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How to classify the diversity of seventh grade students' mathematical process skills: An application of latent profile analysis

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The researcher applied latent profile analysis to study the difference of the students' mathematical process skill. These skills are problem solving skills, reasoning skills, communication and presentation skills, connection knowledge skills, and creativity skills. Samples were 2,485 seventh-grade students obtained from Multi-stage Random Sampling. Each student was measured by the mathematical process skills test which consists of 44 items in total. The research results indicated that the students can be categorized into three groups: the high mathematical process skills students (2.74%), the moderate mathematical process skills students (40.48%), and the low mathematical process skills students (56.78%). Moreover, from the research results, it also shows that the creativity skills seem to be the problem in every group of students. The detail of the research findings has been discussed in this paper.

Key words: Latent profile analysis, mathematical process skills, problem solving skills, reasoning skills, communication and presentation skills, connection knowledge skills, creativity skills.

INTRODUCTION

Mathematical process skills are important capabilities for mathematics' learning (Norhatta and Tengku, 2011). In order to be good in mathematics, learners are required not only to memorize contents and to understand the mathematical problems. They also need to have mathematical process skills to help them learn. The skills can enhance students' ability to solve a problem correctly and logically, according to a given concepts. Students will be able to use the skills to communicate and present mathematical language more effectively (Takahashi, 2007; Giganti, 2004; Jonassen, 2004; Brown, 2003). Conversely, if students lack these skills, they will not able to apply mathematical process skills to solve problems under different circumstances or situations. They will lack the

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connective knowledge that would further their ideas of creativity and other important skills for an advance mathematics (Moscardini, 2009; Ketterlin-Geller et al., 2008; Brown, 1994; Cawley and Parmar, 1992).

According to the theory, mathematical process skills are capabilities that are applied to mathematics for various situations. The mathematical process skills are contributed by these factors; 1) problem-solving skills 2) reasoning skills 3) communication and presentation skills 4) connection knowledge skills and 5) creativity skills (Thailand Ministry of Education, 2008; NCTM, 2000; Lyon, 2001; Alice and Shirley, 1994; Confrey, 2007; Sak and Maker, 2006). The results of mathematical process skills development given by previous studies indicate that, lacking these skills cause big problems not only in Thailand but also worldwide. Many studies found out that mathematics' issues are directly related to mathematical process skills from time to time (Brendefur et al., 2013; Jordan et al., 2009; Morgan et al., 2009; Duncan et al., 2007; Meissner, 2000; Lester, 1994). A great number of studies are aimed in developing mathematical process skills, but most of them have found that there were major problems in terms of applications of these skills realistically. This is due to the fact that the skills were very complicated while most of the previous studies were done by using composite score measurement (Ning and Sun, 2011; Ramdass, 2011). The students with high capability in mathematical process skills do not necessarily have high potential in all of the skills; meanwhile, for those students who have a low capability in mathematical process skills in some of the skills but not at all. Therefore, it is very difficult to indicate which skills are low or high, or should be improved. Moreover, Piaget's cognitive theory (1952) also explained that each of the skills was dependent on developmental age. Additionally, some of them influence the development of the other skills. Therefore, it can be concluded that mathematical process skills are not necessarily developed at the same time (Raghubar et al., 2010). In addition, the contents taught in schools do not allow students to improve these skills at the same time (Fisher, 1990; Brown et al., 1990). With these inconveniences, they cause problems in measurements strategies that it is impossible to indicate students' outstanding or poor skills of each student. Moreover, learners developed the skills with similar strategies without the consideration of individual differences; academic disability issues have caused long-term problems even though there is developed doubt of the continuous evolution of innovation.

