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Teaching a concept with GeoGebra: Periodicity of trigonometric functions*

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Being one of the major subjects in high school mathematics curriculum, trigonometry links algebraic, geometric and graphical reasoning. The aim of this study is to investigate the effect of GeoGebra in the teaching of the concept of the periodicity of trigonometric functions. In this study, it is investigated how effective is the dynamic mathematics software GeoGebra being used in the teaching of the periodicity of trigonometric functions, which is taught based on "formulas" in the context of "traditional" mathematics education. The aim is to analyze and to compare the effect of the traditional teaching and the computer assisted mathematics teaching on students' conceptual learning about the periodicity of trigonometric functions. The design of this study is chosen as a quasi-experimental, and the working group is 36 tenth grade high school students from a public high school in Istanbul. 15 days after the instruction period, participants filled in a 5 questions test. The answers of students are analyzed using descriptive statistics. According to the results of the study, with the aid of computer mathematics education is more effective on students' learning than traditional mathematics education.

Key words: GeoGebra, concept teaching, trigonometry, periodicity.

INTRODUCTION

Trigonometry teaching

Being one of the major subjects in high school mathematics curriculum, trigonometry forms the basis of many advanced mathematics courses. The knowledge of trigonometry can be also used in physics, architecture and engineering. Furthermore, trigonometry links algebraic, geometric and graphical reasoning (Weber, 2005). The importance of trigonometry teaching is emphasized in different countries such as United States of America, Australia, United Kingdom and Turkey, and trigonometry should be supported by real life problems (NCTM, 2000; Delice and Roper, 2006; Stupel, 2012; MEB, 2013b). In general, trigonometry teaching begins with the ratios between the sides of right angle triangles. Then trigonometric functions are represented on the unit

circle and later on, the algebraic properties of the trigonometric functions and their relations are given (MEB, 2013a, b).

In the mathematics education literature, trigonometry is considered among the difficult subjects which students experience learning difficulties and numerous researches have revealed students' misconceptions about trigonometry (Doğan and Şenay, 2000; Orhun, 2000; Doğan, 2001; Demetgül, 2001; Delice, 2003; Fi, 2003; Kang, 2003; Durmuş, 2004; Ng and Hu, 2006; Aydın, 2007; Fiallo and Gutierrez, 2007; Steckroth, 2007; Tatar et al., 2008; Akkoç, 2008; Gooya and Rabanifard, 2008; Kültür et al., 2008; Gür, 2009; İpek and Akkuş-İspir, 2010; Kutluca and Baki, 2009; Güntekin, 2010; Moore, 2010). According to Ross et al. (2011) deep understanding of trigonometry requires the ability to flip between abstract, visual and concrete representations of mathematical objects, and students are particularly handicapped by their inability to formulate and transpose algebraic expressions. In addition, the subject is confounded by inter-relationships between functions (Ross et al., 2011).

In the relevant literature, there exist a lot of studies that investigate the effects of different approaches on students' or pre-service teachers' achievement and perceptions of trigonometry. Among them, in the studies where technological tools (such as graphic calculator, computer software) are used, it is revealed that the use of technology in trigonometry teaching affects positively students' achievement and learning. For instance, Blackett and Tall (1991) examine the effect of an interactive computer graphic software on students' trigonometry learning, and investigate how that effect change according to the gender of students. In their study, they also revealed the fact that the computer helps students (of either gender) lacking versatility in linking numerical to visual skills (Blackett and Tall, 1991). In his experimental research, Autin (2001) deduces that the graphic calculator supported mathematics teaching is more effective than traditional mathematics teaching in the teaching of complex functions such as inverse trigonometric functions. Choi-Koh (2003) investigated the patterns of one student's mathematical thinking processes and described the nature of the learning experience that the student encountered in trigonometry as he engaged in independent explorations within an interactive technology environment. He concludes that the use of technology helped the student to advance his thinking processes from the intuitive, to the operative, and, finally, to the applicative stage (Choi-Koh, 2003). In addition to these studies, two experimental researches have been conducted: Mafi and Lotfi (2012) investigated the effect of the software called COTACSI (a software created for the subject of trigonometry) on students' learning of trigonometry. Meanwhile, Zengin et al. (2012) used the dynamic mathematics software GeoGebra in their research. In both studies, it is deduced that the computer assisted mathematics education is more effective on students' learning than traditional

mathematics education.

