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What makes a mathematical task interesting?

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The study addresses the question of what makes a mathematical task interesting to the 9th year students. Semi-structured interviews were carried out with 15 students of purposive selection of the 9th year. The students were asked to recall a task they found interesting and engaging during the past three years. An analysis of the tasks was made with respect to the context, cognitive demand, and task structure, while interview data were analyzed using the Theory of Didactical Situation (TDS) and Mathematical Task Framework (MTF). The students recalled a total of four teacher-designed tasks. All of the tasks offered a high level of challenge, elements of sharing and some freedom of choices related to the aspects of the tasks. The results showed that in most cases the students pointed out the target knowledge as the reason for a task to be interesting and engaging, followed by the way the content was dealt with in the classroom.

Key words: Mathematical tasks, interest and engagement, 7-9th year, Theory of Didactical Situation (TDS), Mathematical Task Framework (MTF).

INTRODUCTION

Mathematics teachers do not merely want their students to learn, but also to *enjoy* learning. The starting point of this study is the idea of optimizing the conditions necessary to bring about subject-related aspects of interest and engagement. In other words, the study is designed to explore content-related student interest and engagement. Interest and engagement are motivational factors (Ames, 1992), essential for learning (Dewey, 1913). Throughout the study, interest and engagement are treated as mutually supportive, to open up a wider range of associations and to draw up a richer body of research. Despite the importance of these elements, there is a decrease in interest and engagement in mathematics, starting from grade 5 in the Swedish school context (Skolverket, 2003). There are many studies on interest and engagement in school settings (Hidi, 1990; Michelsen and Sriraman, 2009), however, there is a lack of studies relating to the concepts to mathematical content, especially from a student perspective. Recent findings show that teachers' strategies to enhance engagement often focus on the general aspects rather than task specific mathematical content (Mitchell, 1993; Nyman and Kilhamn, 2015). Based on that, my research seeks to study the students of a teacher who deliberately intend to interest and engage them by focusing on mathematics.

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Overall, the aim is to capture students' views in the light of the theoretical framework deriving from Brousseau (1997) using terminology from the Theory of Didactical Situations (TDS), in combination with the Mathematical Task Framework (MTF), which will be taken up in greater detail as the work progresses.

Aim and questions

The aim is to investigate what features of the students' tasks the 9th year identifies as interesting and engaging. The main questions are:

1. What tasks do they identify as interesting and engaging when the teacher has deliberately brought the mathematical content into the foreground?
2. What features are interesting and engaging in those tasks?

Previous research and theoretical framework

Interesting and engaging mathematical tasks

A mathematical task has been defined as a single problem or a set of problems that focuses student attention on a mathematical idea (Stein et al., 1996). The type of task used in the classroom has an influence on the level of students' engagement (Hiebert and Wearne, 1993; Smith et al., 2000). Previous research shows that specific features in the design of mathematical tasks can contribute to interest and engagement in mathematics (Ames, 1992; Sullivan et al., 2013). Mathematical tasks can be designed with the purpose of being "interesting to the students, incorporate a rationale for them to engage, provide some challenges, reduce the risk of failure, and for which success provides the motivation for further engagement" (Sullivan et al., 2013).

Task design - context, level of challenge and structure

According to Hilbert (1900), a good mathematical task that has the potential to enhance student engagement is intriguing, based on clearness and ease of comprehension but is at the same time challenging. Three different elements of task design that may influence student interest and engagement are: Context, the level of challenge, and task structure.

Context

Context is defined as a situation in which a task is embedded (Borasi, 1986), and is one way for the teacher to enhance engagement and make a mathematical idea interesting (Sullivan et al., 2013). Choosing a meaningful context helps students to understand mathematical ideas (Brousseau, 1997). In this study, *context* is used as the choice of the situation in which the mathematical idea is embedded. This situation can either be real life related or purely mathematical. Sullivan et al. (2013) opines that tasks designed around real life contexts can make mathematics 'come alive' for students, showing them a purpose for what they are studying and making mathematics more engaging for them. Mathematics embedded in such contexts can have practical, personal and/or social relevance, and are engaging when they appeal to adolescent curiosity, an opportunity to discover their place in society, giving a sense of power and understanding of democracy (Sullivan et al., 2013). When Hodge et al. (2007) tested tasks involved in the decisions or judgment making on whether the installation of airbags in cars impacts on car safety or exploring the impact of treatments for different diseases, they found support for students' mathematical engagement and learning.

Real-life contexts do not have to be realistic. For instance, a group of tasks known as Fermi problems involve imagination and making estimates of physical quantities, such as the number of people who can fit into a classroom. This problem is an example of when "[...] the context is potentially engaging for what follows and the mathematical decisions that the students make in finding a solution" (Sullivan et al, 2013). Including real life context can be beneficial for learning (Johansson, 2015). However, as Boaler (1993) states, it is not enough to put a task in real life settings in order to engage students in mathematics.

