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Developing a leveling framework of mathematical belief and mathematical knowledge for teaching of Indonesian pre-service teachers

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This study explored the characteristics of pre-service primary teachers (PSTs) influenced by mathematical belief and mathematical knowledge for teaching (MKT) PSTs'. A qualitative approach was used to investigate the levels of PSTs on mathematical belief and MKT. The two research instruments used in this study were an interview-based task and a questionnaire. Data collected from 6 (six) pre-service primary teachers which had completed all courses in teacher education and have joined the practice of teaching in elementary schools. Research results show that there are five levels of mathematical belief and mathematical knowledge for teaching, starting from 0 to 4, with different characteristics. The differences are based on belief systems and indicators of MKT.

Key words: Mathematical belief, mathematical knowledge for teaching (MKT), levels, pre-service primary teachers (PSTs).

INTRODUCTION

The philosophical view of teacher's role in teaching and learning oriented at teaching impact divides teachers into three types (Ernest, 1989). First is the role of teacher as an instructor representing the instrumentalist philosophy that emphasizes skills mastery with expected abilities, and it is the lowest level. Second is teacher with explanatory role representing the Platonist philosophy that has the orientation of conceptual understanding with holistic knowledge. Third is teacher as facilitator representing the philosophical view concerning problem solving, which is the highest level of a teacher's role. Such a role of teacher as a practitioner, according to Ernest (1989) and Philipp (2007) is affected by his or her belief.

Educators of teachers should understand and recognize not only the belief of pre-service teachers, but also the way they obtain it because the way teachers gain their belief has influence on their belief. Hence, eventually the transformations to be made can be identified (Philipp, 2007). Considering the weakness of pre-service teachers and teachers in the contents of Arithmetic and Geometry, namely weak content knowledge (Hinton et al, 2014) and weak pedagogical knowledge, according to Thompson (as cited in Philipp, 2007), it is important then that research considers the interrelatedness of belief and knowledge simultaneously.

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The leveling of mathematical belief and MKT will be in the form of comparison of the characteristics of mathematical belief and categories of MKT. The designed categories of MKT are suited to research results demonstrating that PSTs who do not master content and cannot identify concept central to the topic are unable to identify the interrelatedness of content (connection) and to make representation, as well as having weak pedagogical knowledge (Somayajulu, 2013). Therefore, according to Moursund (2005), mathematical maturity is not basic knowledge of the special field of mathematical content; rather, it is the development of mathematical content.

THEORETICAL FRAMEWORK

Conceptions of Mathematical Belief

There are various definitions of belief. Ernest (1989) has defined belief as a view or conception of the nature of mathematics, a naturalistic model or perspective of teaching mathematics, and a model or perspective of mathematics learning process. Another definition says that belief is a combination of inferences on various phenomena and their nature (Törner and Pehkonen, 1999). According to Philipp (2007), in general there is no agreement for an established definition of belief. Very often, the construction of belief is related to value and knowledge. Belief is a right/wrong construction in a certain context, while value in general refers to what is desired/undesired, and knowledge is a belief assumed to be true based on facts from one's own experiences.

The categories for Mathematics teachers' belief have been formulated by Beswick (2012) to include a belief about the nature of Mathematics, a belief about Mathematics teaching, and a belief about Mathematics learning. The lowest level has the instrumentalist view of the nature of Mathematics; teaching Mathematics means focusing on content with an emphasis on results; learning Mathematics is the mastery of skills; and students take the passive attitude. In the moderate level is the Platonist view of the nature of Mathematics; teaching Mathematics focuses on content with an emphasis on experience; learning belief is that students actively construct knowledge. The highest level is the question solving view with a belief that teaching should focus on each individual ability; learning is believed as an activity of exploring knowledge independently according to interest.

Mosvold and Fauskanger (2013) add to the three categories, a belief about knowledge or epistemological belief. In addition, the instrumentalist belief views Mathematics as knowledge in the form of content memorization, the Platonist belief sees Mathematics as knowledge with content understanding, and the problem solving belief views Mathematics as knowledge with adjustment and differentiation.

