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Full Length Research Paper

Unsteady flow of a Newtonian fluid induced by eccentric-concentric rotation of a porous disk and the fluid at infinity

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The unsteady flow of a Newtonian fluid produced by a sudden coincidence of the axes of a porous disk and the fluid at infinity has been investigated while they are initially rotating with the same angular velocity about non-coincident axes. The exact solutions of both the velocity field and the components of shear stress on the disk have been obtained with the use of the Laplace transform technique. During the transition from the initial time to the steady flow, the influence of both the suction and blowing has been scrutinized. It has been proved that the suction through the disk tends to thin the boundary layer near the disk while the blowing causes a thickening of the boundary layer. It has been shown that the horizontal components of the force for small times are absolutely greater in the case of suction. Furthermore, it has been pointed out that the flow reduces to a helical motion in the steady flow.

Key words: Newtonian fluid, unsteady flow, suction and blowing, eccentric rotation.

INTRODUCTION

Erdoğan (1997) was the first to study the unsteady flow generated by non-coaxial rotation suddenly while a disk and a fluid at infinity are initially rotating about a common axis. He obtained an exact solution to this type of flow for a Newtonian fluid.

Many studies corresponding to the same unsteady motion have been examined by many authors considering various variations in the problem. Hayat et al. (2001) investigated the problem in the presence of a magnetic field for a Newtonian fluid and a porous disk. Siddiqui et al. (2001) studied the MHD flow of a second order Rivlin-Ericksen fluid while the disk is porous with suction or blowing through it and found a solution by means of a perturbation approach for small values of the material modulus α_1. Hayat et al. (2003a) examined the MHD flow of a third grade fluid for a porous disk and obtained the velocity profiles with the use of a numerical method. Ersoy (2005) studied the problem for a Maxwell fluid and acquired an exact solution. Asghar et al. (2007a) took into account the flow of a Newtonian fluid with slip condition for a porous disk under the influence of a magnetic field. Asghar et al. (2007b) studied the MHD flow of a second grade fluid in the case of a porous disk moving with a uniform acceleration. They obtained both an exact solution numerically and an approximate solution analytically. Guria et al. (2007a) investigated the MHD flow including Hall effects of a Newtonian fluid under the application of both the suction and blowing. Recently, Maji et al. (2010) scrutinized the flow in a porous medium and found an exact solution in the case of a Newtonian fluid.

The time-dependent flows caused by a disk that executes non-torsional oscillations have also attracted the attention of several investigators. Erdoğan (2000) studied the unsteady flow of a Newtonian fluid emerged from an oscillation motion of the disk while the disk and the fluid at infinity are initially rotating about a common axis.

Hayat et al. (2003b) investigated the flow in the case of a porous disk under the effect of a magnetic field. Hayat et al. (2004a) re-examined the same flow for arbitrary periodic oscillations. Hayat et al. (2004b) studied the flow for a second grade fluid in the case of a porous disk and acquired a solution by employing a perturbation method for small values of the material modulus α_1. Later, this work was extended to the MHD flow by Hayat et al. (2007). Wang and Wu (2007) examined the flow of a third grade fluid subjected to a magnetic field with suction/blowing through the disk for both slip condition and no-slip condition. Guria et al. (2007b) took into account the flow of a Newtonian fluid due to an oscillating porous disk and obtained an exact solution. Hayat et al. (2008) studied the MHD flow including Hall effects of a Newtonian fluid while the porous disk is executing oscillations. For the other studies attributed to the flows produced by non-coaxial rotations of a disk and a fluid at infinity, the reader may also consult the paper by Ersoy (2010). We also refer to a recent paper by Islam et al. (2011) for unsteady exact solutions corresponding to the flow of a Newtonian fluid.

In this paper, we investigate the unsteady flow of a Newtonian fluid stemmed from coaxial rotation suddenly while a porous disk and the fluid at infinity are initially rotating non-coaxially. The initial condition is the solution obtained by Erdoğan (1976) who considered the steady flow of a Newtonian fluid when a porous disk and the fluid at infinity are rotating non-coaxially. An exact solution is acquired for both the velocity field and the horizontal components of shear stress. All the results obtained are graphically presented and discussed. Ersoy's solution (2003) can be obtained as a special case of this analysis by taking the suction/blowing velocity to be zero.

BASIC EQUATIONS

Let us consider a Newtonian fluid filling the semi-infinite space $z \geq 0$ in a Cartesian coordinate system. The axis of rotation of the disk located at $z=0$ and that of the fluid at infinity are in the plane $x=0$. The disk and the fluid at infinity are initially rotating with the same angular velocity Ω about the z- and z'-axes, respectively. The z'-axis parallel to the z-axis cuts the point ($x=0$, $y=\ell$). The disk suddenly starts to rotate with its initial angular velocity about the z'-axis. Therefore, the initial and boundary conditions are

$$u=-\Omega y+\hat{f}(z), \quad v=\Omega x+\hat{g}(z), \quad w=\text{constant at } t=0 \text{ for } z \geq 0, \tag{1}$$

$$u=-\Omega(y-\ell), \quad v=\Omega x, \quad w=\text{constant at } z=0 \text{ for } t>0, \tag{2}$$

$$u=-\Omega(y-\ell), \quad v=\Omega x, \quad w=\text{constant at } z\to\infty \text{ for } t\geq 0, \tag{3}$$

where u, v, w denote the velocity components along the x-, y-, z-directions, respectively. The functions $\hat{f}(z)$ and $\hat{g}(z)$, obtained by Erdoğan (1976), are given by

$$\frac{\hat{f}}{\Omega\ell}+i\frac{\hat{g}}{\Omega\ell}=1-e^{\sqrt{2}\left(b-\sqrt{b^2+i}\right)\zeta}, \tag{4}$$