Level of challenge

Challenging tasks can be seen as potentially engaging. Smith and Stein (1998) have developed the Mathematical Task Framework (MTF), a framework for analysing the level of challenge. This framework focuses on a task's cognitive demand, implying that the demand increases from (1) "Memorization" to (2) "Procedures without connections", followed by (3) "Procedures with connections" and at the highest stage there are tasks labeled (4) "Doing Mathematics". At the lower levels of cognitive demand, when memorizing and carrying out procedures without connections, a student can write down the answer based on the definition or on algorithms, or because they have previously seen analogous answers. Smith and Stein (2011) point to examples such as stating decimal and percentage equivalents for a fraction as tasks with a lower level of challenge. The third level requires students to use different procedures to develop an understanding of mathematical concepts and ideas. In order to reach this level of cognitive demand, students must select suitable strategies to solve and provide explanations. Stein et al. (1996) have identified various patterns of student engagement when students worked with tasks on the highest level of cognitive demand, that is with tasks that were set up to encourage “Doing mathematics”. In summary, according to MTF, a task is of the highest level, (4), if it:

1. Requires complex and non-algorithmic thinking. There is no predictable approach explicitly suggested by the task instructions.
2. Invites to explore and understand the nature of concepts, processes and relationships.
3. Demands self-monitoring or self-regulation of student’s own cognitive processes.
4. Requires relevant knowledge and experience, and to make appropriate use of them.
5. Opens up for analysis of task constraints that may limit possible solution strategies and solutions.
6. Includes the unpredictable nature of the process leading to the solution(s) will require considerable cognitive effort.

A task that serves as an illustration of high cognitive demand in year 4 is: “A fourth-grade class needs five leaves each day to feed its 2 caterpillars. How many leaves each day would they need to feed 12 caterpillars?” (Smith and Stein, 1998). This task was found to be cognitively demanding by 10 year olds, based on the assessment results and the analysis of empirical data using the MTF analysis guide (Smith and Stein, 1998). The analysis showed that only 6% of pupils in year 4 found a way to solve it. It is important to remember that there are individual differences in what is seen as challenging and demanding. A task of this sort might be less demanding to students in 7 to 9^{th} year, which indicates that a task’s cognitive demand varies depending on the age group. In the forthcoming study the level of challange of the student-chosen tasks will be investigated.

Task structure

A task can also be analysed in respect to its level of openness. Emanuelsson (2001) has drawn upon a model that makes it possible to categorise tasks using a scale of 0 to 3 regarding how open they are. In a task structure of level 0 everything is given. If the task structure is on level 1 the answer is open and on level 2 both the answer and the method are open. Level 3 leaves all aspects open, the problem, the method and the answer(s), inviting the student to make most of the decisions. The level of openness depends on the elements of investigation included in the task design.

Another part of task structure is scaffolding (Wood et al., 1976; Emanuelsson, 2001). Instead of being structured as one whole text or story, a task can be scaffold by a division into smaller parts or questions along the way, to build a bridge between what the students already know and the target knowledge. Scaffolding can be used as an aspect of task structure that enhances engagement (Henningsen and Stein, 1997).

A task can have a routine or non-routine structure, where non-routine tasks are puzzle-like rather than straightforward, very likely to be unfamiliar to students, as defined by Mullis et al. (2003) and used in the Trends in International Mathematics and Science Study (TIMSS) studies. For instance, a routine problem can be structured as “How many 3 L bowls are needed to fill a 9 L jug?” in comparison to a non-routine problem, “How can you take exactly 4 L of water out of the bowl using a 5 L and a 3 L jug?” Non-routine problems have, as in this case, a higher level of challenge and “above those needed for solution of routine problems, even when the knowledge and the skills required for their solution have been learned” (Mullis et al., 2003).

There are other variables possible to consider as influential on student engagement, for instance the teacher’s personality; personal interest of the students; peer support; classroom culture, the use of ICT and so on. However, this study is aimed to maximize the chances of finding subject specific details that makes a task interesting and engaging, and connecting the concepts to mathematical ideas. Therefore, this paper focuses on students’ views on content matter in relation to different features of tasks and the empirical study is limited to the tasks and what the students say about them.

Interest and engagement in light of the theory of didactical situations

Brousseau (1997) introduces the notion of the *didactical situation* in which the interplay between the teacher, the student and the mathematical task takes place. The teacher introduces a task and then steps back and refrains from interfering, by not suggesting the answer or any step in the solution of the task (Brousseau, 1997; Brousseau and Gibel, 2005). *An adidactical situation* is the moment when students accept the tasks as their own. The teacher sets the students up to accept the challenge of an engaging mathematical situation whose conditions are given in advance, such as “conditions, rules, goals, and above all the criteria for success and to do it without his/her help, but as their own responsibility" (Brousseau 1997, p. 230). The devolution of an adidactical situation is the time during which the students start to treat the task as their business. When an adidactical situation has devolved, the students work with the task driven by their own curiosity and ambition.