Latent Profile Analysis (LPA) is a new statistical analysis method used to search and categorize members into a subgroup. Its outstanding advantages are 1) Able to identify group prominence causing obvious differences. 2) Able to apply to a group with unknown subgroup. 3) Identify probability to categorize members into a group by using maximum likelihood method. 4) Decreases measurement error. 5) Affects less of missing data and 6) the complex structure variables increase tendency to categorize (Enders, 2010; Thompson, 2007; Schafer and Graham, 2002; Bray et al, 2006). LPA was developed by Lazarsfeld and Henry in 1968. It has been widely applied to many fields of studies including social science. This is because interested variables to be studied of each individual are different in both known groups, such as sex (Ganley and Vasilyeva, 2011; Patterson et al., 2003; Rodd and Bartholomew, 2006; Effandi and Normah, 2009) and school size (Lamb and Fullarton, 2000; Willms and Raudenbush, 1989; Fredriksson et al., 2013; Humlum and Smith, 2014), and unknown groups. The findings from this analysis method will help to indicate each group's characteristics and differences. Moreover, it gives more accurate information on the classification which will later give obvious information to fulfill the needs of each group.

As mentioned from the issues and theories, the researcher studied the different structures of mathematical process skills by applying the LPA. The researcher found that this method has high possibility to show differences of these skills individually which will later assist in determining the skills of each group of students correctly as well as in designing the most appropriate learning strategies for students difference groups. Moreover, it is an application of the most effective analysis method used for complex structure variables that were difficult to study.

Research objectives

1) To compare mathematical process skills of seventh-grade students classified by gender and school size.
2) To classify mathematical process skills of seventh-grade into groups by using LPA

Research hypothesis

The synthesis of theories and study of previous researches indicate that males in early ages show the highest level of development in comprehension. At this age, the male students are able to learn about abstract subjects and problem solving better than female (Piaget, 1952). Similar finding is in many studies on related subjects (Ernest, 1991; Rogers, 1995; Burton, 1995). The findings of these researches were confirmed by all of them. Therefore, the researcher developed the hypothesis; mathematical process skills of male students are different from those of females. Moreover, supportive classroom environment theory (Bloom et al., 1965; Dunn et al., 1989) and similar finding of the research result show that large schools in Thailand are able to access good study materials which enhance learning management easier compared to small and medium school size. Therefore, there is a doubt that school sizes are likely to have different effects on mathematical process skills.

The revision of literature on the mathematical process skills showed that there were many indicators that created structure's complexity (Kilpatrick et al., 2001) and these indicators brought difficulty to classify the skills into group, include the study result in terms of heterogeneous population (Magidson and Vermunt, 2004; Muthén, 2001) and cause each individual to have different readiness to learn and be developed. These different of learning and self-development get affected from individual characteristics and surroundings. Therefore, the researcher really believes that students' characteristics are varied and can be classified into more than one group.

LITERATURES REVIEW

Mathematical process skills

Mathematical process skill is learners' ability to apply mathematic concepts to respond or solve problems in a given situations or tasks related to mathematics (Grouws, 1999). Activities used for learning mathematics in classrooms must focus on students' engagement in order for them to improve their own mathematical process skills (Reys et al., 2007; Walle, 2001). When learning mathematics, learners need to have basic mathematical knowledge and skills for future study (Schwartz, 2005). The main focus should be to develop necessary skills for intelligent enhancement which will lead students to live their lives happily and effectively in communities. The mathematical process skills consist of 5 dimensions; 1) problem-solving skills (PRS) are defined as ability to apply mathematical knowledge, problem-solving strategies, problem-solving process, and current experiences to get answers. The situations and problems for mathematics should be able to stimulate students' interests and come up with various ideas, ways and means to solve the problems (Nisbet and Putt, 200; Lyon, 2001; Adair, 2000) 2) Reasoning skills (RES) are ability to explain main concepts and scopes to draw conclusions or make decisions in different situations and contexts by using their own logic (O'Daffer, 1990; Alice and Shirel, 1994; Kulik and Rudnick, 1993) 3) Communication and presentation skills (COM) are ability to apply mathematical terminology and signs to present and explain relationship and situation of problems by speaking, writing, demonstrating, and picturing. It is ability to comprehend relations of concepts, to translate them into meaningful writing, and to be able to easily evaluate concepts that will be presented (NCTM, 1989; Kennedy and Tipp, 1994; Adams, 2010). 4) Connections knowledge skills (CON) are ability to connect mathematical knowledge with obstacles, as encountered in a situation. In other words, it means an ability to relate mathematics with world's phenomenon (Confrey, 2007; Usiskin, 2007; Niss et al., 2007) and it can be done by analyzing and applying mathematical knowledge. 5) Creative skills (CRE) are ability to develop or achieve knowledge foundation or innovation to establish or restructure thinking process from mathematics situation into new way of problems' solving (Pelczer and Gamboa, 2011; Sak and Maker, 2006; Loewen, 1995; Gerhard, 1971). It is also extending existing ideas to be more varied in order to solve problems and develop new things.

