and procedural errors.

Key words: Inner product space, conception learning, mathematics education.

INTRODUCTION

The purpose of mathematics education is to ensure that all students develop the highest level of learning ability by advancing their analytic thinking. Nevertheless, a majority of students experience difficulty in mathematics, and, consequently, this affects their success in other areas. Students often find mathematics difficult because they have trouble learning abstract concepts. However, such difficulties can be eliminated or reduced by concretizing these abstract concepts and using appropriate examples during education.

Even though as a whole, mathematics courses are perceived to be difficult, this does not apply to all mathematical topics and concepts. Further, the difficulty level of all mathematic concepts is not the same. Indeed, students consider some topics relatively more difficult than others. Research studies identifying topics that students find "easy" or generally "difficult" have been considered significant in steering education and guiding planners and teachers (Gürbüz et al., 2011).

To make a definition or concept consistent in mathematics, one must understand it by learning its features through its own mathematical terms rather than through memorization while actually as being unaware of its content (Dilber et al., 2000). Permanent and functional learning in mathematics is possible only by balancing operational and conceptual knowledge (Noss and Baki, 1996). Redundancy in the number of concepts learned and the inability to associate these concepts with the

*Corresponding author. E-mail: hulyab@yildiz.edu.tr.

information available have further increased students' learning difficulties at the university level (Kar et al., 2011). When the balance between operational and conceptual knowledge is stabilized, students can attain success in learning concepts.

Specifically, we can categorize the purposes behind the teaching of linear algebra, taught in the first year in the Department of Mathematics, under two headings: the first purpose is to enable its application in other mathematical fields; some of this course's topics have a wide range of application; for instance, in analysis, differential equations, and probability. We also see possible application of linear algebra in physics, biology, chemistry, psychology, sociology, and in all branches of engineering. The second purpose is to introduce axiomatic mathematics to students. This purpose also ensures that students have a better understanding of abstract concepts and improve their skills in this regard.

Linear algebra is an abstract field of mathematics used to introduce MA and PhD levels in Turkish Universities until 1960. After that, it became an undergraduate course. The importance of linear algebra topics can be discussed in two dimensions. The first is the application of linear algebra not only to mathematics departments but also to other departments in the arts and sciences, engineering, and even the social sciences faculties. And the second is creating a base for abstract lectures introduced in the sophomore year. Since linear algebra is one of the most important subjects in mathematics, the basis of abstract algebra, students are required to learn at a higher level than previously (Ozdag and Aygor, 2011). Linear algebra and calculus are the two main mathematical subjects taught in science universities. However, this teaching has always been difficult. In fact, during the last two decades, it has become an active area for research in mathematics education in several countries. In most universities, science-oriented curricula contain calculus and linear algebra.

Mathematics education research first developed works on calculus; however, in the past 20 years, many studies have been conducted on the teaching and learning of linear algebra (Dorier, 2002). In the first phase of his study, prepared based on his personal experiences of lecturing in linear algebra, Haddad (1999) discussed difficulties that students experience in learning linear algebra based on three different perspectives: the nature of linear algebra, teaching of linear algebra, and how students learn linear algebra. He categorizes the root causes of students' inability to learn linear algebra as follows: students' inability in sufficient abstract thinking despite the course being abstract, the axiomatic character of linear algebra, and students' is having inadequate math bases (Tatar, 2006). Conceptual exercises and introducing concepts in an exploratory way allow linear algebra to be understood abstractly.

Indeed, it is quite easy to relate to the basis of linear algebra. Linear algebra expresses itself to students in a clear, strong way, and teachers should convey linear algebra to students just as it guides teachers themselves. Only such an approach can make this class dynamic and beneficial. Still, with this approach, understanding some concepts and applications may take some time (Uhlig, 2002). Hillel divides the languages used in linear algebra into three basic sections:

1. "Abstract language" of the general abstract theory
2. "Algebraic language" of the R^n theory; and
3. "Geometric language" of two- and three-dimensional spaces.

A teacher who drifts from one language to the other during the course of the teaching without clearly warning students is not aware that symbolizations become a problem for students. What students are mostly confused by, or they cannot understand, is the transition from abstract symbolization to algebraic symbolization while working in the R^n space (Aydin, 2009). One of the basic challenges in learning linear algebra is related to viewpoints and structures that can be used to symbolize abstract concepts (Dias, 1995). Students should be able to distinguish concepts from their symbolizations and drift from one to the other. Activation of all these abilities depends on the teachers' attitudes. This is the main theme of studies on learning and teaching linear algebra (Dorier et al., 2000).