Computer assisted mathematics instruction

Nowadays, the impact of technological improvements increases in all areas of our lives; hence the education cannot stand out of that impact. The rapid increase in knowledge producing and in the number of students per teacher cause many problems in education, and triggers the integration of new solutions. In this context, the integration of new technologies, which plays an important role in improvement of the educational quality, to the education practices in schools becomes compulsory (Aktümen and Kaçar, 2003). Therefore, use of these technologies has drawn the attention of researchers and educators, and a new domain called "Computer Assisted Instruction" has come up. The computer assisted instruction can be defined as the use of computers in educational settings with the following aims (Baki, 2002):

1. Students can recognize their lack of knowledge and performance by interplay with computers
2. Students can control their own learning by obtaining feedback from computers
3. Students' motivation can be increased by the presence of graphics, audios, animations and shapes in the computers

The improvements in technology and the computer assisted instruction approach affect also mathematics instruction in schools (Akkoç, 2008). The mathematics instruction where computer assisted cognitive tools are frequently used is called "computer assisted mathematics instruction" (Baki, 2002).

NCTM has emphasized the importance of technological tools in mathematics instruction. It is stated that if the technological tools especially computers are used efficiently and truly to teach mathematics concepts, it will have a rich learning environments to improve students mathematical thinking and thinking skills (NCTM, 2000). Therefore, appropriate use of computers in mathematics instruction may deepen mathematical understanding (Tall, 2002).

Computers may be used for work with various mathematics concepts, including formulas, constructions and proofs, and it can also be used for accessing information and communicating with others mathematically (Wiest, 2001). Whatever the uses of computers in mathematics, the focus should be on higher order thinking with an emphasis on inquiry, reasoning, and engagement in worthwhile mathematical tasks (Wiest, 2001). Different computer software play different roles in the development of students' thinking skills (Kutluca, 2013); but their common aim should be as to provide students an environment where they can pretend to be like mathematicians. Otherwise, if students use computers as calculators for even simple mathematical problems,

their thinking ability may be limited.

Dynamic geometry software

Educational software in mathematics education can be classified in five categories (Arslan, 2006):

1. Dynamic geometry software
2. Electronic spread sheets
3. Symbolic calculator software
4. Graphic drawers
5. Others

Dynamic geometry software (Cabri, GeoGebra, Geometer's Sketchpad etc.) focus on the relationships between geometric shapes such as points, lines, circles and various manipulations can be obtained by using dragging property of these programs (Kabaca et al., 2010). Dynamic learning environments provide new opportunities in mathematics learning and dynamic tools support especially "learning by doing" and "the process of explore" (Kabaca et al., 2010). In contrast to the "traditional" instruction environments that can be called "paper-pencil" environment, dynamic geometry software provides students with potential opportunities in terms of making assumptions, testing and exploring theorems and relations (Güven, 2002).

The use of dynamic geometry programs is suggested in many countries' mathematics curricula. In Turkey, in the latter elementary mathematics curriculum, it is clearly stated that students can do interactive investigations on dynamic geometrical shapes formed in different dynamic geometry software (MEB, 2013a). The dynamic environments where beyond the geometry, other mathematical domains like algebra or analysis that can be studied are called "dynamic mathematical software" (Kabaca et al., 2010). One of the most popular computer software with that property is GeoGebra.