Mathematical knowledge for teaching

Knowledge originates from belief assumed to be true (Phillip, 2007) and in educational context, teacher's practices are influenced by content knowledge (CK) and pedagogical content knowledge (PCK) owned (Shulman, 1987). CK is associated with a number of facts or concepts of subjects, including the understanding of the content of what and why as well as the facts and concepts, and how knowledge is structured in particular science. While the PCK is a special blend of content and pedagogy as a form of professional understanding, i.e. how particular topics, questions or issues are organized, presented and adapted to the diverse interests and abilities of students, and served for teaching. Shulman's theory progresses to encourage the emergence of other teaching theory, when teachers find it difficult to teach Mathematics. This is because teaching Mathematics requires special knowledge because of the scientific complexity in Mathematics. The specialties of teaching knowledge are important to identify that the duty of the teacher is unlimited, not only to provide the true or false information, but also to diagnose the fault of students and provide appropriate referrals (Ball et al., 2008), and help students develop ideas and mathematical reasoning.

The teaching knowledge according to Ball et al, (2008: 395) is called MKT, that is the "..*Mathematics knowledge that teachers need to carry out theirwork as teachers of Mathematics..*". This mathematical knowledge used by teachers in the classroom is to make teaching and student development. Some landslide projects are done to develop knowledge of Mathematics teaching, especially in the so-called MKT. Among them is the LMT project at the University of Michigan who develops the MKT models derived from the concept Shulman to clarify the difference between CK and PCKP and develops the measurement of MKT (Ball et al., 2008). Furthermore TEDS-M project also develops a MKT measurement with the specific terms Mathematics CK as Mathematics content knowledge (MCK) and Mathematics PCK as mathematics pedagogical content knowledge (MPCK) in teacher education to include the measurement of belief (Tatto et al., 2008). Identifying knowledge of Mathematics teaching is also carried out at the University of Cambridge with the term Knowledge Quarter (Rowland and Turner, 2007). The results of the teaching practice of teachers in the classroom generate 17 codes which are then grouped into four categories, namely foundation, transformation, connection, and contingency. According to Petrou and Golding (2011) one of the categories present in the knowledge Quarter is contingency that has combined CK and PCK as suggested by Shulman, namely to identify different interactions of teacher knowledge.

Category MKT in LMT project provides equal portions and balance between MCK and MPCK, while knowledge Quarter has been put CK mastery as a foundation towards MCK and MPCK mastery in contingency. The study of Somayajulu (2013) shows the effect on the content knowledge by pedagogical knowledge. The study makes the level based on the results of tests in analyzing students' thinking. Level of PSTs are (0) Naive, (1) Developing, and (2) Mature.

Purpose of the research

In Indonesia, most educators of PSTs have a master degree in education. Teacher candidates study for 8 semesters and get the compulsory subjects of social sciences, pedagogy, language, art, science, and Mathematics.

The course includes the study of Mathematics, namely Mathematics I which focuses on Arithmetic, Mathematics II which focuses on Geometry, and Learning Mathematics. In the first half of semester 7, all PSTs join the practice of teaching in elementary schools by acting as a classroom teacher who administers all subjects. The class teachers must have competence as stipulated in the Regulation of the Minister of National Education of the Republic of Indonesia NO.16 2007 on Standards of Academic Qualifications and Competencies of Teachers related to professional competence. They are also required to master all subjects in elementary school.

The purpose of the research is to develop a conceptual framework of empirical level of mathematical belief and MKT which are arranged based on existing literature, including categories that emerge from each of these levels.

RESEARCH METHOD

The study was conducted in the Tarbiyah Faculty of The State Institute in Indonesia. The students had completed all courses in teacher education and have joined the practice of teaching in elementary schools.

A qualitative approach was used to test the characteristics of the draft of the leveling of mathematical belief and MKT formulated for PSTs. Data were gathered through interview-based task conducted to last semester students and were analyzed using constant comparative method or grounded theory in order to obtain a theory (Glaser and Strauss, 1967). The method to determine the sample was purposive sampling. Finally, 6 (six) PSTs with good communication ability and heterogeneous result of study (Cohen et al., 2000) in State Institute of teacher education were selected. The tasks given were in the form of open-ended questions with the test instrument of MKT modified from Cheang et al (2007) research results and have been validated by trials with reliability value 0.755. The questionnaire to identify PSTs' mathematical belief was developed from Tatto et al. (2008). In addition, triangulation was used by conducting in-depth interview to the sample. Results of test and questionnaire distributed to pre-service teachers were analyzed by identifying the aspects of mathematical belief and MKT.