Tasks are, as shown by recent theory development within the framework of TDS (Rezat and Strässer, 2012), an important dimension in the didactical interplay. The point of departure in this study is recognizing the task as an important dimension in teaching and learning mathematics, and in the case of problem solving tasks can be a pathway into evoking student interest and engagement (Shoenfeldt, 1992). In addition to the context, level of challenge, and structure in the tasks, this study will also identify the *target knowledge in the task*, described by Brousseau (1997) as the mathematical knowledge aimed for. It is possible that the students might remember a task in connection to the specifics of the target knowledge, that is, the mathematical content dealt with in the task.

METHODS

Participants

The choice of participants were based on optimization of the conditions for researching content-related student interest and engagement within the public school system in a Swedish school of mixed ability and socioeconomic background. In the effort to investigate what students find interesting and engaging in a task, the students were interviewed and the task(s) analyzed with respect to the target knowledge and the task design. Students were chosen from two different classes with a particular skilled teacher, this is due to advice gained from studies where teachers have been identified as suitable if they are seen as highly competent by the local community (Clarke et al., 2006). The teacher in this case has been teaching for over 20 years, with a doctoral student in mathematics education, and supervises professional development courses. According to this teacher's students and colleagues, she holds high standard lessons. Furthermore, in order to interest and engage students, this teacher puts mathematical content in the foreground. The following conditions are fulfilled to optimize the possibilities of finding interesting and engaging tasks:

1. This teacher *intends* to actively engage the students in mathematics. She chose to take on the most challenging, non-engaged classes at a school at the beginning of the 7th year and teach them through the 7-9th year. She encourages students to engage in tasks and participate in her lesson activities. The two classes had, according to the teacher, come far in their development of interest and engagement compared to when she first started to work with them in the 7th year.
2. The target knowledge is central when tasks are designed with the mathematical ideas in the foreground. The tasks are deliberately designed to be potentially engaging in order to learn the target knowledge.
3. During the interviews students rated the teacher's competence as high or very high.

The teacher was interviewed for 40 min, during which she described the students in grade 9 as engaged on a whole class level, with a range of individual variations. Her two classes were at the same level and once a week they were mixed, working in smaller groups, choosing the group themselves according to which final year grade they aimed to achieve at the end of the 9th year: A (the highest), C or E. The teacher was asked to select the student participants for this study by choosing 4 or 5 students from each group. This choice was based on the teachers' knowledge of her students' ability to verbally express their reflections. She asked them if they wanted to participate in the interview and all those asked, 5 from each group, agreed. The students were coded as A1-5, E1-5 and C1-5.

Data collection

This study was designed to reveal examples of interest and engagement in mathematical content. The data consists of semi-structured interviews with selected students (n=15). Individual interviews lasting 4 to 11 min with each student were audio-recorded and transcribed verbatim. Each interview was divided into four parts. Part one, students were asked to give an example from any lesson in the 7-9th year of an interesting and engaging mathematical task (or in some cases one or several tasks or topic areas). The key question was: *Do you remember something interesting and engaging you have done in mathematics with your teacher?* Follow up questions were about what was found interesting and engaging in the task(s). Also, the students were given an opportunity to give further examples of interesting and engaging tasks. Questions that were posed were: "*What makes the task interesting and engaging?", "What interests and engages you in a task?* In the fourth part of the interview the students were asked if they had any suggestions to how a teacher can enhance interest and engagement.

When it comes to ethical considerations, it was clarified for the students that they were not facing a test situation. They were informed that the answers or reasoning about interest and engagement or task solutions would not affect their grades or be recognizable by the teacher or the readers on an individual level. Consent forms were filled in before each interview, read out loud and explained by the interviewer to each student. The audio data was transcribed for the analysis of students' reasoning about the tasks. Parts of the transcripts involving students' reasons for choosing a certain task were selected to be analysed.

Data analysis - the three phases

The analysis of semi-structured interviews is based on treating mathematical tasks as items (Goldin, 2000). After the individual semi-structured interviews with the students, the teacher provided the tasks that the students had brought up. Those tasks were analyzed in three phases:

1. Phase one is the researcher's analysis of the tasks concerning the target knowledge, the context, the level of challenge and the task structure. The analysis was made in order to answer research question 1, that is, what tasks students identify as interesting and engaging.
2. Phase two focuses on students' utterances. The transcripts of student interviews were analyzed in order to shed light on research question 2, which is, what students identify as

Table 1. An overview of each student's task choice.

Task 1: Human size doll	Task 2: Statistics poster	Task 3: Spatial geometry	Task 4: Population
A1, A2, C2, C3, C4, E1, E3, E4, E5	A4, C1, C3, E3	A2, A3, C1	A4, A5

Table 2. Summary of task features.