GeoGebra

GeoGebra is dynamic mathematics software for all levels of education that brings together geometry, algebra, spreadsheets, graphing, statistics and calculus in one easy-to-use package (URL1). Being an open source software under the GNU general public license, GeoGebra is a dynamic mathematics software for teaching and learning mathematics from middle school through college level, and it is as easy to use as dynamic geometry software but also provides basic features of computer algebra systems to bridge some gaps between geometry, algebra and calculus (Hohenwarter and Preiner, 2007). GeoGebra provide to see graphical, numerical and algebraic representations of mathematical object on the same screen. Therefore, different representations of the same object are assembled dynamically and any change in one of these representations is automatically transformed to the other ones.

The basic objects in GeoGebra are points, vectors, segments, polygons, straight lines, all conic sections and functions in x and with GeoGebra dynamic constructions can be done like in any other dynamic geometry system (Hohenwarter and Fuchs, 2004). These constructions may be altered dynamically by dragging free objects and furthermore, it is possible to enter coordinates of points or vectors, equations of lines, conic sections or functions and numbers or angles directly (Hohenwarter and Fuchs, 2004). Shortly, GeoGebra is an open source dynamic mathematics software that can be used at any level of mathematical instruction.

Purpose of the study

As computer assisted mathematics instruction is generally supported, the necessity of this study where the effect of GeoGebra, Turkish and open source, user friendly and useful dynamic mathematics software on students' learning about the periodicity of trigonometric functions. As seen in the literature, there are some studies about trigonometric functions, their conceptual meanings and operations with these functions. In contrast, few researches have found the concept of the periodicity of trigonometric functions (Dreyfus and Eisenberg, 1980; Shama, 1998). In the solution of trigonometric equations or inequalities, one has to investigate and represent graphically functions with trigonometric parts, or to calculate the area between the graphs of trigonometric functions, and thus knowledge of the periodicity of the function simplifies this task greatly (Stupel, 2012). This concept is taught as a value that can be calculated at the end of some algebraic operations, and it is rarely associated with its visual representation (Weber, 2005). Therefore, based on the fact that students may have difficulties because of the operational teaching of the concept of the periodicity of trigonometric functions, the aim of this study is determined and to investigate the effect of GeoGebra in the teaching of the concept of the periodicity of trigonometric functions.

METHODOLOGY

Working group

The participants of this study are 36 tenth grade students from a public school in Istanbul. These participants have been chosen by using convenience sampling techniques (Yıldırım and Şimşek, 2008). The students were divided in two groups (control and experiment) according to their classrooms.

Design of the study

The quasi experimental design with post-test control group is used randomly divided (like changing of students' classrooms). As the

Table 1. Post-test questions used in the study.

Parameters
1. What is a period? Give an example
2. For the function $cos^n(ax + b)$, which of n, a and b affects the periodicity? How?
3. What is the period of the following function $sin^5(2x - 4)$?
4. What is the period of the following function $f(x) = cos^9(\frac{3x+5}{2})$?
5. What is the period of the following function?
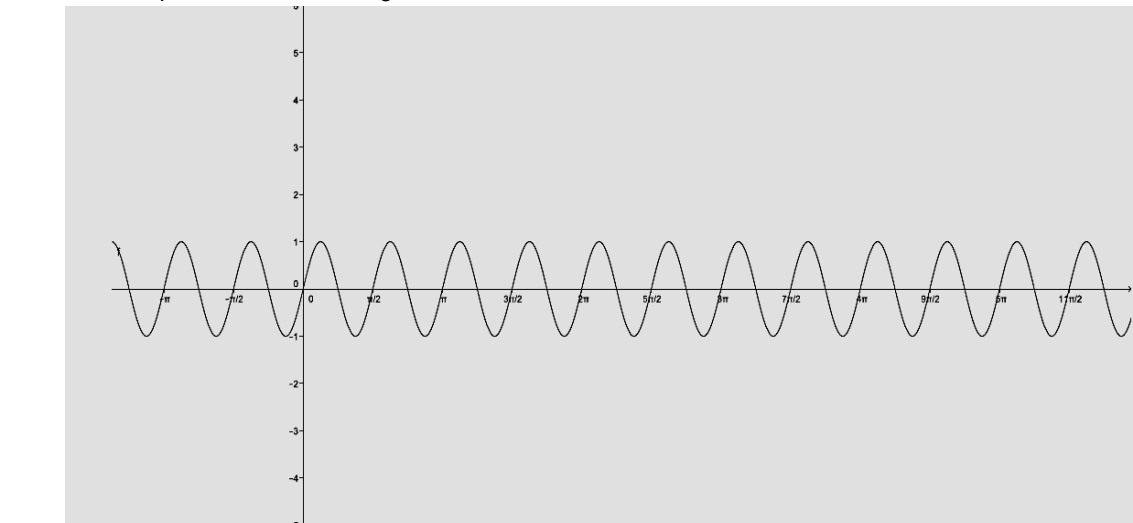
participants of the study haven't yet seen the "periodicity of trigonometric function" subject, the level of knowledge of the participants about this subject is considered equal. Thus, post-test control group modal has been chosen. In this modal, a pre-test is not necessary (Baştürk, 2009).

