

Full Length Research Paper

# A planar microstrip metamaterial resonator using split ring dual at Ku-Band

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This paper introduces a new planar microstrip metamaterial resonator, the novelty of this paper lays in its unit cell design. The unit cell is formed by connecting metallic traces of two edge coupled split ring resonators to form the infinity symbol on one side of the substrate, and an array of conducting wires on the other. An RLC equivalent model of the structure is also proposed, it can be advantageous to use this model to identify the resonant frequency along with the root of the negative permeability and negative permittivity. The model shows resonance at 17 GHz. The structure was designed and simulated using EM solver Ansys HFSS, the extracted s-parameter matrix was analyzed to determine the effective permittivity, permeability and index of refraction. The structure shows negative values for effective  $\epsilon$ ,  $\mu$  at resonant frequency 16.5 GHz. At frequencies where both the recovered real parts of  $\epsilon$  and  $\mu$  are simultaneously negative, the real part of the index of refraction is also found to be negative.

**Key words:** Microstrip metamaterial, negative refraction, permittivity, permeability, RLC circuit.

## INTRODUCTION

Metamaterials, first named and theoretically discussed by Veselago (1968), are studied widely throughout the world. These are an artificially engineered material showing electromagnetic properties not readily found in naturally occurring material, such as, property of negative refractive index and artificial magnetism (Sabah, 2010; Mahmood, 2004; Sulaiman et al., 2010; Smith et al., 2001; Sharma et al., 2011). Recently work is done in direction of making a perfect lens using metamaterials (He-Xiu Xu et al., 2013a, b).

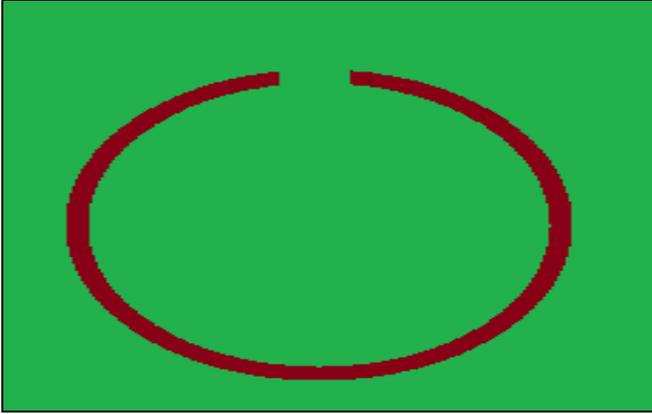
Metamaterials are often characterized in terms of their effective material properties, such as effective electric permittivity and effective magnetic permeability. Any one of these parameters, or even both of them may be simultaneously negative. The former is known as single negative material (SNG), if only effective permittivity is negative it is called Epsilon negative material (ENG),

whereas if only effective permeability is negative it is called as Mu-negative material (MNG). The latter is referred to as left-handed metamaterials (LHM), double negative (DNG), or negative refractive index material (NRIM).

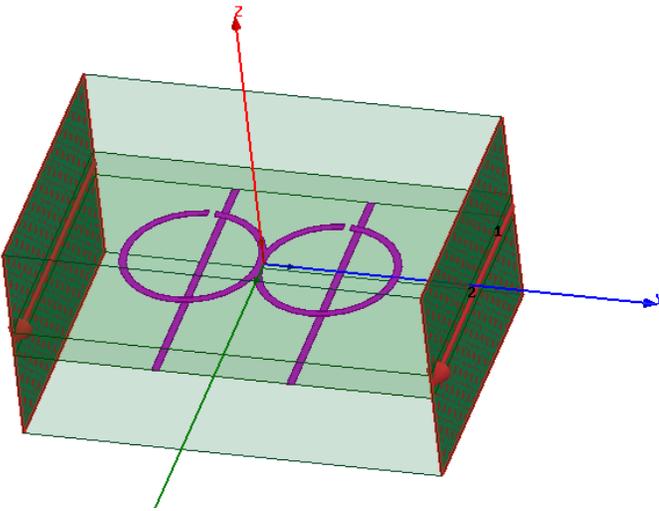
Artificial plasmas show negative effective permittivity for all frequencies smaller than plasma frequency of the Plasmon medium (Pendry et al., 1996). Effective negative permeability can be obtained in the well known Split-ring-resonator structure, but only for a narrow magnetic resonant frequency band (Pendry et al., 1999). In past few years, metamaterials has been a naive topic of interest among the research fraternity. Over these years various innovative structures have been reported.