In this research, during tests, university students were observed making mistakes because they could not exactly understand abstract concepts regarding inner product spaces. Therefore, this study aimed to reduce these mistakes to a minimum level. If an instructor knows in which topics students experience which difficulty, he or she will provide a better understanding through the selection of appropriate teaching methods thus minimizing the learning challenges. This study's purposes are to determine which difficulties undergraduate mathematics students encounter in learning concepts of inner product space and enlighten concerned instructors.

METHODOLOGY

This research sample involved 35 first-year students at the Department of Mathematics, Faculty of Arts and Sciences, Yildiz Technical University, during the 2011 and 2012 spring semesters. The study was conducted to determine what difficulties students encounter in the matter of inner product space and to analyze their lack of knowledge. In this research, students were asked to answer five questions about inner product spaces. The topic of inner product spaces had been explained to all students participating in the research. Questions were selected from concepts frequently encountered in different years and necessary for learning linear algebra course material at the university. Experts in the field checked the questions for reliability and validity. The examination time was 60 min.

Problem used in research

The following questions were presented to students in order to

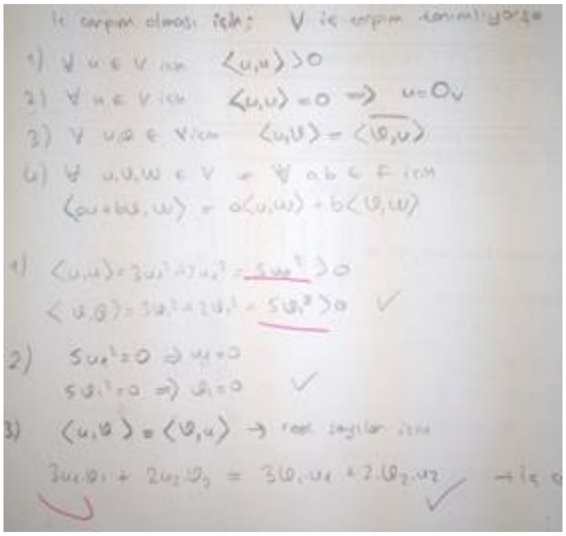

Figure 1. Student A's answer to the first question.

obtain this study's research data. The examination consisted of five questions on the concept of inner product spaces. The questions involved concepts that students could encounter in the later years of study; further, they are necessary for learning linear algebra course material. In the evaluation of the information obtained, students' answers were classified and tabulated as true, false, and blank.

1. i) Write the definition of inner product.
   ii) Let $u = (u_1, u_2)$, $v = (v_1, v_2) \epsilon\ R^2$ be. Show that $\langle \mathrm{u}, v\rangle = 3u_1v_1 + 2u_2v_2$ is an inner product.
2. i) Write the definition of orthogonal set.
   ii) Find the value of a for which the set $\{ [1\ \ 0\ \ \mathrm{a}],\ [-1\ \ 0\ \ 1],\ [0\ 1\ 0] \}$ is orthogonal in R^3 .
3. i) Write the definition of orthogonal complement.
   ii) Let W be the subspace of R^4 spanned by the vectors $\mathrm{S} = \{[1\ 0 - 1\ 1],\ [1\ 1\ 0\ 1]\}$. Find the orthogonal complement ($\mathrm{W}^{\perp}$) of a subspace W.
4. i) Define the matrix of the inner product $<,>$ relative to the basis S.
   ii) Find the matrix of the inner product $\langle \mathrm{x}, \mathrm{y} \rangle = \mathrm{x}_1\mathrm{y}_1 - 2\mathrm{x}_1\mathrm{y}_2 - 2\mathrm{x}_2\mathrm{y}_1 + 5\mathrm{x}_2\mathrm{y}_2$ relative to the basis
   $\mathrm{S} = \{(\ 1\ \ 4\ ), (\ 2\ \ -3\ )\}$.
5. i) Define the inner product denoted by the matrix with respect to standard basis.
   ii) Find the inner product denoted by the matrix $C = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$ of R^2 with respect to standard basis.