From what have been mentioned, it is obvious that the mathematical process skills and knowledge are to be developed in students. The development process can consist of many factors and has a complex structure. In order to increase mathematical process skills, every dimension should be developed and equally focus on all of the skills.

Latent Profile Analysis

Latent Profile Analysis (LPA) is considered one of the latent structure models. It can help to classify population characteristics that are homogenous from a heterogeneous population (Magidson and Vermunt, 2004; Muthén, 2001). It is a very important tool to estimate the probability to classify members into groups (O' Connor and Colder, 2005; Vermunt and Magidson, 2002) and to identify members with same characteristics into same group. With this technique of classification, number of a group is unknown as well as group combination patterns. Indicators can be both continuous and categorical (Wang and Wang, 2012). LPAs results give both quantitative information and similar characteristics within a group as well as differences of characteristics among each group. Recently, many researchers apply this method to psychology study (Pastor et al., 2007), teenagers behaviors (Herman et al., 2007), in marketing (Wedel and Kamakura, 2001) and educational (Grunschel et al., 2013). Latent Profile Analysis model is sometimes called mixture cluster model (McLachlan and Peel, 2000) and latent cluster analysis (Vermunt and Magidson, 2002).

General sets of criteria were used to determine the model with the optimal number of profiles as recommended by Vermunt (2008) and Jung and Wickrama (2008); 1) we examined the following fit indices: the Akaike Information Criterion (AIC), the Bayesian Information Criterion (BIC), and the Adjusted Bayesian Information Criterion (Adjusted BIC; Nylund et al., 2007). 2) We examined Entropy values which standard value should be more than .70, indicating acceptable classification accuracy (Jung and Wickrama, 2008). 3) We examined the Lo-Mendell-Rubin Likelihood Ratio Test (LMR-LRT; Lo et al., 2001) to compare the relative fit of a k profile solution with a k-1 profile solution. A significant

LMR value ($p< .05$) suggested that the current model with k profiles fit the data better than the k-1 profiles model and 4) we examined the Bootstrap Likelihood Ration Test (BLRT) (McLachlan and Peel, 2000; Nylund et al., 2007) to evaluate an appropriate model for the conceal group and to choose the best model.

METHODOLOGY

Participants

The data were collected from 2,485 seventh-grade students who enrolled in the academic year of 2014, in the North-east area of Thailand. This information was reported by the participants and confirmed by the parental consent letters. This is a heterogeneous group consisting of 1,073 male (43.18%) and female 1,412 students (56.82%) from 63 schools. If the categorization was divided according to the schools' size, there were 397 students (15.98%) from small schools, 850 students (34.21%) from medium schools, and 1,238 students (49.82 %) from large school. The sample was selected through multi-stage random sampling.

Research instruments

The data collection instruments used in this research were 1) problem-solving skills test, reasoning skills test, communication and presentation skills test, and connection knowledge skills test. The tests were 4 multiple choices of 40 items. The discrimination indexes by using B-index were between 0.23 – 0.89 and the reliability of each test by using Lovett's method was between 0.67 - 0.89. 2) Creativity skills test consists of 4 open questions. The discrimination indexes by using Whitney and Sabers method were between 0.33 -0.68 and the reliability by using Cronbach coefficient (α) was 0.77. The analysis of construction validity was done by using confirmatory factor analysis; 5 mathematical process skills consist of empirical data with χ^2 =6.869, df=3, χ^2 / df =2.29, p-value= 0.076, CFI =0.998, TLI = 0.993, RMSEA = 0.023, SRMR = 0.008