This paper presents design and simulation of a new planar microstrip metamaterial resonator, exhibiting negative index of refraction. In comparison to the papers

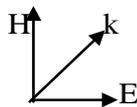
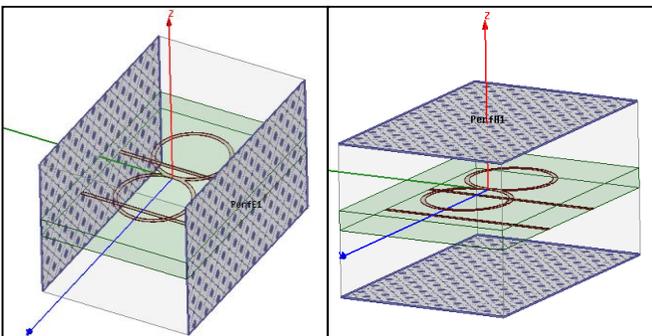
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**Figure 1.** A Circular Split-Ring Resonator Structure (SRR).



**Figure 2a.** Structure of Infinity Shaped Metamaterial (ISM) kept in a waveguide with wave-ports.



**Figure 2b.** Schematic diagram of ISM, showing the boundary conditions: PEC and PMC boundaries respectively.

cited here, the novelty of this paper lies in the unit cell design, where two edge coupled circular split-ring resonators are connected to form the infinity symbol and an array of straight wire conductors is also used. The overall size of the metamaterial unit is very small 6 mm x 8 mm. Since the geometry of the structure resembles the shape of mathematical symbol 'infinity' it will be referred to as Infinity Shaped Metamaterial (ISM) in rest of the paper. The structure was simulated using Ansys HFSS and the extracted s-parameter values ( $S_{11}$  and  $S_{21}$ ) was analyzed to calculate index of refraction. The results promises a bandwidth of around 8 GHz (~8.5 to 16.5 GHz).

Many researchers have done the analysis of SRR unit cells and SRR arrays, and shown that SRRs behave as LC resonators that can be excited by external magnetic flux. The analysis of SRRs by accurate circuit models can be effectively used to estimate the behavior of SRR structures in a simple, efficient manner. Also, explicit relationships between electrical parameters, dimensions of the SRR structure and its frequency dependent transmission/reflection behavior may be found.

Another method called NRW technique (Suganthi et al., 2012) was also used to calculate effective permittivity and effective permeability from s-parameters, using which refractive index can be calculated. The results obtained from all techniques were compared and found in satisfactory agreement with each other.

## DESIGN SETUP

Figure 1 above shows a single Circular Split-Ring Resonator, and Figure 2a shows the unit cell of Infinity Shaped Metamaterial (ISM).

It has been shown in various papers that a single SRR provides magnetic resonance and supports negative effective permeability (Pendry et al., 1999), in this paper it can be seen from Figure 2a that, ISM can be formed by connecting traces of two SRRs in edge coupled configuration in the shape of mathematical symbol 'infinity'. This structure behaves as 2 SRR's connected in series. Figure 2a also shows the ground plane, which is composed of an array of straight wire conductors instead of a continuous sheet of copper. These straight wire conductors are placed directly beneath the slit of the SRRs lying on other side of the substrate; they will provide a virtual path for the currents to continue flowing in the split rings.

As suggested by Pendry et al. (1996), the electric field should be parallel to the wire while the magnetic field should be perpendicular to the SRR. To retrieve the scattering parameters the radiation setup of the structure is done in an air filled waveguide. The electric (PEC) and magnetic (PMC) fields are defined over the walls of the waveguide in such a manner that the aforementioned conditions are satisfied, and is shown in Figure 2b. The structure is fed RF signals ranging from 15 to 18 GHz,

Table 1. Parameter table.

Parameters	Values
Substrate (Duroid (tm)) with Thickness	0.786 mm
Relative dielectric constant	1.1
Radius of outer circle of the ring	2 mm
Radius of inner circle of the ring	1.8 mm
Width of split/Width of wire conductor	0.2 mm

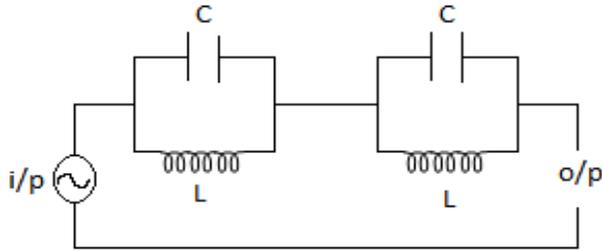


Figure 3. Simplified equivalent circuit of ISM unit cell.