FINDINGS

Assessed qualitatively, this study's findings were created and interpreted based on the written answers to the questions posed to the students, who concentrated only on performing the operations. Therefore, they tried to solve the questions by ignoring concepts outside of the operation in the given question. However, it is impossible to arrive at the solution of the questions related to the concepts' properties without fully understanding the concept itself. Some answers by five students (Student A,

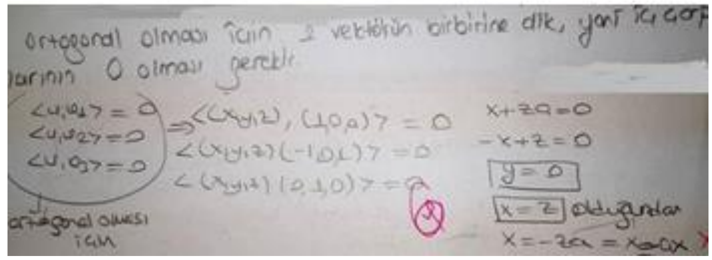

Figure 2. Student B for to the second question.

Student B, Student C, Student D, and Student E) who tried to solve the questions without fully understanding the concepts are illustrated below through a scanned copy of their answer sheet (please note that we have not made any changes to the answers of all the students). Student A's answer to the first question is shown in Figure 1

Although Student A wrote the definition of inner product space correctly, the student could not apply the example to the fourth condition of inner product space conditions. When we examined other students' papers, we observed that a majority of students wrote the definition correctly; however, like Student A, they could not apply the fourth condition to the example. Some students tried to take its complex conjugate by ignoring that the field is one of the real numbers in the third inner product space condition; and some other students chose the elements of inner product space incorrectly.

Student B's answer of Student B for to the second question is shown in Figure 2.

Although Student B knows the definition of orthogonal set, the student used the definition of orthogonal complement, confusing the two definitions while performing the operations. When we examined other students' papers, we observed that a large portion of students confused the definitions of orthogonal set with the orthogonal complement. Student C's answer to the third question is shown in Figure 3.

Although Student C knows the definition of orthogonal complement, the student could not solve the equation system due to lack in the background knowledge. When we examined other students' papers, we observed that, like Student C, the vast majority of students could not solve the equation system. Furthermore, although some students did solve the equation system, they were unable to reach the conclusion because they selected the wrong arbitrary values. Because some students could not select the element from R^4, they could not establish the equation, and, therefore, they could not reach the conclusion. Student D's answer to the fourth question is shown in Figure 4.

Although student D knows the matrix definition of inner product according to a base, the student could not apply the definition to the example. Further, the student could not contemplate applying the defined inner product

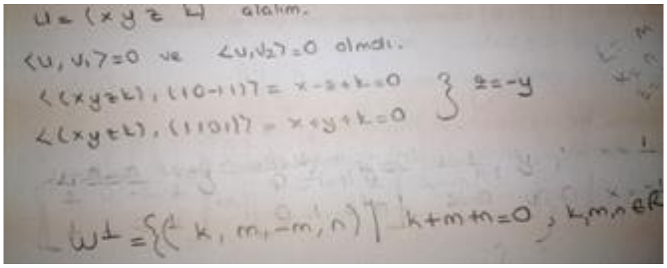

Figure 3. Student C's answer to the third question.

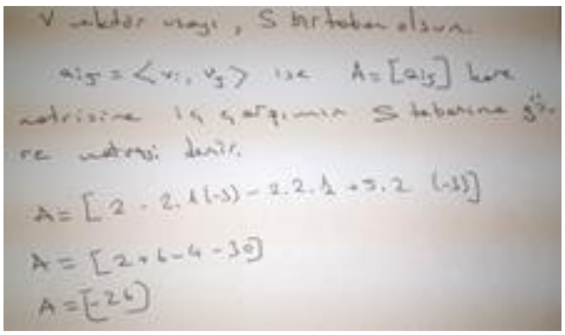

Figure 4. Student D's answer to the fourth question.

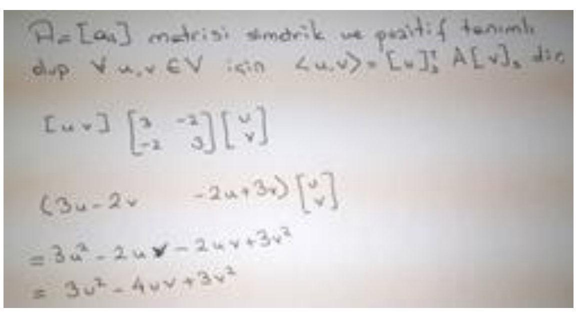

Figure 5. Student E's answer to the fifth question.

separately to the vectors in S base for the elements of a_{11} ,$a_{12} = a_{21}$, a_{22}. When we examined other students' papers, we found similar errors. Additionally, some students performed an error in the operation. Student E's answer to the fifth question is shown in Figure 5.