Data collection and analysis

The data collection was done from 16th May to 15th June, 2014. The data were divided into two parts 1) measures mathematical process skills by using the test and 2) collected demographics consist of gender and school size by using data recording, recorded the individual data of gender, school size, and mathematical process skills score were divided into 5 areas: 1) problem solving 2) reasoning 3) communication and presentation 4) connection knowledge and 5) creativity. Each area had a score of 10 in total. If there was missing data more than 10%, it would not have been analyzed (Palardy, 2003). Instead, the missing data would have been replaced by Mplus program version 7.1 (Muthén and Muthén, 2009).

To answer the research hypothesis, the researcher had analyzed the data as follows; 1) analyze the data to study interaction effect between gender and school size that influence the mathematical process skills; it was done by using Two-Way MANOVA. If there had no interaction effects, the researcher would analyze separately the main effect thus: analyze the effect of gender by using Hotelling T^2, analyze effect of school size by using One-Way MANOVA, and analyzing Post hoc test by using Bonferroni method, and 2) analyze information to classify the group of mathematical process skills using LPA by considering fit index and deciding about appropriate groups include; AIC, BIC, Adjusted BIC, Entropy, LMR-LRT, LMR, and BLRT.

RESEARCH RESULTS

The results of mathematical process skills comparison according to gender and school size

The results of multivariate analysis of variance (MANOVA) showed that 1) covariance matrix of the 5 mathematical process skills was not identity matrix (Barlette's Test of Sphericity: Likelihood Ratio = .00, p-value =.00). Therefore, each of the five areas was related to each other so they were appropriated to apply MANOVA. 2) According to Variance - Covariance Matrices, each group presented different value (Box' M = 262.10, p-value = .00) which broke the agreement of equality of variance - covariance Matrices. 3) Test of the variability of each variable was done by using the Levene's test. It was found that the variability was not different in problem solving skills (F=2.02, p-value = .07) and communication and presentation skills (F=0.46, p-value = .08). For reasoning (F=3.31, p-value=.07), connection knowledge (F=10.03, p-value =.00), and creativity skills (F= 10.03, p-value = .00), they were found to have different group variability with statistic significant level of .01 which exceeded the assumption homogeneity of variance, therefore the researcher decided to apply Pillai's Trace (Table 1).

According to interaction effect results, H_0 was not rejected; therefore, gender and school size have no relevance to mathematical process skills. Moreover, if comparing the mean of mathematical process skills between male and female, statistical significant showed the result of level .05 only with problem-solving skill. The comparison of school size was different in all of the skills except for mathematical communication skills. From the overall, the large schools have better mathematical process skills means than those in medium schools except for connection knowledge skills. Additionally, creative skills of the large schools were better than in the small schools (Table 2).

From the results of group discrimination of mathematical process skills found, the 3 profile groups are an appropriate model and have the suitable capability to meet the target criteria (Log likelihood = -22953.385; AIC = 45950.770; BIC 46078.767; ABIC = 46008.867; LMRT = 624.623; BLRT = 637.939; E_k = 0. 763). Mathematical process skills of each area in profile 1 were low in all skills; therefore, it is categorized into the lowest skills group compared to other 2 groups (56.78%). Profile 2 had all indicators in the moderate level except for the

Table 1. Means and standard deviation of mathematical process skills.

Variable		Small		Medium		Large		Levene's test
		Male	Female	Male	Female	Male	Female	
PRS	$\overline{X}$	4.53	4.56	4.43	4.13	4.83	4.57	F=2.02
	S	2.31	1.95	2.01	2.02	2.07	2.14	p-value=.07
RES	$\overline{X}$	3.93	4.17	4.17	3.84	4.21	4.24	F=3.31
	S	1.80	1.91	1.99	1.89	2.04	2.19	p-value=.00
COM	$\overline{X}$	3.35	3.58	3.61	3.50	3.67	3.69	F=0.46
	S	1.71	1.77	1.77	1.70	1.72	1.79	p-value=.80
CON	$\overline{X}$	3.02	3.09	2.97	2.89	3.21	3.39	F=10.03
	S	1.56	1.61	1.58	1.54	1.83	2.04	p-value=.00
CRE	$\overline{X}$	0.98	1.15	1.03	0.98	1.30	1.22	F=10.03
	S	0.91	0.80	0.89	0.89	1.10	1.04	p-value=.00