Data collection tool

In this study, a post-test is used to investigate the effect of GeoGebra on the explore process of the periodicity formula of the trigonometric functions. The participant students filled the test approximately 15 days after the course. The post-test consists of five conceptual and operational questions about the periodicity of trigonometric functions (Table 1).

Implementation process

This research is conducted with two classes of tenth grade students from an industrial vocational high school. These classes have equal classroom size of 18 students. Their academic achievement can be considered at the average level but some students have low skill level of calculation. In one of the classroom, the course is done with GeoGebra assisted mathematics instruction where students only followed the teacher who used GeoGebra for demonstration and drawing graphs, whereas in the other one, the expository teaching technique is used. In both of the two classes, the same teacher conducted the courses.

GeoGebra assisted mathematics instruction process

The course began with a brief discussion about the concept of "periodicity" and it lasted for 4 to 5 min. Students tried to give examples of the concept from the real life. Later a few graphs of trigonometric functions were drawn with GeoGebra, and a projector was used in order to let students follow. Students were asked to determine which parts of the graphs were repeating and to calculate the length of these parts. The periods $sin x$ and $sin^2 x$ were calculated by students and the periodicity of the functions of odd and even power was presented.

Then the graphs of $sin x$, $sin 4x$ and $sin 8x$ were drawn. Their periods and the change in periods were asked. Students realized the decrease in period as the coefficient of x was increasing (Figure 1). In order to show that the period of the sine function doesn't depend on the number added to or subtracted from x, the graphs of some functions like $sin(x + 4)$, $sin(x + 12)$ and $sin(x - 12)$ were drawn (Figure 2). Same procedures were followed for the cosine and tangent functions, and same results were deduced. Students' findings at each step were noted on the board. Finally, students tried to form a formula for the period when a generic function $sin^n(ax + b)$ was given. The course was finished by solving algebraic questions about the periodicity of trigonometric functions.