Although student E knows the inner product definition of matrix according to a base, the student performed an operation by taking a vector formed as $[u, v]$ instead of taking two vectors formed as $u = (u_1, u_2), v = (v_1, v_2)$. This resulted because the student did not know how to select the element in R^2. When we examined other students' papers, we observed that some students wrote the definition incorrectly, and some performed an error in the operation. Overall, students' answers were grouped as true, false, and blank and then converted to table form.

As can be seen from Table 1, although the vast majority of students correctly wrote the definitions required in part i) of the questions, they could not apply these definitions to the examples given in part ii) of the questions. In other words, although 76% of students wrote definitions correctly in answer to part i) of the questions, only 36% could give correct answers to part ii). Students could not apply these definitions to examples even though they knew the definitions. In the second and third questions, where background knowledge is necessary, many students could not solve the questions correctly because they could not recall their background knowledge. In other words, students do not know how to combine their background knowledge and new information. These two cases indicate that students wrote definitions from memory but did not understand them conceptually.

CONCLUSION AND RECOMMENDATIONS

In his study, Tall (1993) classified learning difficulties in mathematics as follows:

1. Insufficient understanding of basic concepts
2. Inability to formulate verbal problems mathematically, and
3. Inability in algebraic, geometric, and trigonometric skills.

Moore (1994) examined difficulties that university students experience in learning how to do mathematical proofs, and he identified these difficulties as follows:

1. Understanding the concept
2. Mathematical language and notation, and
3. Starting the proof.

Besides that, students' perception of the methods of mathematics and proofs influence how they go about the steps in the proof. Similar to what Tall (1993) and Moore (1994) reported, this study determined that students cannot quite form conceptual definitions in their minds, have difficulty in understanding concepts, and cannot perform applications involving the use of such concepts. We can easily see this by comparing parts i) and ii) of the questions.

Harel (1989) studied causes of students' learning difficulties related to basic concepts in linear algebra and how a program should be designed to overcome the difficulties. In this study, reasons for students' difficulties are as follows: first, concepts are abstract structures; second, their application areas are unusual for students; and third, most students have yet to learn proof and axiomatic methods. Moreover, the author mentioned the importance of visualization in overcoming learning

Table 1. Number of true, false and blank answers with their respective percentages.

Variable	Q1				Q2				Q3				Q4				Q5			
	i.		ii.		i.		ii.		i.		ii.		i.		ii.		i.		ii.	
	F	%	F	%	F	%	F	%	F	%	F	%	F	%	F	%	F	%	F	%
True	33	94	20	57	28	80	13	37	24	68	10	29	28	80	12	34	20	57	8	23
False	2	6	9	26	3	9	12	34	8	23	13	37	1	3	13	37	7	20	14	40
Blank	-	-	6	17	4	11	10	29	3	9	12	34	6	17	10	29	8	23	13	37

difficulties by stating that the basic concepts in linear algebra are not shown geometrically. In other words, students will have difficulties learning these concepts if they have not visualized them correctly. Further, the author stated that this is the primary cause of difficulties in learning abstract concepts.

This study is consistent with the author's view, in that it determined that students are not fully able to conceive the definition of a concept; they have difficulties in understanding a concept and they are unable to apply the concept in problems.

This is a consequence of memorizing the concept without actually understanding it. In this study as well, students' learning difficulties in linear algebra were found as follows: failing in abstract thinking although the topics require abstract thinking, poor conception of definitions, incapacity to interpret verbal expressions, and inability in the readiness level.

Both in mathematics and other areas, learning difficulties that students often experience are identified, and studies can be conducted in order to resolve these difficulties. Instructors should follow this type of research on the basis of the subject and must be aware of the kinds of difficulties that their students encounter in the subject that they teach. To minimize learning difficulties that students experience the following methods have to be adopted:

1. Instructors should explain concepts used in definitions through examples.
2. They should emphasize on which field the inner product space lies and show how elements should be selected.
3. In examples that define an inner product different from the standard inner product, they should dwell on the newly defined inner product and solve the examples.

In addition, instructors, while lecturing on a subject, should remind students of the basic information connected to the subject and increase students' readiness level. Instructors should reshape their courses by paying attention to these basic matters. In short, they should provide mathematics teaching by balancing operational and conceptual knowledge and should use materials that reduce the abstractness of the concept discussed.

Conflict of Interests

The authors have not declared any conflict of interests.

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