Spatial dependency and contextual effects on Academic Achievement

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This study investigated the influences of district-related variables on a district’s academic performance. Arkansas augmented benchmark examination scores were used to measure a district’s scholastic achievement. Spatial analysis fit each district’s performance to its geographical location; spatial autocorrelation measured the amount of influence one district’s scores has on its neighbours. Regression, both ordinary least squares regression (OLS) and spatial auto regression (SAR), quantified how much a district’s academic scores were accounted for by the proportion of white students enrolled, the appraised value of property within a district, and the proportion of students receiving federally assisted lunches. The OLS model was able to account for 30% to 60% of the variation in scores. When ethnicity was predominately white, the district’s scores were higher; the more federally assisted lunches a district’s enrolment received, the lower scores tended to be. Spatial analysis indicated that a district’s performance was highly influenced by the surrounding districts. Major findings showed that, for 2008 data, once fit to an OLS regression model, the spatial dependency completely disappears for mathematics responses, but not literacy. Similar results were seen in 2009 and 2010, though they were not as systematically patterned.

Keywords: Arkansas education, regression, spatial analysis and spatial autocorrelation.

INTRODUCTION

In 2002, the "no child left behind" act changed the face of public education. As an effect, schools are being held accountable for the performance of students on standardized tests. Local report cards give a testimony of the performance of schools and students within a district and are required by each school district to be available to the public (U.S. Department of Education, 2003). This change in educational policy sparked new interest into what factors influenced a district’s performance. As the smallest unit of assessment, the student is of most interest, but classroom, teacher, school, district, regional, state, and ultimately national characteristics all influence the ability of the examinations to capture the performance skills of the students.

A weighted mean of a school’s scores within a district determines the district’s overall academic performance. Ideally, this average score would solely reflect scholastic achievement, yet it was being influenced by other factors. A measure of these outside influencers would provide a better understanding of a district’s overall performance. Three important factors considered in this study are: the proportion of white students enrolled, the total appraised value of all furniture, equipment, buildings and real estate within a district, and the proportion of students who

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received federally assisted lunches.

An emerging factor in education is also the effect a single district’s academic performance may have on the surrounding districts. Correlation quantifies the relationship and dependency between two variables: spatial autocorrelation quantifies how a measurement at one location may be affected by measurements at surrounding locations. By assigning each district’s assessment score to its geographical location, the measure of influence one district’s performance has on neighboring districts can be quantified. In the state of Arkansas, clusters of high and low performing districts are present, and single districts exist which are surrounded by districts with opposite performance levels. Only in the past decade researchers began to study the influences of geographic location on performance levels between and among districts; each found some amount of spill over. By accounting for spatial autocorrelation, the present study seeks to accurately measure the extent to which outside factors may correspond to academic achievement for school districts in Arkansas.

Background

Reeves (1998) began a research series on district accountability in Kentucky. The study identified four relative variables: the effects of median household income, percent of students on free or reduced lunches, per student spending, and enrollment. As a response to these covariates, Kentucky school districts’ accountability index scores [KIRIS] were used. Spatial autocorrelation was measured using Moran’s I test statistic, and a generalized linear model [GLM] was applied. Results of the GLM indicated that the percent of students on free or reduced lunch had the strongest influence on a district’s accountability score, while median household income had no effect. After fitting the model, spatial autocorrelation among districts was decreased. Reeves (1998) noted that the influences of neighboring districts increased as grade level increased.

This study generated the question: Should spatial analysis be a principle component in educational study? Reeves and Pitts and Reeves (1999) followed up with the previous research, and the KIRIS scores were again used as the dependent variable in the study. Socio-economic factors were the focus of influencers, including rural-metro differences in districts, median household income, percent of students on free/reduced lunch, teen birth rate, and enrollment. Similar to the results of Reeves (1998), the median household income was not significant and percent of students on free and reduced lunch had the biggest impact on the KIRIS scores. As the grade level increased, the effects of SES also increased. Geographic clustering was seen in Kentucky school districts based on the assessment scores, and the strength of the correlation was measured with Moran’s I test statistic. In many cases, when contextual effects were controlled, the spatial autocorrelation disappears. The researchers concluded, “Results indicate that including contextual effects as explanatory variables reduces the spatial autocorrelation and provides a more reliable measure of school and school district performance.”