Task	Target knowledge	Context	Level of challenge	Openness	Routine/ non-routine	Scaffolding
1	Scale, measurement, proportionality	Real-life	4	1	NR	2 parts, 7 steps in part 1 and 12 steps in part 2
2	Descriptive statistics, diagrams and charts, percent	Real-life	4	2	NR	5 parts (concepts, methods, communication, reasoning, problem solving), 3-6 steps in each section
3	Geometrical shape, volume, scale, use of π	Mathematical	4	1	NR	3 parts about task and assessment with small steps explaining each part
4	Estimates of physical quantities	Real-life (Fermi)	4	3	NR	2 parts (the story and criteria for the grades)

interesting and engaging in those tasks. The focus is on looking for words in students' utterances related to task features, to see if details in the target knowledge, the context, the level of challenge and the task structure will be revealed. Examples of student utterances are included in the results to illustrate what students identify as interesting and engaging.

3. Phase three is a thematic analysis, to characterize task features mentioned by the students as interesting and engaging themes devolved in an iterative process, as well as connecting features found in the first two phases in relation to TDS.

RESULTS AND ANALYSIS

In the course of 15 interviews, a total of four tasks were recalled by the students. These tasks are presented and the overall results are summarized. Table 1 illustrates individual choices of the students, and Table 2 summarizes the analysis of the task features. Thereafter, task features are analyzed and parts of the transcripts were presented, illustrating what the students found interesting and engaging in the chosen task.

The tasks identified by the students as interesting and engaging

The tasks identified as interesting and engaging are (1) the human size doll, (2) statistics poster, (3) spatial geometry and (4) population on an island. All of the tasks turned out to be designed and revised by a group of teachers.

The human size doll

This was a geometry task from year 8, with scale, measurements and proportionality as target knowledge. This task required a doll, a ruler or similar instrument of measurement, paper and pen, calculator and what the teacher referred to in the instructions as a "quick mind". The task consisted of two parts. In part one, the students worked on their own, to produce an image of a three-dimensional doll (Barbie or Action man). A body part of the doll was chosen to serve as a unit of measurement. The students were instructed to find out what a reasonable size of that body part is in reality and calculate the scale of the measurements. From these measurements a scale was chosen. A discussion on which measurements turned out to be realistic followed.

In part two, they were instructed to choose a measurement of the doll's body as a unit. A scale based on the real human measurement of the same body part was suggested, for example for a person (180 cm) and the doll (15 cm), the scale being 1:12, since 180/15=12. The students were then instructed to use this scale through the whole task and a poster of the human size doll was made based on the scale. At the end the measurements were summarized in a table and presented to year 6 students at the same school, discussing how unrealistic the dolls were.

Statistics poster

This task was based on the topic of descriptive statistics from year 9, dealing with representations, such as different kinds of diagrams and charts. The students had a short introduction followed by five lessons to work on the task, with no additional homework during this period. Instructions were to choose a question the pair of students would like to investigate, a question statistically possible to answer. A poster was to be produced in pairs, representing the results of the investigation. Independency was encouraged at all stages of the task. After each lesson the students were encouraged to reflect over what they had done and how much advancement they had made since the previous lesson. The poster was handed in and presented in a 2-minute poster-session, in pairs, to a small group of younger students and to the teacher. The students assessed each other's work according to a list of goals and requirements they were given, for example to know what is meant by table and diagram and what the difference is, using different types of charts and graphs and position measurements. Concepts, methods, communication, reasoning and problem-solving skills were all outlined and exemplified in the description of goals and requirements.

Spatial geometry

Task 3 was a spatial geometry task from year 8, dealing with the concept of volume. It was an individual task where the aim was to create a geometrical shape with a volume of 5 L. The shape had to incorporate several different solids (cube, cuboid, pyramid, cylinder, cone, prism or others), including one that involved the use of π in calculations. The students had to make a sketch of the shape from different perspectives and present calculations in a detailed and comprehensible way. In order to get a higher grade, a sketch to scale was to be made, so that the shape could be reconstructed from the sketch. Correct mathematical language was to be used and calculations were supposed to be presented in a clear way. Materials provided were paper, glue, ruler, calculator and adhesive tape. Assessment of the task rested on the calculations of volume.

Placing the world population on an island

This was a Fermi-problem focusing on the concept of area and estimations. It was dealt with in year 7 and was one of the first tasks the teacher introduced to the students and was used for assessment. The theme was "Is the world really overpopulated?" A discussion between two girls was presented on the issue of the world population. The two girls argued about how much space it would take if the whole world population were squeezed in a small area. Is there an island where the whole world would fit in at once, the first girl wonders? No way, says the second girl. The students were asked to agree with one of the girls, by investigating if it was possible to fit in the world population on an island and present their investigation to the rest of the class.

Overall results

All of the students stated that they are, to some extent, interested in mathematics and engaged in tasks. The data presented in Table 1 includes the four tasks the students recall as interesting and engaging. There is no ranking order in which the students chose the task. Five students, A2, A4, C1, C3, E3, chose more than one task to illustrate interest and engagement, while students C5 and E2 could not name any specific tasks and are therefore not included in Table 1.