Expository teaching process

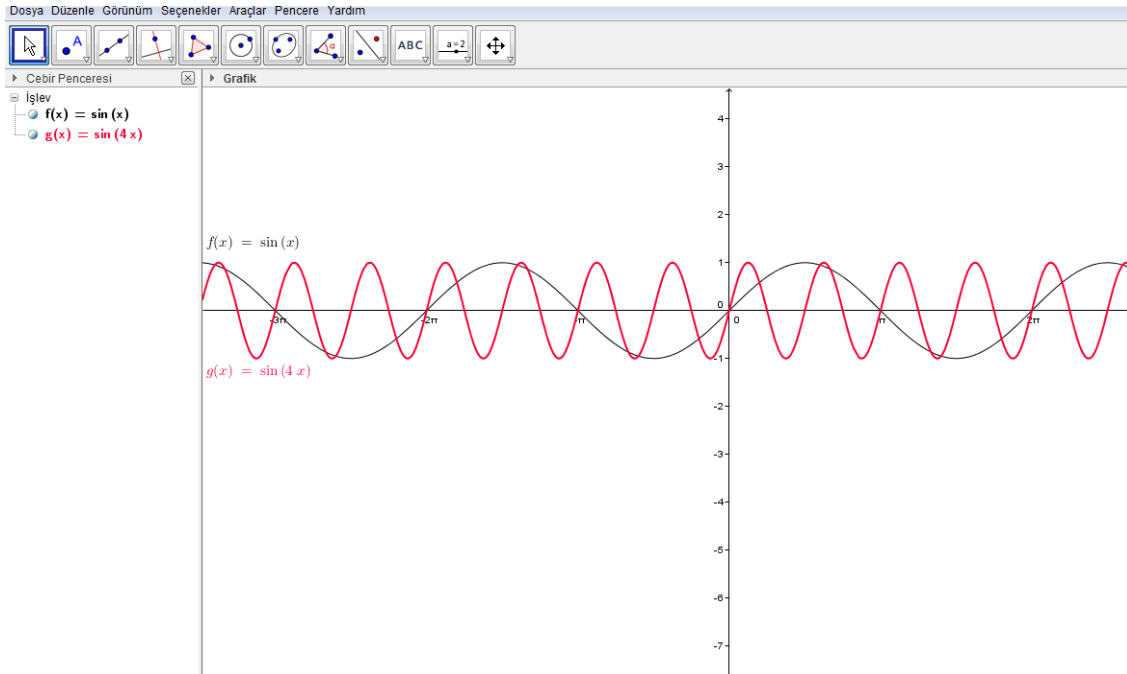The course began with a brief discussion about the concept of

Figure 1. The graphs of $f(x) = \sin x$ and $g(x) = \sin 4x$ with GeoGebra.
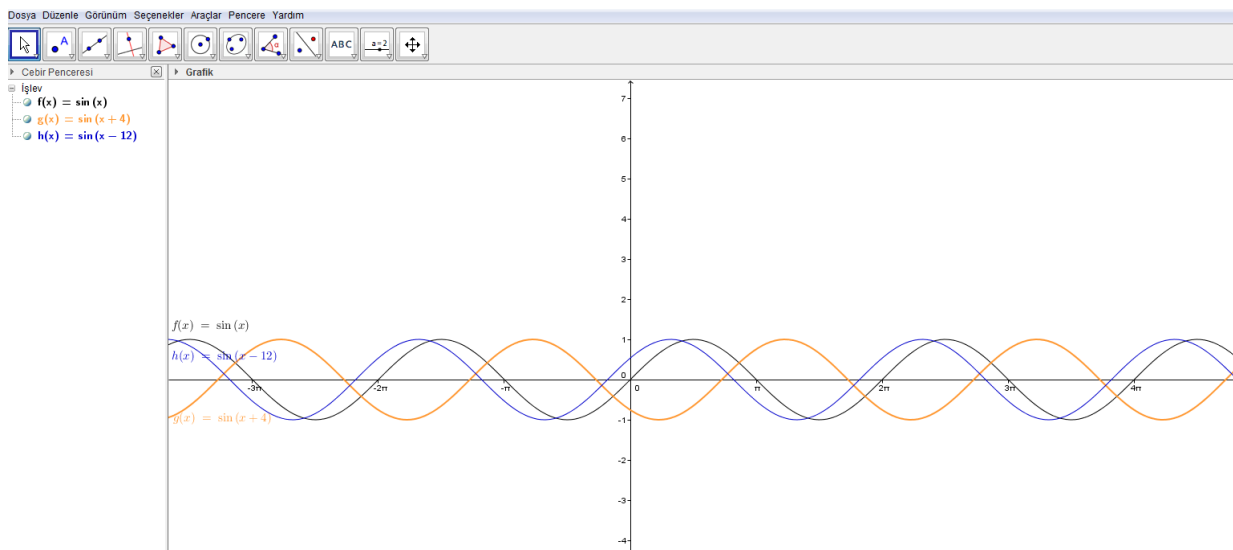

Figure 2. The graphs of $f(x) = \sin x$, $g(x) = \sin(x + 4)$ ve $h(x) = \sin(x - 12)$ with GeoGebra.