Assumption: Box's M = 262.10 , F= 3.47 , df1= 75 , df2 = 3201418.66 , p-value =.00; Bartlett's test of sphericity : Likelihood Ratio = 0.00; Approx. Chi-Square = 3223.518; df = 14 , p-value = .00.

Table 2. Mathematical process skills comparison.

Multivariate Tests							
Source	**Statistics**	**Value**	**F**	**Hypothesis df**	**Error df**	**p-value**	**Post Hoc**
gender*size	Pillai's Trace	0.006	1.440	10	4952	.156	
size	Pillai's Trace	0.025	6.171	10	4958.00	.000	
sex	Hotelling's Trace	0.005	2.253	5	2479.00	.047	
Tests of Between-Subjects Effects							
Source	Variables	**Type III Sum of Squares**	**df**	**Mean Square**	**F**	**p-value**	
	PRS	30.729	1	30.729	7.036	.008	
	RES	2.238	1	2.238	0.544	.461	
gender	COM	0.136	1	0.136	0.045	.833	
	CON	3.510	1	3.510	1.108	.293	
	CRE	0.414	1	0.414	0.428	.513	
	PRS	91.447	2	45.724	10.524	.000	L>M**
	RES	33.042	2	16.521	4.028	.018	L>M*
	COM	17.780	2	8.890	2.915	.054	-
size	CON	82.795	2	41.398	13.196	.000	L>M**, L>S*
	CRE	33.272	2	16.636	17.419	.000	L>M**,L>S**

S : Small schools; M : Medium schools; L: Large schools; ** statistical significance level .01; * statistical significant level .05.

creative skills; therefore, it was categorized into the medium skills group. Lastly, Profile 3 had highest means in all skills except for the mathematical creativity; therefore, it was categorized into the high skills group (2.74%) (Table 3).

DISCUSSION

The comparison of mathematical process skill results between gender showed that the males have higher problem-solving skill than the females. This may be explained through the interpretation of the theory of development which stated that male between 11-15 years has a higher capability in comprehension to learn about abstract subjects related to problem-solving than female (Piaget, 1952). The result mentioned previously was similar to the study of Rodd and Bartholomew (2006) and Ross et al. (2012) which found that male students have an advantage in problem-solving skill and in risky tasks. Moreover, Patterson et al. (2003) and Osafehinti

Table 3. Means of mathematical process skills used to classify 3 groups.

Indicators	Class 1		Class 2		Class 3	
	Mean	S.E.	Mean	S.E.	Mean	S.E.
Problem Solving (PRS)	3.322**	0.080	5.910**	0.110	8.224**	0.154
Reasoning (RES)	3.020**	0.081	5.334**	0.092	8.342**	0.212
Communication (COM)	2.933**	0.052	4.309**	0.091	6.782**	0.227
Connections (CON)	2.646**	0.045	3.514**	0.088	7.466**	0.424
Creative (CRE)	0.924**	0.028	1.297**	0.038	3.049**	0.237
Count	1411		1006		68	
Proportion	0.568		0.405		0.027	
Mean probability	0.903		0.861		0.939	

Note: ** statistical significance level 0.01.

(1988) found that male students are likely to work better in mathematical tasks while there is no difference in other skills between male and female (Githua and Njubi, 2013; Alyman and Peters, 1993; Geary, 1996; Tate, 1997).