As shown in Table 1, most students could give examples of one or several interesting and engaging tasks. The target knowledge in all student-chosen tasks is within the field of geometry and statistics. However, there is evidence that other topics are interesting and engaging: Five students (A4, A5, C4, C5, E2) stated that they found probability and percentages engaging. A4 and E2 found algebra (equations) as generally engaging, A5 found it especially interesting to solve systems of equations or working with two unknown. Two of the students (C5 and E2) could not exemplify interest and engagement by describing a task they had worked on; they spoke only about interest and engagement in general, but did not come up with any examples during the course of the interview. Another student, C3, also mentioned the coordinate system, graphs and diagrams when talking about the statistics poster, and gave an example explained by the teacher on the board, about how to represent and compare the velocity of different vehicles.

Analysis of students' utterances on why tasks are interesting and engaging

The following section consists of representative student quotes analyzed thematically. Thematic analysis is used to discover patterns related to existing themes (Miles and Huberman, 1994), from TDS and MTF. The themes that emerged connect interest and engagement to the target knowledge, the didactical situation, the context the target knowledge is embedded and the aspects of task structure.

Interest and engagement in relation to the target knowledge

One particular task feature that stood out was the target knowledge and it was mentioned most frequent, early in the students' utterances. In Task 1 scale, measurements and proportionality were brought up particularly early in their utterance by the students from all the three groups (A, C and E):

I: Can you give an example? An example of a task that was interesting?
A1: It was… What is it called? [When we worked] with scale, sort of. We got… Sort of… A Barbie doll, […] and we were sort of supposed to make it in a certain scale. We were supposed to see how proportional it was.

Similarly, to student A1, student A2 first pointed out the target knowledge, in this case scale, and only thereafter the didactical situation in which the task was dealt with:

I: What task or tasks did you find interesting? What task did you find engaging?
A2: Lets see. What was it about… That's right, it was scale. We got a Barbie doll and drew it to see if it really could be…

In the above citations from the interview with students A1 and A2, the target knowledge was to a high extent at the fore for the students, which is shown by how they described the task. This is similar to the excerpts from the same students presented earlier in this section. In the next excerpt, it is visible that the target knowledge was the primary focus in relation to interest and engagement:

I: Can you tell about an interesting task? When you were engaged?
E4: We worked with scale. A doll… Or… We had a doll. And we were supposed to draw it ten times bigger. Draw so it would be exactly 10 times bigger.

In case of E4 both the ratio of the drawing and the process of drawing itself were mentioned. The students clearly knew that the task was about scale and not about the Barbie doll. The doll was perceived as a tool in order to reach the target knowledge. So even when both the doll and the situations were mentioned, the target knowledge still appeared at some point in the descriptions:

I: What task… Or what tasks were interesting and engaging according to you?
E5: We were in a group and then we were supposed to make a doll in… Scale? What is it called? I don't remember how it was but anyway, it turned out to be 2 m tall. Something like that, really tall.

The target knowledge in Task 2, bar charts, was mentioned almost instantly, in combination with the presentation and the choice of topics:

C3: We were supposed to choose topics and make a chart, a bar chart. Also to explain the results for example like…
Which music is the most popular on Spottily right now and eeh...

C3 mentions the chart when talking about Task 2, a task in which the target knowledge has a central position, and thereafter the context he/she chose was pointed out. It was clear in the instruction of Task 2 what type of statistical representations the students should use. Students also found Task 2 interesting and engaging due to the target knowledge from the lower-level group:

I: Why was it [the task] interesting?
E3: Actually, I do not know. Maybe because it is… about diagrams and that, I think that is pretty fun.
But if the task had more text in it I think it would be harder, because I had to pick out part that were like… important and I think it… It is harder.

This student stated that diagrams are fun, and can make the task interesting. This is an example of how the target knowledge at the fore can make the task interesting and engaging. The level of challenge is also affected if the task was perceived as interesting and engaging. Here the clear focus on the target knowledge with no superfluous text made the task manageable and therefore interesting to the student. The target knowledge in Task 3 was also recalled when the task was described, being the concept of volume and the way the student recalls working with a solid:

I: Can you tell me about an interesting task you have worked with? When you felt engaged?
A2: I think it was in 8th grade. We were supposed to work with volume.
An individual task with a shape that contains 5 L.

The geometric solids and the fact that they could chose different shapes were mentioned:

A3: In geometry we got to…sort of…different shapes. We were supposed to build three different… You know what I mean? […] Cube, cylinder. 5 L in total.

In group C, the target knowledge of area and shapes emerged, and other details, such as 3D, π and calculations:

I: Are you interested in mathematics?
C1: Mm…Yes.
I: Are you engaged? Do you feel engaged during math lessons?
C1: Yes. I usually am quite engaged. […] There are some things that I am really good at, or best at, but I

have missed a lot…
I: What is your best topic?
C1: Best topic…
C1: It must be the one with that… 3-Dimensional.