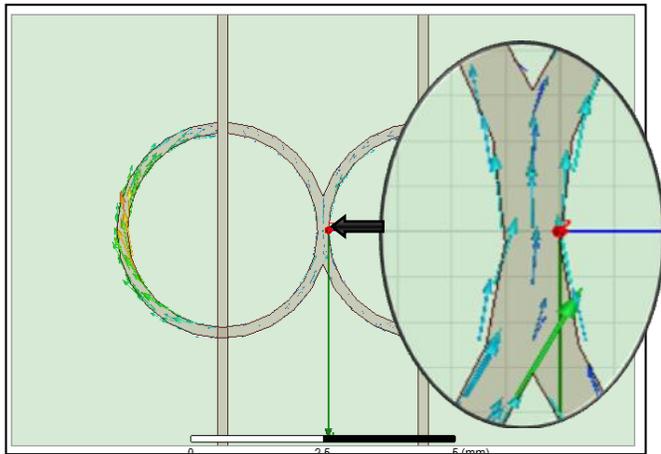


Figure 4. Distribution of current over the metallic rings, also showing the addition of current in the area common to both rings.

with the help of wave-ports (Figure 2a). The physical parameters of the structure are mentioned in Table 1.

Thus, the proposed structure ISM is a composite of split-rings and array of wires; both components are required to obtain negative effective permittivity and negative effective permeability in a single structure.

### Two port equivalent circuit model of ISM

The simplified two-port equivalent circuit representation suggested for ISM unit cell is shown in Figure 3, where  $L$  is the self-inductance of the metal loop, which can be

computed by the expressions given in (Mondher et al., 2011). The model parameter  $C$  is the capacitance computed for split ring calculated as:

$$C = C_{pp} + C_s$$

where  $C_{pp}$  and  $C_s$  are parallel plate and surface capacitances, respectively. The resonant frequency of the structure can be calculated by  $f = 1/2\pi LC$ . For simplicity of design and calculations, the effect of coupling between strip used as ground and metallic SRRs is neglected; similarly the mutual coupling effect between the two SRRs connected to form 'infinity' is also neglected. Since the two metallic traces are connected together in series w.r.t the feeding, the current flowing in two rings must combine additively in the area common to both the rings. The same is verified by plotting the distribution of current over the metallic rings using HFSS, and is shown in Figure 4.

The electrical parameters of the model are computed as:

$$L = \mu_0 r \left[ \log \left( \frac{2r}{g} \right) + 0.9 + 0.2 \left( \frac{g}{2r} \right)^2 \right] \quad (1)$$

where,  $L$  represents the inductance of SRR,  $g$  represents the width of the split, and  $r$  is the average or mean radius. To calculate the total capacitance, a simple analytical approximate expression may be used. First, the surface capacitance is determined analytically by using analytical expressions for the electric field of a split-ring, and is given by (Mondher et al., 2011):

$$C_s = \frac{2\epsilon_0(t+w)}{\pi} \log \frac{4r_i}{g} \quad (2)$$

where,  $C_s$  is the surface capacitance,  $\epsilon_0$  permittivity of free space,  $w$  represents the width of the metallic split ring,  $t$  thickness of the metal used for split ring,  $r_i$  inner radius of the split ring, and  $g$  width of the split.

Secondly, the gap capacitance or parallel plate capacitance of the split is computed as:

$$C_{pp} = \frac{\epsilon_0 \epsilon_r A}{g} \quad (3)$$

where,  $C_{pp}$  is the parallel plate capacitance (of the split),  $\epsilon_r$  relative permittivity,  $A$  is area of the plate of capacitor

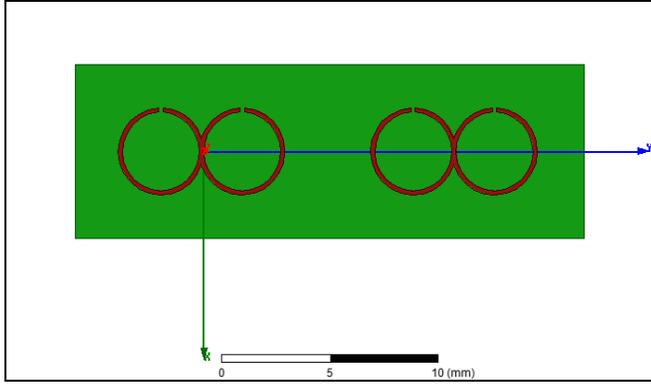


Figure 5. Array of two ISM elements.