According to the results of the schools' size comparison, the finding shows that there are differences between mathematical process skills among schools with different sizes. The reason is that students who come from different schools size have better access to better facilities, tools and latest technology to enhance learning. The students in large schools are found to have better problem solving, reasoning, knowledge connection, and creativity skills than in medium and small schools. This might be because the mathematical contents in high school level are more abstract, hence it makes understanding more difficult for students. Therefore, the students lack the creativity to make it concrete (Dossey et al., 2002; Fisher, 1990; Brown et al., 1990). There are also similar researches such as the research of Fredriksson et al. (2013) and Humlum and Smith (2014) which found that school size has direct effects on students' learning outcomes. Moreover, in Thai schools' systems, the larger schools have more advantages such as sufficiency of teaching materials which create more direct teaching and reinforce the effectiveness of the teachers (Thailand Ministry of Education, 2013; Bicer et al., 2013). Moreover, there are some problems of the small schools such as receiving fewer budgets, having not enough teachers, and teaching unfamiliar subjects. These factors lead to inequality in education (Girevoch, 1996; Hiroshi, 2003; Soares, 2003). However, the research found that there was no difference in communication skills. The reason may be that these skills are based on learners' ability to communicate with signs and mathematical languages (Fennell and Rowan, 2001; Mummer and Sheherd, 1993; Morgan, 1999) and these skills are universal and everyone can understand in the same way even though learning contexts are different.

From the results of student's classification according to skills, the findings can prove the hypothesis. This is because the structures of mathematical process skill are complex and consist of many sub- variables. Moreover, individual differences and level of development with age give the inequality of each skills area (Raghubar et al., 2009). In addition, creativity skills seem to be the problem in every group. The problems occur due to the fact that these skills need an in-depth comprehension and real experiences (Brunkalla, 2009; Mann, 2005; Renzulli et al., 2000; Tomlinson et al., 2001) to create new outcomes. Some researchers explained that creativity is a product of new experiences that must be different from others in order to respond and fulfill the needs to solve certain problems (Sternberg and Lubart, 1991; Park, 2004). Hence, these skills are developed slower than other skills.

The development of problem-solving and reasoning skills for every group can be observed and the result indicated that these two skills are related to each other (Barbey and Barsalou, 2009; Ball and Bass, 2003; Kilpatrick et al., 2001 Battista, 1990). This result probability comes from a problem-solving process; the learners must point out the reason and give their supports appropriately (Yurt and Sunbul, 2014; NCTM, 1991). Therefore, these two skills are the skills that must be taught hand in hand and not as a separate skill. Thus, if students can solve problems; they will be able to give any reasons of how to solve them. Conversely, if students cannot solve the problem, they will not be able to give any reason. Therefore, development of these two skills can possibly do at the same time.

Suggestions

In order to measure the mathematical process skills to evaluate students' ability, the evaluation itself should be divided into sections and each indicator must be specified. The results of this technique can be evaluated to get an improvement in terms of learning environments to enhance the strength of each individual. Therefore, in order to distribute each skill, the activities should be promoted differently.

The development of mathematical process skill between male and female could be developed in class activities to promote reasoning skills, communication skills, connection knowledge skills, and mathematical creativity skills. For the problem-solving skills, there should be separation of gender. Additionally, the small and medium size schools should be rapidly improved to decrease the gap. Therefore, the organizations should take this into consideration and do something as soon as possible.

The result of mathematical process skills classification showed that creativity skill was the problem in every group because the mathematical contents are mostly abstract that are related to creative skills. Therefore, it required a longer time to develop. However, the guidelines of how to develop the skills can be done in various ways. For example, create a classroom where students can use their imagination more than contents with individual differences awareness. Schools may assign specialists to give suggestions to students on their tasks or projects that aim to share their ideas. The application of LPA with social variables is very interesting.

The method was able to indicate complex structure variables and show differences between variables. If latent profile analysis is applied together with a new method of analysis, such as Latent Transition Analysis (LTA) and Mixture Structural Equation Model (Mixture SEM), it will give various types of information. This information will indicate factors concerning the relationship of the variables. Moreover, if Longitudinal Analysis is applied, the probability of information relating to dynamic variables will be achieved easier. Therefore, it will result in deeper details. Moreover, it could be developed more in this study that mostly matches with problems.

Conflict of Interests

The author(s) have not declared any conflict of interests.

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