And then you are supposed to use to… π… to calculate…
What is it called? I do not know. [Shows area of a square on the paper].

The student talks about interest and engagement in relation to being successful, being good or even best at mastering the content. In connection to the target knowledge, that is the use of *π*, another reason for Task 3 as being interesting was the way the target knowledge was made comprehensible, as student C1 describes:

C1: …it is [interesting and engaging] when you know beforehand, such as here with π, here you know that you will multiply it. You sort of know what you are supposed to do.

In this utterance, the use of *π* was mentioned in connection to an interesting and engaging task. In relation to Task 4, calculation and presentation were brought up:

I: Can you come up with a similar example…With an example that made you feel like you were engaged, at least?
A5: Eeeh, it was a long, long time ago. We… How was it? Like…We had a task and we had to calculate…. If there was only one country in the world, would all the population of the earth fit in this island or in this country?

Or as A4 describes the same task:

I: What was it about and why was it… Yes, why was it interesting? A4: We were supposed to present about a country, I think it was. And how big the surface was and such.

A5 mention the target knowledge, since he talks about "calculations" and A4 describes the target knowledge when talking about the area as "how big the surface was". This can be seen in light of the task instructions, where the emphasis on participation in the presentation and understanding how to solve the task can be interpreted as being an aspect of the didactical situation rather than the target knowledge. It was said that they would be graded on mathematical creativity (showing how the task is solved); knowledge about calculation (how mathematics is used to reach the solutions); communication (how the others in the class comprehend students calculations and conclusions); and participation (how active the student is during group work).

Interest and engagement in relation to the didactical situation

In addition to the target knowledge the students brought up features of the didactical situation as reasons for interest and engagement. These features were related to the activities in the task, such as calculating, drawing and other practical aspects. In Task 1, a student from group C brought up the didactical situation and what was practically done during the lessons:

I: What task did you find interesting to work with?
C2: We were supposed to make a picture.
We had to measure different parts of a doll and make a really huge doll out of it. We measured every centimeter, in scale.

According to this student, the didactical situation consisted of making a picture. The student also talks about the target knowledge measurements, with making the picture in the foreground. The presentation of the poster at the end was one of the reasons the students were interested and engaged in Task 4:

I: Why were you engaged?
A4: I was engaged because I knew I would have a presentation later. […] It is about the task where you do not sit alone and calculate, alone, but you are supposed to maybe present later.

When the activity, the presentation, was pointed out as engaging (by the same student that brought it up in Task 2), new form of presenting was emphasized.

I: What was it about the task that was so engaging, then? A5: We were supposed to present, about a country. […] And we were supposed to present it to [teacher's name], who was a little bit new to us so…

The type of presentation was selected by the students, as stated in the instruction of the task, and that part as well as the selection of the questions to investigate, involved free choice. The fact that it involved elements of presentation is, in my interpretation of the interview data, what made it interesting and engaging. This was also noted in the descriptions of Task 1, where students drew upon drawing and calculations as interesting and engaging:

I: Why was it interesting?
C4: It was interesting because…
You had to draw so everybody had something to do every time.
It [the scale] was 1 to 15.

Drawing and calculating were pointed out as engaging by several students, for instance E1:

> I: Why was this task interesting? E1: …because we could draw and calculate.

In the case of Task 3, first the practical aspect of the task is mentioned as one of the reasons for being engaging, followed by the target knowledge and the description of what solids the student worked with:

> C1: Because we worked… A little bit practical, so…
> We took some paper and made a square.
> We were supposed to make an area and then form three different shapes with that. Threesome…
> Triangle, cylinder and we were supposed to sort of put them together.

Interest and engagement in relation to the context

One student mentioned the context, the Barbie doll, as the reason why Task 1 was interesting:

> I: Why was it an interesting task? What made it…?
> E5: It was my favorite task because it was about the Barbie doll.
> It was about… we were supposed to make a Barbie doll that was supposed to look like… I mean…
> It was supposed to be really big.

Even despite the real-life context, it is hard to imagine that Barbie dolls are appealing to this age group, compared to the effect it might have on initiating interest and engagement in younger children. The context provided a personal connection, a development of a mathematical model, modeling the big doll - the human size doll, and the relations between the proportions of the doll and the human body in connection to the student's own body.

Task 3 had a purely mathematical context, the students worked with geometrical shapes, although an association to a realistic context, a water tower, was made by the illustration provided on the instruction sheet. Since the total volume of the body is to be only 5 L, the context is not real-life, but realistic in a sense of being a prototype of a water tower. It is not the context of a tower prototype the students point out as interesting and engaging, but the three different types of solids they chose. Task 3 was of high cognitive demand, with questions that allowed or required investigation through the use of materials, data gathering, testing and, most importantly, choice of solids and calculations. In other words, Task 3 was pointed out as interesting and engaging because it was explicitly mathematical, with the target knowledge in focus. Task 4 had a real-life context, although in comparison to Task 1 and 2 was not a realistic one, providing an imaginary situation connecting the question of overpopulation with the size of the available land. Students got to choose an existing island.