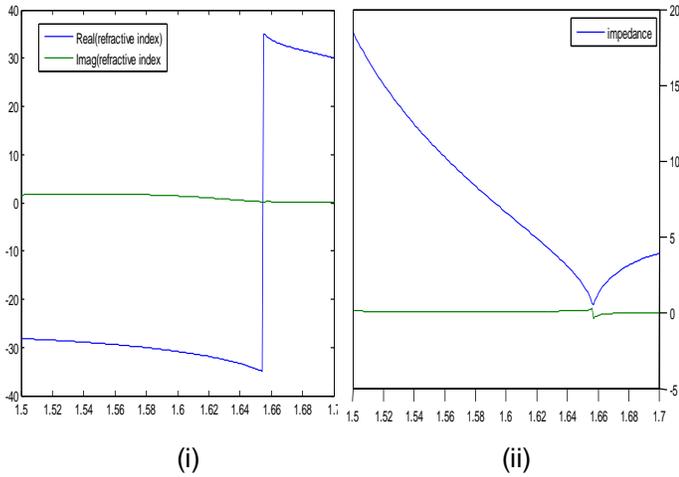


Figure 6. (i) Refractive Index (ii) Wave impedance versus Frequency.

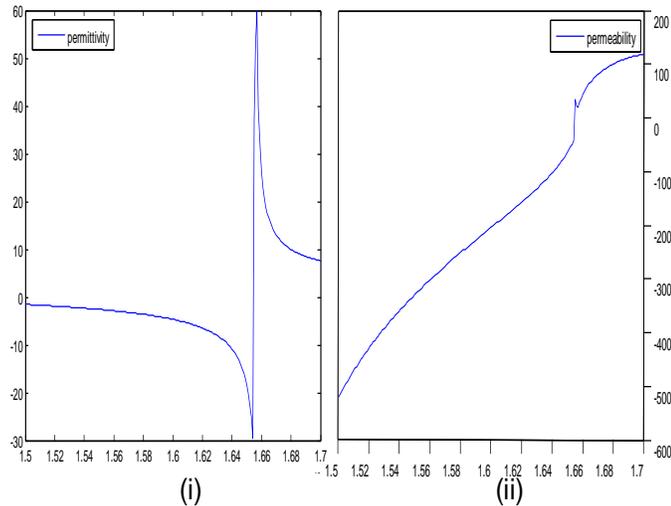


Figure 7. (i) Effective permittivity, (ii) Effective permeability versus Frequency in GHz.

(here  $A=t * w$ ). Using the above formulae, the value of inductance and capacitance was calculated, and the resonant frequency using these values comes out to be  $\sim 17$  GHz.

A linear array of two elements (shown in Figure 5) was then developed and analyzed, the coupling effects between two ISM unit cells can be described by a parallel RC circuit in the shunt branch. This coupling equivalent circuit is connected in series between two ISM blocks. The coupling parameters  $C_m$  and  $L_m$  can be computed by similar approaches used for the computation of the parameters  $C$  and  $L$  mentioned previously. The effect of mutual coupling between two ISM elements was found to be too small, and had a very little effect on the resonant frequency. Hence, it was not considered here.

### SIMULATION AND RESULTS

The ISM structure was designed and simulated using EM solver Ansys HFSS. With extracted s-parameter matrix, value of refractive index  $n$  and wave impedance  $z$  was calculated using the following equations (Sabah, 2010; Smith et al., 2001).

$$n = \frac{1}{kd} \cos^{-1} \left\{ \frac{1 - S_{11}^2 + S_{21}^2}{2S_{21}} \right\} \quad (4)$$

$$z = \pm \sqrt{\frac{(1 + S_{11})^2 - S_{21}^2}{(1 - S_{11})^2 - S_{21}^2}} \quad (5)$$

The value of effective permittivity  $\epsilon$  and effective permeability  $\mu$  then may be computed as  $\epsilon_{eff} = n/z$  and  $\mu_{eff} = n * z$ .