The results of tests and questionnaires were analyzed with PSTs' aspects of the mathematical belief and MKT. Level of PSTs is determined by applying the method of qualitative analysis to determine the level, and then validated by experts in Mathematics education. It is beneficial to compare the framework arranged by the existing framework in the literary source, which eventually can add new knowledge and contribute to the field (Table 1).

The research focuses on identifying mathematical belief and MKT of PSTs and describing the levels. The PSTs provide solutions showing their knowledge of mathematical content, where procedural counting ability is separated from the ability to memorize and to define (Hinton et al., 2014). Level 0 and level 1 show that PSTs do not master pedagogic knowledge because they have instrumentalist belief with an inclination to prioritize more the ability to memorize formulas than concept understanding. Level 2 and 3 have mastered the content and are in the development stage of pedagogical content knowledge. These levels believe that PSTs need to understand more the concept than to memorize it. Level 4 believes that Mathematics presented through an exploration of questions independently can develop students' mathematical knowledge corresponding with their ability.

RESULTS

Based on data from the task-based interviews and questionnaire on 6 PST students, it is acquired conceptualization verification in every part of the level of mathematical belief and MKT. The characteristics of the levels of mathematical belief and MKT constitute different aspects for each level. The differences lie on MCK and MPCK according to the levels of mathematical belief, namely instrumentalist, platonist, and problem solving. Nevertheless, there are similar characteristics in the levels. For instance, PSTs with the same belief level have different levels of MKT. At the levels 1 and 0, PSTs tend to have instrumentalist belief, but at the level 0 it is not developing. They have difficulties in procedural counting and are unable to give representation of a mathematical idea. At level 1 with the same belief, PSTs have mastered a little mathematical concept. Ascending to the levels of 2 and 3, here PSTs tend to have the platonist belief about explaining content and understanding mathematical content as holistic knowledge. Comparing level 2 to 3, PSTs have mastered concept and are able to give representation, but not thoroughly able to connect mathematical ideas and topics. The highest level, level 4, or mature, is where PSTs have problem solving belief and master MKT. The descriptions for the characteristics of each level are as follows:

Level 4: PSTs believe that Mathematics can be presented by problem solving which facilitate various students' abilities.
MCK: Able to master counting and to define a concept, to represent a complete picture, and to identify mathematical ideas.
MPCK: Able to relate among the topics, to represent with the aim of explaining comprehensively and meaningfully,

Table 1. The draft of leveling of mathematical belief and MKT.

Mathematical Knowledge for Teaching (MKT)				**Levels of Mathematical Belief and MKT**	**Levels of Mathematical Belief (Ernest, 1989; Beswick, 2012; Mosvold and Fauskanger,2013)**
Mathematical Content Knowledge (Ball et al, 2008; Tatto et al, 2008)	**Mathematics Pedagogical Content Knowledge (Ball et al, 2008; Tatto et al, 2008)**	**Knowledge Quarter(Rowland and Turner, 2007)**	**Somayajulu (2012)**		
The ability to count procedural		*Foundation*	*Naïve*	0 Not Developing	Instrumentalist
The abilities to memorize & to define			Developing	1 Not Yet Developing	
The ability to represent mathematical ideas	The ability to select examples or representations to explain	*Transformation*		2 Quite Developing	Platonist
The ability to make connection of mathematical ideas	The ability to understand mathematical structure and topic	*Connection*		3 Developing	
The ability to make connection of students' ideas and mathematical ideas	The ability to anticipate students' unexpected responses	*Contingency*	Mature	4 Mature	Problem Solving

to identify students' responses comprehensively, and to provide appropriate feedback.

Level 3: PSTs believe that Mathematics needs to be understood, not merely memorizing formulas. They tend to believe that teaching with contextual approach and case-study can be obstacle in teaching and learning because it makes poor students unable to compete with the good ones.
MCK: Able to master counting and able to define a concept, to provide complete representation, and to identify mathematical ideas.
MPCK: The connection between mathematical concept and topic is complete, able to relate it with another topic; representation to explain is completeness and meaningfulness; unexpected students' responses are not completely identified.