Chen and Ferguson (2002) conducted spatial analysis with the 3-year grand averages of district achievement scores in 4th, 8th, and 12th grades in Massachusetts. Spatial autocorrelation between the accountability scores was detected and a simultaneous spatial autoregression [SAR] model was implemented instead of an ordinary least squares regression [OLS] regression model. Both school-related and non-school related variables were studied with great caution to ensure the absence of multicollinearity.

Teachers’ maximum and average salaries and superintendents’ salaries were also of interest. It was found that increases in $1,000 to both maximum teacher salary and superintendent salary tended to raise MCAS scores by one seventh of a point and one twentieth of a point, respectively. Other school variables of interest included internet access, budgeting, and percent of students with limited English speaking skills. Having the greatest impact was the language variable; each percentage increase in the number of students with limited spoken English decreased test scores on average by a tenth of a point. Economic variables included income, government aid, household makeup, race, and political affiliations. As a result, scores increased by approximately half a point per every $1,000 increase in per capital income; a one percentage increase in the number of families with two parents were associated with an increased scores by one-seventh of a point, and for each percent increase in number of recipients receiving Temporary Aid To Families With Dependent Children [TAFDC] payments scores decreased over a point and a half. Other variables studied- environmental hazards within districts, political party identification, and voter turnout- had no effects.

Brasington (2005) studied public- and private-school competition, and concluded that a school district’s performance does affect that of its neighbouring districts. Thus, it was suggested to use a SAR model with the percentage of students passing all four sections of achievement exams as the dependent variable. Higher pass rates were detected in districts with higher amounts of two-parent families, higher income districts, and also higher paid teachers. Districts having lower pass rates often had more students from a minority group and, interestingly, teachers with higher levels of education. Fittings SAR model increased the R² coefficient of determination, measuring how well the dependent variable was accounted for by the covariates, from 0.49 to 0.56.

Rodriguez-Pose and Tselios (2012) concluded, “Regions with similar educational conditions tend to cluster, often within national borders.” Through exploratory spatial data analysis, the report addresses educational inequality across 102 regions in Western
Europe. The level of education tended to be highest in rural areas and lowest in more urban areas.

Student achievement across regions of Italy and Spain were assessed by Agasisti and Cordero-Ferrara (2013). Regional heterogeneity was present in both regions. In Italy, where governmental system had a stronger control on educational policy, geographic location had a higher influence on students’ achievement, whereas in Spain, having a more autonomous government, the special influences was not as strong.

Strategy and Hypothesis

Taking into account the previous studies, the current research continues the study of the effects of geographic locations and various demographic variables on academic achievement scores. The 2008 through 2010 Augmented benchmark examination mean score for each school district in both mathematics and literacy was used as the dependent variables of interest. The following covariates were used in fitting the regression models: the proportion of students who received free or reduced lunch, the proportion of students classified as white, and the proportion of students who received free or reduced lunch, the total appraised value of a district’s assets measured the worth of physical properties; and the proportion of white students indicated the amount of ethnic diversity within a district.

METHODS

Arkansas Benchmark examination

Under the No Child Left Behind Act, students in grades three through eight were required to take standardized examinations in both mathematics and literacy; those in grades five and seven were also assessed in science (U.S. Department of Education, 2003). The Augmented Benchmark Examination was implemented to fulfill this requirement (Arkansas Department of Education, 2009). Performance levels were defined for each subject and for each grade, profiling a score as advanced, proficient, basic, or below basic. Appropriate modifications were made for students with disabilities. The Arkansas Department of Education supplied a complete description of performance level and accommodations made for students with a disability (Arkansas Department of Education, 2009).

For analysis, scale scores from the Arkansas Augmented Benchmark exams were aggregated from each school to the district level. The mean scaled score for a district was calculated by taking a weighted average of all schools’ scores within a district, weighted by the total number of students processed within each school.

Scores were reported to the public based on a performance level, rather than actual scores, shown in Table 1. Different range definitions existed for each subject and grade level due to the varying content. Therefore, it was not appropriate to compare scaled scores of different subjects or grade levels (Arkansas Comprehensive Testing, Assessment, and Accountability Program (ACTAAP 2008).

Covariates

The proportion of students who received free or reduced lunches, the proportion of white students, and the total appraised value of a district were selected to summarize key characteristics of a school district as they potentially impact student achievement. The proportion of students who obtained financially aided lunch gave an index to the amount of poverty within a district; the total appraised value of a district’s assets measured the worth of physical properties; and the proportion of white students indicated the amount of ethnic diversity within a district.