The condition  $\text{Im}\{n\} \geq 0$  fix the choice for sign of 'n'. Similarly the condition  $\text{Re}\{z\} \geq 0$  fixes the choice for sign of 'z'. An improved parameter retrieval method given in (Liu and Wang, 2012) is as follows:

$$n = \frac{\ln \left( \frac{S_{21}}{1 - S_{11} \frac{z-1}{z+1}} \right)}{ikd} \quad (6)$$

where,  $k$  is wave number,  $d$  is thickness of ISM unit cell.

We calculate  $z$  first, and then  $n$  can be calculated from Equation (6). All of the above formulae were programmed in MATLAB 2009a to obtain the required plots. Refractive index versus frequency curve using Equation (6) and wave impedance using Equation (5) are shown in Figures 6 and 7:

After calculating  $n$  and  $z$ , the value of effective permittivity and permeability was computed and the graphs versus frequency are shown below:

The graphs above suggests metamaterial behavior of ISM at  $\sim 16.5$  GHz. Although the results above are calculated using well known techniques, one more

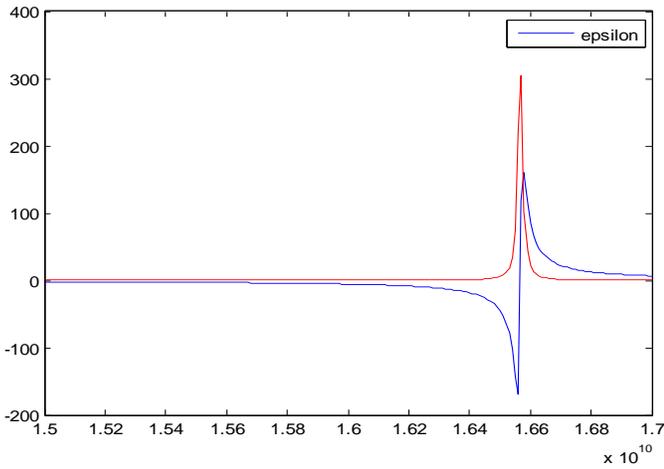


Figure 8. Effective permittivity versus Frequency in GHz.

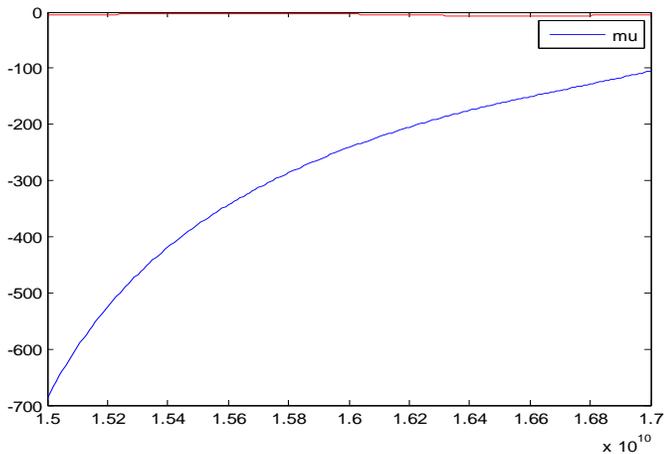


Figure 9. Effective permeability versus Frequency in GHz.

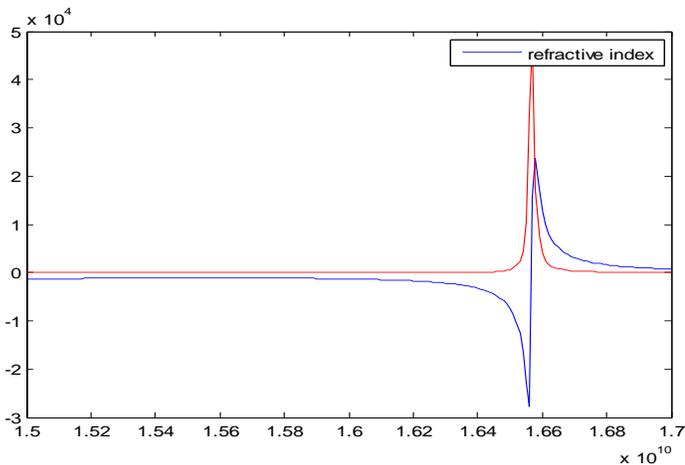


Figure 10. Refractive Index versus Frequency in GHz.