Level 2: PSTs believe that Mathematics needs to be understood, not merely memorizing formulas. They tend to believe in teaching with contextual approach.
MCK: Able to master counting and to define a concept; picture representation is quite comprehensive; and mathematical ideas are identifiable limitedly.
MPCK: The connection between mathematical concept and topic is not quite complete and limited in connecting it to another topic; representation is almost comprehensive and meaningful; students' responses are identified less completely.

Level 1: PSTs believe that Mathematics is about counting and memorizing formulas. They tend to say that a collection of formulas should be memorized in order to be successful in learning.
MCK: Able to master counting and to define a concept; picture representation is not complete; and mathematical ideas are identified limitedly.
MPCK: The connection between concept and topic is very limited; representation is very limited and incomplete; students' responses are identified very limitedly.

Level 0: PSTs believe that Mathematics is about counting and memorizing formulas. They tend to say that Mathematics is hard to study.
MCK & MPCK: PSTs still make errors in counting, are very limited to define a concept, unable to give representation; mathematical ideas are limited, and unable to respond to students' obstacle.
A single characteristic of each level is established from constant comparison of the characteristics of two PSTs at the same level; for example, Wina, Cita, and Dina. Cita and Dina have the same belief of mathematical content to do in relation to mathematical procedure, for example $2\frac{1}{4}:\frac{1}{2}$. It is, however, different when doing the problem in relation to the ability of Mathematics pedagogical

Table 2. Results from task based interview.

Level	Participants	Results from test & questionnaire			Results from interview
		MCK Mean	MPCK Mean	Questionnaire	
0	Wina	0.2	0.7	Instrumentalist	Instrumentalist
1	Cita	1.4	0.8	Platonist	Instrumentalist
1	Wida	1.4	1.2	Instrumentalist	Instrumentalist
2	Eri	1.4	1.2	Platonist	Platonist
3	Dina	1.4	1.3	Problem Solving	Platonist
4	Nisa	1.8	2.8	Platonist	Problem Solving

content.

Imagine you are teaching fraction division. In order to make it more meaningful to students, you relate it to the real world using word question. The story you create is concerned with the question of $2\frac{1}{4} \div \frac{1}{2} =$

Wina: I have 2 and $\frac{1}{4}$ chocolate bars, then my friend comes, and I share with him a half of it. The result of the division is what my friend has got. How many has she got?
Researcher: Then, what's the result?
Wina: (drawing two bars of chocolate divided equally . . . and drawing $\frac{1}{4}$ bar of chocolate divided equally) thus becomes 1..$\frac{1}{8}$
Researcher: Please check by calculating it directly, how does it differ from$2\frac{1}{4} \div 2$?
Wina: I'm confused, Ma'am.
When checking Wina's responses to questionnaire, it is found that she has the tendency towards instrumentalist belief. The interview is followed by one with Cita.

Cita: Erik has $2\frac{1}{4}$ of cake (with a picture), then $\frac{1}{2}$ of it has been given to Erika. How many of the cake does he have now?
Researcher: Well, then how many of the cake do you think he has now?
Cita: $4\frac{1}{2}$, Ma'am (looking at the result of the calculation of $2\frac{1}{4} \div \frac{1}{2}$
Researcher: Why can it be $4\frac{1}{2}$? Can you explain from the case of the cake?
Cita: By dividing it into two . . .
Researcher: Can you explain the difference between the case of the cake and the equation of $2\frac{1}{4} \div 2$?
Cita: When directly calculated, the result is indeed different, Ma'am, but I am confused if I have to explain it with pictures . . .
Cita's responses to questionnaire of mathematical belief have an inclination towards the Platonist belief

Dina: Andi has two pieces of cake and $\frac{1}{4}$ of a cake. He will share $\frac{1}{2}$ of the cake with his friends. How many kids will get the shares of the cake?
Researcher: Can you explain the question with a picture?
Dina: (drawing a part of the cake) each cake is divided by $\frac{1}{2}$, Ma'am, so two pieces of cake will be received by four kids and the last one gets a smaller piece.

Researcher: So, how many kids will get the cake?
Dina: 4 kids, with the last one gets $\frac{1}{2}$ part.
Researcher: Doesn't your question ask how many kids?
Dina: Um… then it's $4\frac{1}{2}$ kids (confused..)
Dina's responses to the questionnaire of mathematical belief show she is inclined towards the problem solving belief.

Summary of the results of test and questionnaire for 6 PSTs is in Table 2.