The data were obtained from the Arkansas department of education [ADE] Data Center. The proportion of white students was calculated as the total number of white students divided by the total number of enrolled students within a district. The proportion of students receiving free or reduced lunch represented the combined number of students receiving free lunches or receiving reduced price lunches divided by the total number of enrolled students within a district. The total appraised value was the combined value of all furniture, equipment, buildings and real estate within a district.

Statistical Analysis

Spatial mapping involved the construction of a matrix, W, to record the structure of neighbouring regions. Two types of spatial weight matrices were used for comparison: (1) binary and (2) row standardized; these were indicated as B and R, respectively, in the results. A matrix of zero’s and one’s constructed a matrix of neighbouring districts, where \( w_{ij} = 1 \) if the two regions were adjacent and \( w_{ij} = 0 \) if the two regions were not adjacent. This matrix was symmetric along the diagonal. A row standardized weight matrix divided each “weight,” or distance between neighbours, by the row sum. This standardization created a relative distance, between zero and one, for each neighbour rather than an absolute quantity.

Spatial autocorrelation measured the degree of dependency among variables in a geographical space. The following equation includes two components needed when calculating this spatial statistic, that is. The closeness between two observations, \( w_{ii} \), and the measured relationship between the observation and its neighbours, \( a_{ij} \).

\[
S = \sum_i \sum_j w_{ij} a_{ij}
\]

This global spatial autocorrelation statistic quantified the interaction of the location of the regions and value of variables at each region with a single statistic, summarizing the correlation over the entire region. Moran’s I statistic evaluated the global spatial autocorrelation by linearly relating the variable of interest and the weighted sum of values of the neighbours, shown in the following equation:

\[
I = \frac{n}{\sum_i \sum_j w_{ij}} \sum_i \sum_j w_{ij} (x_i - \bar{x})(x_j - \bar{x})
\]

Where \( w_{ij} \) is the spatial weight, \( x_i \) is the observed value at location \( i \), and
Table 1. Performance Level Classifications for 2008 Benchmark examination scores.

<table>
<thead>
<tr>
<th>Mathematics</th>
<th>Literacy</th>
</tr>
</thead>
<tbody>
<tr>
<td>BB</td>
<td>BB</td>
</tr>
<tr>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>P</td>
<td>P</td>
</tr>
<tr>
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<td>A</td>
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<td>0-329</td>
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<tr>
<td>409-499</td>
<td>330-499</td>
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<tr>
<td>500-585</td>
<td>500-653</td>
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<tr>
<td>586-999</td>
<td>654-999</td>
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<tr>
<td>0-494</td>
<td>0-353</td>
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<tr>
<td>495-558</td>
<td>354-558</td>
</tr>
<tr>
<td>559-639</td>
<td>559-747</td>
</tr>
<tr>
<td>640-999</td>
<td>748-999</td>
</tr>
<tr>
<td>0-543</td>
<td>0-381</td>
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<td>673-866</td>
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<td>0-506</td>
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<tr>
<td>655-699</td>
<td>507-699</td>
</tr>
<tr>
<td>700-801</td>
<td>700-913</td>
</tr>
<tr>
<td>802-999</td>
<td>914-999</td>
</tr>
</tbody>
</table>

Note: BB: Below Basic, B: Basic, P: Proficient, A: Advanced.

\[ \bar{x} \text{ is the mean value of observation.} \]

The test assumed no spatial autocorrelation was present. The Moran’s I test statistic is interpreted similar to correlation. Positive values indicated areas of positive correlation or similar measurements, and negative values indicated areas of negative correlation or dissimilar measurements. Values close to zero indicated areas that are spatially random, or that have no spatial autocorrelation (Anselin, 1995).

Regression analysis, Multiple linear regression analysis was used to fit a linear combination of the covariates to a single dependent variable in the following equation:

\[ Y = \beta X + \varepsilon, \]

where \( Y \) is a vector of the response variable,

\[ \beta = [\beta_0 \beta_1 ... \beta_m], \]

is a vector of the parameters,

\[ X = [1 \ X_1 \ X_2 \ ... \ X_m], \]

is a matrix of the covariates, and

\[ \varepsilon \]

is a vector of the error terms.