"periodicity", and it lasted 4 to 5 min. Students tried to give examples of the concept from the real life. Then the graph of $\sin x$ and $\sin^2 x$ were drawn on the board, and their periods were calculated. While drawing graphs, students reacted negatively and claimed that the drawing process was long and boring. The periodicity of the functions of odd and even power was presented. Later, the graph of sin3x was drawn on the board and the relationship of its period with the period of sine function was presented. As the level of students' participation to the course wasn't high, the formula for the periodicity of trigonometric functions

was directly presented and algebraic questions about the periodicity of trigonometric functions were solved.

Analysis of data

The post-test questions are examined separately for two classrooms. Each question categories are formed according to the students' answers and the analysis are done with respect to these categories. For every question, the frequencies and percentages

Table 2. Students' answers with respect to the questions.

Variable		Control group		Experimental group	
		Frequency (f)	Percentage (%)	Frequency (f)	Percentage(%)
Question1	Correct	11	61.11	14	77.78
	Partial correct	2	11.11	3	16.67
	Incorrect	4	22.22	1	5.56
	No answer	1	5.56	0	0
Question2	Correct	4	22.22	13	72.22
	Partial correct	11	61.11	3	16.67
	Incorrect	1	5.56	0	0
	No answer	2	11.11	2	11.11
Question3	Correct	14	77.78	14	77.78
	Partial correct	1	5.56	4	22.22
	Incorrect	1	5.56	0	0
	No answer	2	11.11	0	0
Question4	Correct	1	5.56	14	77.78
	Partial Correct	9	50.00	2	11.11
	Incorrect	2	11.11	1	5.56
	No answer	6	33.33	1	5.56
Question5	Correct	2	11.11	7	38.89
	Partial correct	2	11.11	5	27.78
	Incorrect	2	11.11	3	16.67
	No answer	12	66.67	2	11.11

were calculated and presented.

FINDINGS

The findings of the study are summarized in Table 2. The answers of students in the control group are coded as:

1. Correct answers: 11 students claim that a period is a repetitive pattern and give examples from real life.
2. Partial correct answers: 2 students state that a period is the time passed for the formation of a sound wave.
3. Incorrect answers: 2 students try to give an explanation with regard to the periodic table, and 2 students give no sense explanations with respect to the subject.
4. 1 student did not answer the question.
The answers of students in the experimental group are coded as follows.
5. Correct answers: 14 students claim that a period is a repetitive pattern and give examples from real life.
6. Partial correct answers: 2 students state that a period is the time passed for the formation of a sound wave.
7. Incorrect answers: 1 student claim only that the period is something related to the trigonometry but he did not

give any more explanations.

These findings show that the percentage of correct answers in the two groups are quite high (above of 60%). Despite the fact that students filled in the post-test after two weeks of instruction, they still remember the definition of the periodicity. Explanations of students' answer for the second question are given as:

1. Correct answers: 4 students give the following correct answer: "n is the power. If n is odd then the period is $2\pi/|a|$. If n is even then the period is $\pi/|a|$. If a increases, the period decreases. The period doesn't depend on b."
2. Partial correct answers: 11 students' following answers are considered as partial correct: "the period depends on n. For odd values of n, the period is $2\pi/|a|$ and for even values of n the period is $\pi/|a|$." (4 students); "the period depends on the nature of n" (2 students); "the period depends on n and a" (2 students); "n is the power. If n is odd then the period is $2\pi/|a|$. If n is even then the period is $\pi/|a|$. The period doesn't depend on b. a is in the denominator and it affects the denominator." (1 student)
3. Incorrect answers: 1 student give this answer: "n is power, a is the product, b is the sum". The student did not remember the formula and he tried to make a comment

according to the place of numbers".