technique, the NRW parameter retrieval approach is also used in this paper to reinforce the results already obtained. A separate MATLAB code was developed based on NRW approach to find the medium properties using extracted S11 and S21 parameters. The results obtained using NRW approach shown in Figure 8 to 10 are in satisfactory agreement with those produced from Equation (4) to (6) (shown in Figures 6 and 7). The  $\epsilon_{eff}$  and  $\mu_{eff}$  of the medium are related to S-parameters by the Equations (7) and (8) below (Suganthi et al., 2012):

$$\epsilon_{eff} = \frac{2}{jk_0d} \frac{1-V_1}{1+V_1} \tag{7}$$

$$\mu_{eff} = \frac{2}{jk_0d} \frac{1-V_2}{1+V_2} \tag{8}$$

where  $k_0$  is a wave number equivalent to  $2\pi/\lambda_0$ ,  $d$  is the thickness of the substrate and  $V_1$  and  $V_2$ .

$$V_1 = S_{21} + S_{11} \tag{9}$$

$$V_2 = S_{21} - S_{11} \tag{10}$$

After calculating  $\epsilon_{eff}$  and  $\mu_{eff}$  using above equations, refractive index 'n' can be computed using:

$$n = \pm \sqrt{\epsilon_{eff} * \mu_{eff}} \tag{11}$$

Using MATLAB, graphs for effective permittivity, effective permeability, refractive index versus frequency are plotted and are shown in Figures 8 to 10:

Figure 10 shows negative value of refractive index below ~16.5 GHz. The results obtained from HFSS for ISM unit cell and for linear array of 1x2 ISM elements are shown in Figures 11 and 12.

In Figures 11 and 12 the dip in value of  $S_{11}$ (dB), shows the resonant frequency of ISM unit cell and array of 1x2 ISM elements, respectively. The resonant frequency in both cases is 16.58 and 16.5 GHz, approximately same. The graph (Figure 13(i)) shows the phase of  $S_{11}$  and  $S_{21}$ (Radians) for ISM unit cell and its array. The phase of  $S_{11}$  and  $S_{21}$  crosses each other and shows zero crossing at resonant frequency, which suggests the presence of metamaterial property. Also, the metamaterial property was preserved in case of linear array of two or more elements.

For further analysis a linear array of 10 elements was prepared to observe any deviation in resonant frequency, Figure 14 shows the structure and results for a linear array of 10 elements. From the results (Figure 13 (ii)) it may be observed that the shift in resonant frequency is too small to be considered.

The results obtained for a single ISM element, array of 1x2 ISM elements, and array of 1x10 ISM elements, all