The vector of parameters, \( \beta \), were estimated using the least squares method, which minimized the sum of the squared distance from \( Y \) to the predicted value, \( \hat{Y} \), notably

\[ S(\beta) = \sum \varepsilon^2 = \sum (Y - \beta X)^2. \]

By taking the partial derivative of the least squares function, \( S(\beta) \) with respect to the parameters, the parameters were estimated using the following equation:

\[ \hat{\beta} = (X'X)^{-1}X'Y. \]

In choosing which variables to use in the regression model, covariates were checked for multicollinearity. This occurs when two or more variables are almost perfectly correlated. The coefficient of determination, \( R^2 \) shown in the following equation, measured of the variation in the response which accounted for the by covariates and was useful to determine the validity of the fitted model and how well the model predicted:

\[ R^2 = \frac{\sum (\bar{Y} - \bar{Y})^2}{\sum (Y - \bar{Y})^2}. \]

Spatial simultaneous autoregression. A simultaneous auto regression model, SARerr, takes into account the spatial relationship between observations. The response variable was not only affected by the explanatory variable but also by the geographical relationship between neighbouring responses. A spatial error model assumed that the error term of a fitted OLS model has spatial dependency.

Therefore, this model, provided in the following equation, complemented the OLS regression model with a spatial term, where \( \lambda \) was the spatial coefficient, \( \lambda \) was a spatial weights matrix, and \( \varepsilon \) was the spatial error term:

\[ Y = X\beta + \lambda W u + \varepsilon. \]

Several variants of SAR models are available: spatial error, lagged, and mixed models. The study by Kissling and Carl (2007) concluded that the spatial error model, SARerr, was the most robust; therefore, this was the spatial model of choice in this study.

Procedure

An OLS regression model was fit using the previously described independent and dependent variables. Normality, independence, and correlation of residuals were assessed. Because spatial correlation was detected, a SAR model was also applied for comparison.

The goodness of fit for each model was measured using Akaike Information Criterion (AIC), and a final model was chosen. Using geographical information system (GIS), the mean score for each district was mapped to its geographical spatial coordinates to create a geospatial data file to which the analysis was applied. In the absences of data, the district was omitted from examinations.

RESULTS

Data Summary

The proportion of white students was negatively skewed, shown in Figure 1, with 120 of the total 244 school
districts having more than 90% white enrolment. In 158 districts, more than 75% of the enrolled students were classified as “white,” most of which occurred in the northwestern half of the state, shown in Figure 2. Districts whose student populations were less than 25% white (n=17) were all located in the southwestern region.

Most districts (n=147) had between 50% and 75% of students who received free or reduced priced lunches, while only two districts had fewer than 25% of the student population who receives such aid, shown in Figure 3; in 18 districts, more than 90% of the students received free or reduced lunches. Those districts which had more than
75% of the students receiving aid were located in the south-eastern region, shown in Figure 4. Many of these were also those districts whose student populations were predominantly not white.

The spatial distribution of districts based on total appraised value did not present any visible correlation. The inner 50% of all districts had appraised values between $18.20 million and $57.78 million; outliers of
extremely high appraised values were in the Little Rock, Springdale and Pulaski County school districts, shown in Figure 5, with $576.32, $403.11, and 3.97 million, respectively. These districts also had the highest school enrolments.

Each variable was negatively correlated with the other, Table 2. Small amounts of correlation were between total appraised value and both other variables; strong correlation existed between the proportion of white students and the proportion receiving free or reduced lunch.

Spatial clustering of districts with similar performance levels was visible in each map, Figures (6 to 11). At the 8th grade level, no districts reach the advanced standing for either subject. Though it is inappropriate to compare scaled scores between subjects, comparisons of performance levels are recognized. As grade level increases, ranges of each level become stricter, shown previously in Table 1. Data for districts in white were not available from ADE.

Consistently, districts on the eastern border performed at lower level; in mathematics most often districts at the basic and below basic levels were in this region. Performance in this subject tended to worsen as grade level increased, more districts performing at the basic and below basic levels. In literacy, the spatial arrangement was not as continuous over grades; though districts in the southeastern region performed at lower levels, districts in the northwestern did as well.

A comparison of the spatial plots of scores (Figure 7) and of the covariates (Figures 2 and 4) revealed similarities. Districts at lower performance levels were those which had fewer proportions of white students and more students receiving free or reduced lunch. Similar results were found with all grade levels.

### Data Analysis

The full analyses were completed on each grade, three through eight, in mathematics and literacy for academic years completed in 2008, 2009, and 2010. For each grade, subject, and year combination, Benchmark exam scores were spatially correlated ($p < .001$).

The effects of the proportion of white students, the total appraised value, and the proportion of students receiving free or reduced lunch on a district’s benchmark exam
Figure 6. Spatial distribution of 3rd grade mathematics performance level of districts.