4. 2 students did not answer the question.

The answers of students in the experimental group are coded as follows.

5. Correct answers: 13 students give the same correct answer of the students in the control group.

6. Partial correct answers: 2 students' following answers are considered as partial correct: "The period depends on all of n, a and b." (1 student); "the period depends on n. For odd values of n, the period is $2\pi/|a|$ and for even values of n the period is $\pi/|a|$." (1 student).

7. 2 students did not answer the question.

These findings show that there is a difference between the percentages of correct answers in the two groups. 72% of students in the experimental group gave the correct answer, whereas only 22% of students' answers in the control group are correct. With respect to this question that assesses whether or not the periodicity of trigonometric functions has been learned conceptually, experimental group students' achievement is higher than the control group. Furthermore, one can reveal that most of the control group students (61%) remember the formula of the periodicity but they cannot interpret its meaning. Also among the experimental group, 3 students (16,67%) remember the formula without explicit inter-pretations. Explanations of students' answer for the third question are given as:

1. Correct answers: 14 students used the formula correctly, and did not make any calculation error.

2. Partial correct answers: 1 student confused the formula with respect to the nature of n.

3. Incorrect answers: 1 student confused the place of "n" and "a" in the formula. Thus, it can be deduced that these student memorized the formula but he did not understand conceptually.

4. 2 students did not answer the question.

The answers of students in the experimental group are coded as follows.

5. Correct answers: 14 students used correctly the formula and did not make any calculation error.

6. Partial correct answers: 4 students either confused the formula or made some calculation error.

These findings show that the percentage of correct answers in two groups is very high (above of 75%). Hence, students are very successful at using a formula to answer an algebraic question. Explanations of students' answer for the fourth question are given as:

1. Correct answers: 1 student used correctly the formula, and did not make any calculation error.

2. Partial correct answers: 8 students either made calculation errors or confused the value of a. The low level of skill of four operations in fractions may be considered as the reason of these mistakes.

3. Incorrect answers: 2 students confused the place of "n" and "a" in the formula. Thus, it can be deduced that this

students memorized the formula but he did not understand conceptually.

4. 6 students did not answer the question.

The answers of students in the experimental group are coded as follows.

5. Correct answers: 14 students used correctly the formula and did not make any calculation error.

6. Partial correct answers: 2 students made some calculation error at the last step where they divided two fractions.

7. Incorrect answers: 1 student wrote correctly the formula but he did not make any calculations, he did not replace the numbers with the letters. Thus, one can deduce that this student memorized the formula without understanding it.

8. 1 student did not answer the question

Explanations of students' answer for the fifth question are given as below.

The answers of students in the control group are coded as follows.

9. Correct answers: 2 students calculated correctly the period using the graph.

10. Partial correct answers: 2 students did not find the period even though they placed the intersection points of the graph and x-axis by 90^0, 180^0 and 270^0. The deficiency or the low level of skill of interpretation of graph may be the reason of these mistakes.

11. Incorrect answers: 1 student calculated the period as 180° because of wrong interpretation of the graph. 1 student calculated the period as 360°.

12. 12 students did not answer the question.

The answers of students in the experimental group are coded as follows.

13. Correct answers: 7 students calculated correctly the period using the graph.

14. Partial correct answers: 5 students did not find the period even though they placed the intersection points of the graph and x-axis by 90^0, 180^0 and 270^0. The deficiency or the low level of skill of interpretation of graph may be the reason of these mistakes.

15. Incorrect answers: 2 students calculated the period as 180° because of wrong interpretation of the graph.

16. 2 students did not answer the question.