Interest and engagement in relation to cognitive demand

All of the mentioned tasks were found interesting and engaging due to their high level of cognitive demand. Using MTF terms, Task 1 demanded complex, non-algorithmic thinking, without explicitly providing any predictable approach in the task instructions. It included exploring and understanding the nature of the concepts of scale, the process of reproducing a 3D image and how disproportional the doll is to human sizes. Self-monitoring or self-regulation of one's own cognitive processes when measuring and finding out about human size proportions is required to solve the task. Relevant knowledge and experience when reproducing an image to a certain scale, and making appropriate use of them is also an indicator of the high level of cognitive demand. What is reasonable when it comes to human size and what in measurements of the doll that limit possible solution strategies was analyzed. The process leading to the reproduction requires several steps, which meant considerable cognitive effort for the students. The students recognized this task as interesting and engaging because it was challenging:

> I: Why did you… Do you like this task?
> A1: I like it because… It is a really hard tasks, that… sort of…
> No one can solve and then… […] One really gets to work (A1).

Or as another student puts it: C2: It was hard for me, it was fun.

Task 3 includes exploring and understanding the nature of area and volume and it involves complex and non-algorithmic thinking in relation to the transition between area and volume. There was no predictable approach explicitly suggested by the task instructions; however there was an explicit instruction about working with π and some suggestion of which shapes to choose. Relevant knowledge and experience of the concepts of area and volume as well as of the shapes, making appropriate use of the concepts to end up with 5 L is a part of this task. Students had to analyze task constraints that could limit possible solutions and strategies. Task 2 was also highly demanding, since no predictable approach was explicitly suggested by the task instructions – the student had to formulate a question and chose a way of representing the answers. As a part of this task, students were exploring and understanding the nature of concepts, such as representations, percentages and using different types of diagrams and charts, processes and relationships. During the exploration, self-regulation was central. The students were given free rein to choose representations. In order to represent the answers, the students were given an opportunity to use relevant knowledge and experience appropriately. Task constraints that could limit possible solution strategies and solutions were analyzed while making the choice of diagrams and justifying the choice, as was pointed out by C3 and A4 in the interview:

> C3: Up there we had… Mm… How many percent? We wanted to know how many percent and it is much easier to find out if you look at a bar chart. […] Since there were only four categories and then it is much easier with a circle diagram.

When it comes to cognitive demand of Task 2, the process leading to the solution required cognitive effort, although concepts, methods, communication, reasoning and problem-solving skills were all outlined and exemplified in the description of goals and requirements. Task 4 provided a high level of challenge mentioned as one of the reasons it was interesting and engaging:

> I: So why was this task interesting, why did you engage in it? A5: It was not just… Eeehm… With the help of the map. You were supposed to calculate, like…To really put your mind to it.

The task was interesting and engaging due to the high level of challenge, as student A5 explains, beyond the experience of the tasks they had previously worked with in their textbooks:

> A5: The tasks in the book are just one after another, just calculate - next one calculate - next one.
> This task is not like that. It makes you think, like extra.

Task 4 was of high cognitive demand, it required non-algorithmic thinking, provided no numerical values and no predictable approach explicitly suggested by the task instructions; the students had to explore the task by using relevant previous knowledge and experience of the concept of area and link it to the population size. Since no suggestions of how to make the necessary calculations are given, or even where to start, the unpredictable nature of the process leading to the solution of Task 4 requires considerable cognitive effort for the student. The students analyzed task constraints that could limit possible solution strategies and solutions, for example the choice of the island.

DISCUSSION

All of the tasks brought up by the students had the target knowledge at the fore and were of high cognitive demand, which was recognized as interesting and engaging by the students. Another aspects of interest and engagement in tasks are the didactical situations (Brousseau, 1997) in which they are dealt with, for example if elements of drawing and presenting are included in the task.

The context a task is embedded in was also pointed out as a reason for becoming interested and engaged, but no particular context was chosen over the other. For example, the most frequent selected task (Task 1) with the target knowledge of scale and measurement has a realistic context, as well as Task 2. Task 3 was purely mathematical while Task 4 was related to real-life but not realistic. As long as the context was well-matched with the mathematics in the tasks, the students seem to find the task interesting and engaging.

If judged by the frequency, Tasks 2 and 3 are almost equally interesting and engaging. Task 4, a Fermi problem is similar to the task about how many people can stand up in the classroom (Sullivan et al., 2013) and was brought up by two students from group A. The students became interested in the tasks, by engaging in the combination of understanding the concept of area, estimation strategies and the notion of measurement errors and applying this understanding in thought experiment. This suggests that a task with such a structure has the potential of being used to enhance interest and engagement for students who already have pre- knowledge in the area and strive for challenges.