Figure 7. Spatial distribution of 3rd grade literacy performance level of districts.

Figure 8. Spatial distribution of 6th grade mathematics performance level of districts.

Figure 9. Spatial distribution of 6th grade literacy performance level of districts.

Figure 10. Spatial distribution of 8th grade mathematics performance level of districts.

Figure 11. Spatial distribution of 8th grade literacy performance level of districts.
score was assessed using an OLS model, shown in Table 3. The proportion of white students and the proportion of students receiving free or reduced lunches were significant \( (p < .05) \). Higher benchmark exam scores were present in districts having a higher proportion of white students and a lower proportion of students receiving free or reduced priced lunches. The total appraised value of a district was not significant in predicting the mean scaled benchmark examination scores. In 3rd grade, the chosen covariates accounted for 31% of the variation in mathematics scores and 41% of the variation in literacy scores. As grade level increased, the validity of the model also increased.

Analysis of residuals revealed normality in all models except 4th grade literacy and 5th grade mathematics using the Shapiro-Wilk test statistic. Non-constant variance was not suspected in the plots of residual value versus fitted values. The residuals were also tested for spatial autocorrelation using a row-standardized list; an interesting pattern arose. In models where mathematics scores were the response, Moran’s I test statistic was not significant \( (p > .05) \) and spatial autocorrelation had disappeared. Spatial autocorrelation of the error terms was present in those models whose response was a literacy score for all grade levels, but not for any models whose response was a mathematics score.

A spatial simultaneous autoregressive error model was applied due to the presence of spatial autocorrelation. Two spatial weights matrices were considered, binary and row standardized. Parameter estimates of the fitted SAR model using a binary list, Table 4, had similar significance to those of the OLS model. Models with a mathematics score as the response had little change, if any. For those models with a literary score as the response, the estimates had a substantial change.

The OLS and SAR models were compared using the AIC, shown in Table 5. As expected from the results of the residual spatial autocorrelation, the OLS model best fit those models with a mathematics score as the response, and the SAR model best fit those models with a literacy score as the response, except for 4th grade which only had a small significant amount of spatial autocorrelation in the residuals of the fitted OLS model.

For the 2009 and 2010 data, all original data sets were spatially correlated, and the systematic pattern of residual spatial analysis was not as apparent. Analysis of 2009 Benchmark scores when fitted to an OLS regression model revealed spatial autocorrelation of residuals in fitted models with responses in 4th, 5th, 7th, and 8th grade literature and also 7th grade mathematics. Spatial autocorrelation of residuals in the OLS fitting of 2010 data is only visible in 7th and 8th grade literacy.

**CONCLUSION**

State-mandated examinations are among the key elements in assessing individual students, schools, and districts under the *No Child Left Behind* legislation. Arkansas regularly administers the Augmented Benchmark Examination. The geographical distribution of test
scores revealed clustering districts of high and low performance. Similar dispersions were present in the proportion of white students of a district's enrolment and the proportion of students receiving free and reduced lunches. These two indicators had a significant association with the mean benchmark scores for its district.

As ethnic composition became more predominately white, scores tended to escalate. The increasing numbers of federally assisted lunches correlated with decreased benchmark performance. The overall value of a school district's property provided no indication of a district's benchmark performance.

A linear combination of the proportion of white students, the proportion of students receiving federally aided lunches, and the total appraised value of a district accounted for 30% to 60% of the variation in a district's mean score on the benchmark exam. The OLS models became stronger as grade level increases and, for each grade level, accounted for differences in literacy scores more than for mathematics scores. A linear model fully accounted for the spatial correlation between districts when looking only at mathematics scores of the 2008 examination. On the other hand, districts literacy scores' remained spatially correlated. Data from 2009 and 2010 resembled this data but were not as systematic.

The dependency between covariates, location, and benchmark exam scores should prompt educational analysts to additionally include the influences of spatial correlation of districts when examining state-wide exam scores within a state, especially on the subject of literacy. Outside factors influence a district's benchmark examination performance, as well as its surrounding districts' scores.

Cervini (2008) took a multilevel approach, studying the effects of class, school, municipality, and state level characteristics on mathematics achievement scores in Argentina. The report concluded that all levels were indicators of academic performance; class level factors accounted for the highest amount of variation in scores (15%). This study may also be extended to a multi-level setting with factors from four levels: classroom (including teacher), school, school district, and state.

### Conflict of Interests

The author(s) have not declared any conflict of interests.

### REFERENCES