The analysis of test results shows that Cita and Dina succeed in completing $2\frac{1}{4} \div \frac{1}{2}$ procedural calculation and have same idea by equalizing the division by 2. They are difficult to provide representation between 2 and $\frac{1}{2}$. Dina is able to give representation, but the meaning is still rather inappropriate, namely equalizing fraction measurement with people.

DISCUSSION

The research that was limited to 6 students who were given interview-based tasks shows that 1 person (Wina) is identified to be at level 0, 2 people (Wida, Cita) are at level 1, 1 person (Eri) is at level 2, 1 person (Dina) is at level 3, and 1 person (Nisa) is at level 4 . From the results of the test and questionnaire, Cita, Dina, and Nisa

are identified to be inconsistent. Cita and Dina have platonist and problem solving beliefs. Based on the interview, both of them belong to level 1 and level 3, which is instrumentalist and platonist; meanwhile, Nisa has platonist belief that is categorized into level 4, which is problem solving.

The above findings are consistent with Northcote (2009)'s study that inconsistencies is caused by the complexity of belief system, so that researchers can find inconsistencies between teaching belief and teaching knowledge. One of the approaches to take is checking certain belief in the more ultimate belief system, and from here it determines the role affecting knowledge, and compared to another belief through the presentation of education issues in general, such as choice of action in limited time, resources, standardized tests, and student environment. In addition, teacher's belief concerning either student, social phenomena, and education can also be considered. Thus, the research uses written test in the form of analysis of tests on MKT and perspective of student and education experience.

The characteristics of Wida and Cita are categorized under level 1. Different from Cita, Dina, and Nisa who show inconsistencies on their mathematical, Wida shows consistency that she believes that the ideal teaching and learning is one that teaches memorization first, followed by understanding. Test results on MKT also demonstrate that Wida, like Wina and Cita, still has difficulties in providing representation for the given question. Wida's perspective of mathematics is that one can be ensured to master mathematics if s/he is good at memorizing and counting. On the other hand, Cita and Eri believe that learning should begin with understanding first.

The findings show that Cita, Wida, Eri, and Dina have same results on MCK and different results on MPCK and mathematical belief. Based on intervew with PSTs shown that they have different experiences on teacher education learning. This is in line with the findings of Hinton et al. (2014) that PSTs' number sense skills and computational abilities were not found to be related. Participants can solve mathematical problems using procedures; however they may not know the concepts or the reasons behind the procedures.

The lowest level the respondents achieve is level 0 for Wina. When presented with the above question, Wina provides inappropriate and incorrect representation of fraction in procedural counting. Interview results prove that Wina has a negative view on Mathematics, in which she believes that it is burdened with memorizing formula and counting, and she has bad experience with mathematics teachers. Furthermore, Wina explains that she takes teacher education program because of not her own desire. Study by Hinton et al. (2014) stated that PSTs with lower scores in beliefs also had significantly lower computation scores.

The highest level, which is level 4, is achieved by Nisa, who has actual belief in problem solving and can provide representation for the above question. From the interview, it is found that Nisa has background of vocational high school (accounting program), and since junior high school, she has had a change in her view of how to learn mathematics. Her experiences have taught her that when mathematics is studied with conceptual approach, by exploring everything around us, then we will gain better understanding of a certain concept. This is in line with study by Somayajulu (2013) was demonstrated that education background and work experience were a constant influence on how PSTs viewed the mathematical content.

CONCLUSION AND LIMITATION

This research shows that PSTs have been identified by leveling of formulated mathematical belief and MKT. The different levels are based on the MKT indicators (Ball et al., 2008; Tatto et al., 2008; and Rowland and Turner, 2007) and mathematical belief (Ernest, 1989; Mosvold and Fauskanger, 2013). PSTs have been observed in all levels. Learning needs to balance between mastery of content knowledge and pedagogical content knowledge to gain the highest level of mathematical belief and MKT. PSTs need to master mathematics content simultaneously to achieve mathematical maturity.

It is necessary to conduct extended research in the form of experiment to examine the change of belief and MKT. The extended research involves both instrument and qualitative method. Research using only the written test instrument to look at the relationship between belief and MKT will be difficult because it can not provide measured data validation and the deep effect of educational programs. Mixed method by combining quantitative and qualitative data can be an option.

Conflict of Interests

The authors have not declared any conflict of interests.

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