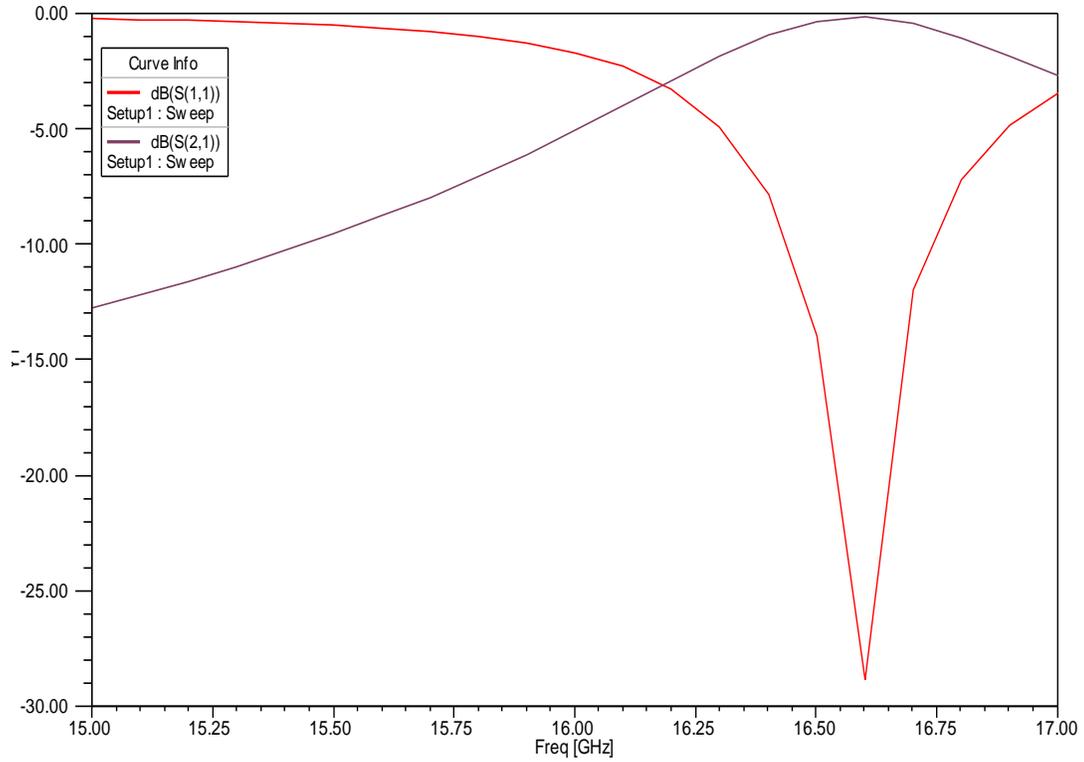


Figure 11.  $S_{11}$ (Red),  $S_{21}$ (Brown) in dB versus Frequency for ISM unit cell.

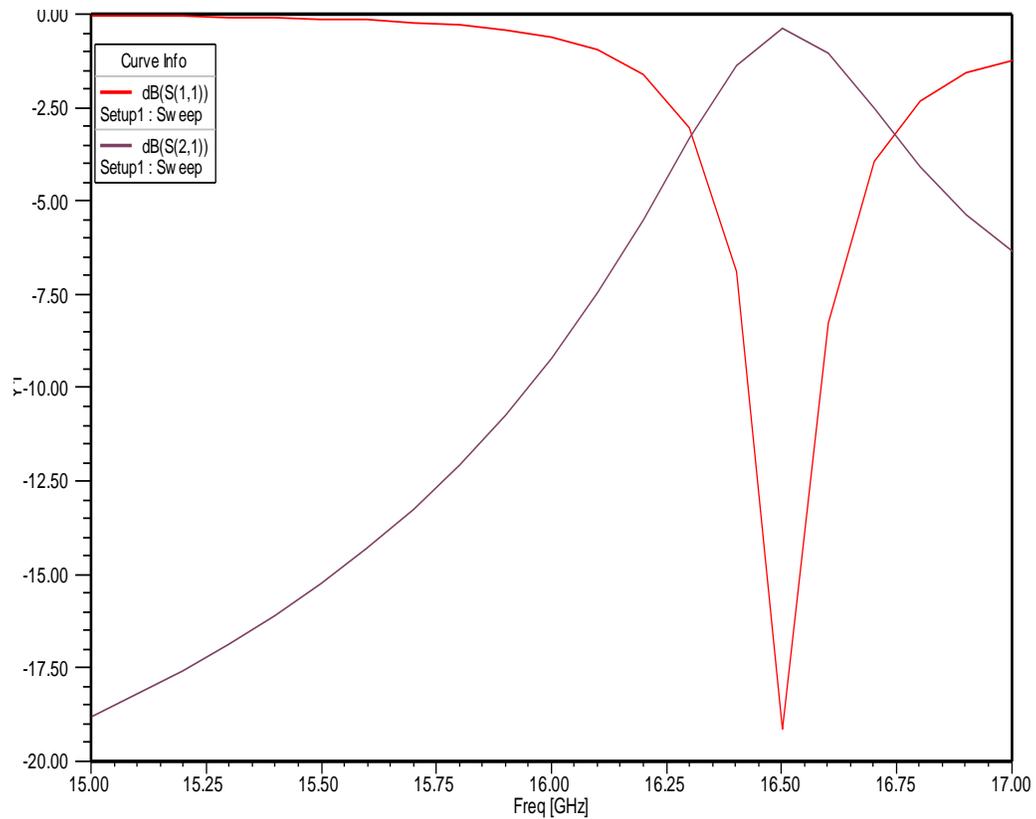
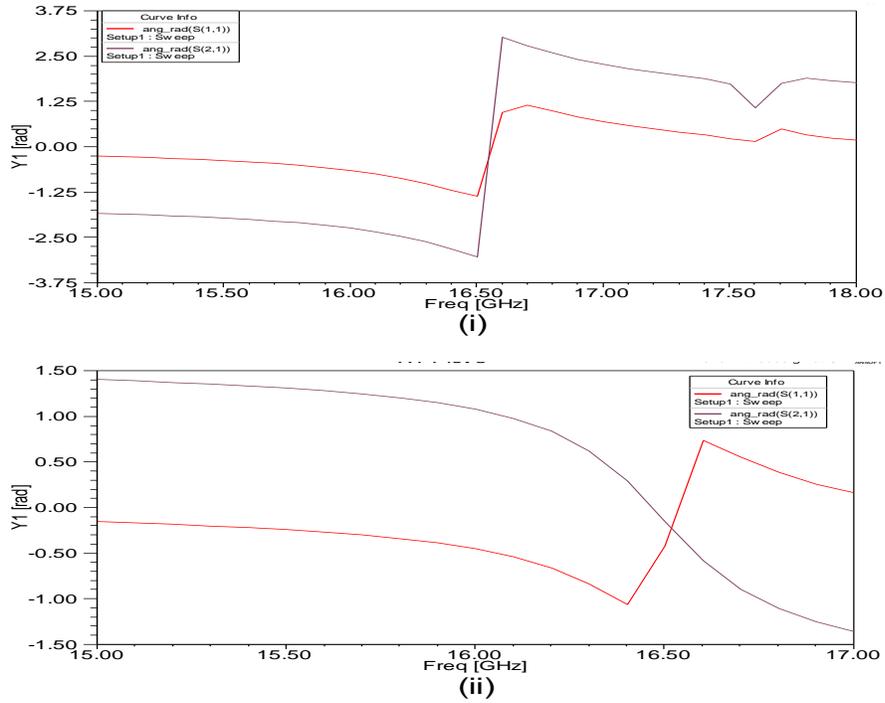
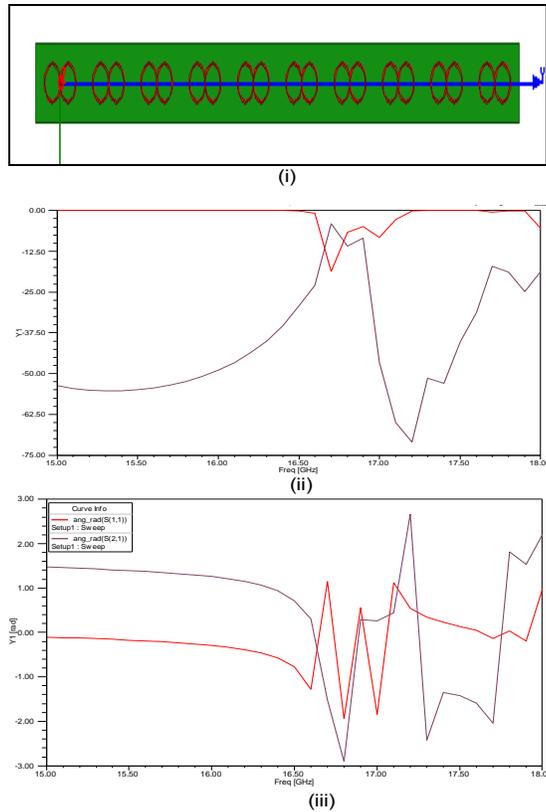


Figure 12.  $S_{11}$ (Red),  $S_{21}$ (Brown) in dB versus Frequency for 1x2 array ISM.



**Figure 13.** Phase of S11 (red), S21(brown) in radians, (i) ISM unit cell, (ii) Array of 1x2 ISM elements versus Frequency in GHz.



**Figure 14.** Linear array 10 elements (i) Structure, (ii) S11(red) and S21(brown) in dB, (iii) Phase of S11(red), S21(brown) in radians.

are in good agreement with each other and shows resonance around 16.5 GHz.

## Conclusion

A new planar microstrip metamaterial resonator using circular split ring dual, connected in the shape of 'infinity' and array of straight wire conductors is presented, it exhibits the property of negative index of refraction at Ku-band. Results obtained using HFSS are verified by coding formulae for refractive index, effective permittivity, and effective permeability in MATLAB and plotting curves versus frequency. The results obtained from all the techniques mentioned in this paper are found in satisfactory agreement. In future, the presented ISM resonator can be incorporated with microstrip antennas to get highly directional beam patterns either by using it as a substrate or by using it as a metamaterial cover kept in front of the antenna; also a physical model of ISM resonator may be fabricated.

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