The time frame does not appear to influence students' choices, since the chosen tasks were dealt with during different periods, 0.5 to 2.5 years previously. For the past three years, these students have been working with different types of tasks, some designed by the teacher and others taken from the textbook, short routine tasks and large ones, thematic tasks that include smaller task units. However, all 15 students recalled one, or several, of the four specific tasks to exemplify their interest and engagement.

In all tasks the students brought up target knowledge rather than any other task features or aspect of the didactical situation. For instance, in Task 1 the students referred to scale rather than to the Barbie doll as interesting and engaging. In Task 2 the students put the mathematics to the fore and the didactical situation in which the mathematics is brought up more in the background. Task 2 provides an opportunity for the students to contextualize the target knowledge on their own terms, and to choose representations based on their questions and the data generated when posing them. It is also noteworthy that two of the four chosen tasks include target knowledge from geometry and that the target knowledge in all four tasks are from only two areas of mathematics. The context is in all cases a pathway, leading the students to the mathematical idea. The students are aware of this, which is shown by the most common way they describe the tasks when they bring them up, by acknowledging the target knowledge, such as scale, prior to the didactical situation. The teacher's focus on bringing the target knowledge to the fore shows up in the students' reasoning in connection to interest and engagement. In terms of TDS, the target knowledge was not only recognized as the reason for becoming interested and engaged. In other words, the meso-level of the didactical contract that is the activity level is important when enhancing interest and engagement in mathematics (Nyman and Kilhamn, 2015). However, in contrast to the mentioned study, the qualities the teacher brings up are all mirrored in the way the students' reason about the tasks. The teacher took the target knowledge as the point of departure when designing the task and managed to engage the students in the didactical situation successfully.

The teacher designed the tasks to provide opportunities for students to make their own choices and included elements of presentation, which turned out to be interesting and engaging. The most chosen task, Task 1, provided the fewest opportunities for student choice and then only in connection to the context it, what doll to choose and which body parts to measure. Task 2 provides a choice of mathematics content, such as which diagram to illustrate the question with, though the choice is dependent on the data generated by the chosen question. In Task 3, the students chose the solids and presented them to the rest of the school by hanging them up in the hallway. Task 4 provides the choice of the island, which indirectly also leads to the choice of area on which to put the world population. Choices related to modeling also had to be made. To give the students choices might be a catalyst for engaging them, allowing the didactical situation to devolve, and for the students to make the task their own. This does mean that the task has to be as open as possible in order to be engaging. The open-ended elements of the tasks, where students made their own choices, challenged students' decision- making. As Boaler (1993) puts it, an activity can start with a context, but then be open for the context to be up to student's own development.

All of the tasks were on a high level of cognitive demand, implying that the tasks require some previous basic knowledge in arithmetic, geometry or statistics. This result supports the early assumption that in order to be engaging a task must be seen as challenging by the students (Hilbert, 1900). Of course, previous knowledge of the subject can affect the student's view on whether a task is challenging or not. This might be the reason why only students from group A, the ones aiming for the highest grade, brought up Task 4 as interesting and engaging. Also, all of the chosen tasks were non-routine tasks, strengthening the findings of Carpenter et al. (1989) that non-routine tasks are interesting and engaging.

A limitation of this study is that the students find it difficult when reflecting on what makes a task interesting and engaging. Not all of the students' could discuss details of the tasks. When addressing the second research question, it was found that the students had difficulty pointing out what made the task they suggested interesting and engaging. Attempts in the interviews to find out more details often resulted in answers like "I do not know", or "everything", about one or several of the chosen tasks. Therefore, in future studies it would be fruitful to approach this question by bringing the task back to the students and making an additional, task-based interview.

Conclusion

The study provides a student perspective on what can make tasks interesting and engaging. The analysis of the interviews shows that the students identified the target knowledge as a reason for being interested and engaged which confirms the teacher's intention. All of the chosen tasks are of high cognitive demand, require previous basic knowledge and are set in different context. They were all examination tasks with an element of presentation and sharing, and open to some extent; leaving room for student's own choices of the conditions in which the task is set.

A didactical implication of the presented results is to have the target knowledge as a starting point in task design, preferably with a well-matched context to make the mathematics visible. All student-chosen tasks had target knowledge from geometry and statistics, suggesting that target knowledge related to those topic areas can be a good starting point and a valuable source when working with interest and engagement on a general level in grades 7 to 9. Concerning the context, the results show that both tasks with purely mathematical context and with a real-life context were chosen. What the students highlighted as interesting and engaging was their influence over the context, choosing what body part of a doll to measure in Task 1, what shape to work with in Task 2, what questions to pose and what type of descriptive statistics to work with in Task 3 and what island to place the population on in Task 4. The study provides an opportunity to expand views on the nature of interest and engagement in mathematics. In future research it would be rewarding to investigate the interestingness of target knowledge of other topics areas. For instance, by interventions including the features highlighted by the students in this study (high cognitive demand, influence over context, presentation), including target knowledge from other topic areas.

Conflict of Interests

The author has not declared any conflict of interests.

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