These findings show that the percentage of correct answers in the two groups are considerably low (below of 40%). Despite the fact that this graph question is more understood by experimental group students than by control group students, the percentage of the correct answers are found to be low because of the lack of students' skills of interpretations of graphs.

DISCUSSION

Based on the findings of the study, for the question of the periodicity of a trigonometric function in algebraic form (question 2 of the post – test), the number of correct

answers of the students that participated in the GeoGebra assisted mathematics instruction is much higher than the students that participated to expository teaching. Even if the students in the control group remember correctly the formula of the periodicity of a function, they do not understand sufficiently the meanings and the effects of numbers in the formula. The reason behind this gap may be the fact that, as Ross et al. (2011) stated in their research, deep understanding of trigonometry requires the ability to flip between abstract, visual and concrete representations of mathematical objects and students are particularly handicapped by their inability to formulate and transpose algebraic expressions.

Most of the students in the experimental group both remember correctly the formula and explain clearly the effects of the numbers a, n and b in the formula. Hence, they understand conceptually the periodicity of the trigonometric function as these students explored by themselves the formula during the course. GeoGebra gave them the opportunity of conjecturing the formula. Therefore, students learned conceptually and formed their own mathematical knowledge. So, they remembered easily the necessary knowledge. This result is similar to the experimental study of Zengin et al. (2012). These researches used GeoGebra in their research, and they deduced that the computer assisted mathematics education is more effective on students' learning than traditional mathematics education. Furthermore, the result of this study about the efficiency of GeoGebra on the learning of the periodicity of trigonometric functions is very similar to the experimental studies done by Blackett and Tall (1991), Autin (2001), Choi-Koh (2003) and Mafi and Lotfi (2012) about the efficiency of different technologies on trigonometry teaching.

According to the findings of the study, for the question of direct application of the periodicity formula (question 3 of the post-test), the number of correct answers of all students is very high. This result may be originated from the fact that, as Weber (2005) also explained, this concept is taught as a value that can be calculated at the end of some algebraic operations, and it is rarely associated with its visual representation. In addition, the finding that students in the experimental group answered correctly as much as the control group students shows that GeoGebra is useful not only for conceptual learning but also for operational learning. In other words, students may improve their skill of operation by the effect of GeoGebra assisted mathematics instruction. The experimental group students' performance for fourth question supports this result. Hence, it can be deduced that better learning conceptually provides better making calculations.

The findings for the fifth question of the post-test reveal the fact that students have difficulties in interpreting the graphs of functions. Even if the number of graphs drawn in the course with GeoGebra is higher than the traditional mathematics course, students in experimental group give also wrong answers for that question. The dominance of algebraic representation in mathematics teaching may cause the difficulties in interpreting graph. In other words, since multiple representation of the periodicity of trigonometric functions is not used often and efficiently in courses, students can explain the meaning of the period in algebraic form but not in visual form. However, the number of students in the experimental group that give correct and partial correct answers for this question is more than in the group. This result yields that, about the graph of trigonometric functions, GeoGebra assisted mathematics instruction is more effective than traditional teaching techniques.

As a result, in this study whose aim is to represent GeoGebra as an alternative way of teaching of the periodicity of trigonometric functions that is usually taught algebraically not visually, GeoGebra assisted mathematics instruction is more effective than traditional expository mathematics instruction. The results of the study may be considered as favorable because the high school type of the working group students is not preferred often by the researchers and the mathematics achievement of students from this type of high school has been found as low (Mumcu et al., 2012). It is recommended that some researches about this concept have to be conducted with students from different type of high schools, and also with pre-service teachers in universities.

According to the results of this study, in mathematics instruction with expository teaching techniques, the relationship between the algebraic form and the graphical representation is often ignored. Therefore, this observation emphasizes one more time that multiple representation of any concept must be always presented in courses. Furthermore, in order to decrease calculation errors in basic operation, in the elementary mathematics education, the skill of operations should be improved.

Conflict of Interests

The authors have not declared any conflicts of interest.

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