where $i=\sqrt{-1}$, $b=\frac{w}{2\sqrt{\Omega\nu}}$, $\zeta=\sqrt{\frac{\Omega}{2\nu}}z$ and ν is the kinematic viscosity of the fluid. Thus, we seek a solution that describes such a flow

$$u=-\Omega y+f(z,t), \quad v=\Omega x+g(z,t), \quad w=\text{constant}, \tag{5}$$

which satisfy the continuity equation. Using Equations 1 to 3 and Equation 5, we have

$$f(z,0)=\hat{f}(z), \quad g(z,0)=\hat{g}(z) \quad \text{for } z\geq 0, \tag{6}$$

$$f(0,t)=\Omega\ell, \quad g(0,t)=0 \quad \text{for } t>0, \tag{7}$$

$$f(\infty,t)=\Omega\ell, \quad g(\infty,t)=0 \quad \text{for } t\geq 0. \tag{8}$$

Substituting Equation (5) into the Navier-Stokes equations, we have

$$\frac{1}{\rho}\frac{\partial p}{\partial x}=\Omega^2 x+\left(\nu\frac{\partial^2 f}{\partial z^2}-\frac{\partial f}{\partial t}-w\frac{\partial f}{\partial z}+\Omega g\right), \tag{9}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y}=\Omega^2 y+\left(\nu\frac{\partial^2 g}{\partial z^2}-\frac{\partial g}{\partial t}-w\frac{\partial g}{\partial z}-\Omega f\right), \tag{10}$$

$$\frac{1}{\rho}\frac{\partial p}{\partial z}=0, \tag{11}$$

where ρ is the density of the fluid and p is the modified pressure. Equations (9) to (11) give

$$\nu\frac{\partial^2 f}{\partial z^2}-\frac{\partial f}{\partial t}-w\frac{\partial f}{\partial z}+\Omega g=C_1(t), \tag{12}$$

$$\nu\frac{\partial^2 g}{\partial z^2}-\frac{\partial g}{\partial t}-w\frac{\partial g}{\partial z}-\Omega f=C_2(t). \tag{13}$$

Since the fluid at infinity has no shear stress, we found that $C_1(t)=0$ and $C_2(t)=-\Omega^2\ell$ with the use of Equation (8).

If we introduce the following dimensionless variables

$$F = \frac{f}{\Omega \ell} + i\frac{g}{\Omega \ell} - 1, \quad \tau = \Omega t, \tag{14}$$

we have

$$\frac{\partial^2 F}{\partial \zeta^2} - 2\frac{\partial F}{\partial \tau} - 2\sqrt{2}b\frac{\partial F}{\partial \zeta} - 2iF = 0, \tag{15}$$

and

$$F(0,\tau) = 0\,(\tau > 0); \quad F(\infty,\tau) = 0\,(\tau \geq 0);$$

$$F(\zeta,0) = -e^{\sqrt{2}\left(b - \sqrt{b^2+i}\right)\zeta} \quad (\zeta \geq 0). \tag{16}$$

SOLUTION OF THE PROBLEM

Let the Laplace transform of $F(\zeta,\tau)$ be $\overline{F}(\zeta,s)$, so that

$$\overline{F}(\zeta,s) = \int_0^\infty F(\zeta,\tau)e^{-s\tau}\,d\tau. \tag{17}$$

Taking the Laplace transforms of Equations (15) to (16), we get

$$\overline{F}'' - 2\sqrt{2}\,b\overline{F}' - 2(s+i)\overline{F} = 2e^{\sqrt{2}(b-\sqrt{b^2+i})\zeta}, \tag{18}$$

$$\overline{F}(0) = 0, \quad \overline{F}(\infty) = 0. \tag{19}$$

The transformed solution is obtained as

$$\overline{F} = \frac{1}{s}e^{\sqrt{2}\left(b-\sqrt{b^2+i+s}\right)\zeta} - \frac{1}{s}e^{\sqrt{2}\left(b-\sqrt{b^2+i}\right)\zeta}. \tag{20}$$

Taking the Laplace inversion of $\overline{F}$ (Oberhettinger and Badii, 1973), we finally obtain

$$\frac{f}{\Omega\ell} + i\frac{g}{\Omega\ell} = 1 - e^{\sqrt{2}\left(b-\sqrt{b^2+i}\right)\zeta}$$
$$+\frac{1}{2}e^{\sqrt{2}\left(b-\sqrt{b^2+i}\right)\zeta}\operatorname{erfc}\left(\frac{\zeta}{\sqrt{2\tau}} - \sqrt{(b^2+i)\tau}\right)$$
$$+\frac{1}{2}e^{\sqrt{2}\left(b+\sqrt{b^2+i}\right)\zeta}\operatorname{erfc}\left(\frac{\zeta}{\sqrt{2\tau}} + \sqrt{(b^2+i)\tau}\right). \tag{21}$$

The dimensionless horizontal components of shear stress in the fluid are

$$\widehat{T}_{xz} + i\widehat{T}_{yz} = -\sqrt{2}\,(b-\sqrt{b^2+i})e^{\sqrt{2}\left(b-\sqrt{b^2+i}\right)\zeta}$$
$$+e^{\sqrt{2}\left(b-\sqrt{b^2+i}\right)\zeta}\left[\frac{\sqrt{2}}{2}\left(b-\sqrt{b^2+i}\right)\operatorname{erfc}\left(\frac{\zeta}{\sqrt{2\tau}} - \sqrt{(b^2+i)\tau}\right)\right.$$
$$\left.-\frac{1}{\sqrt{2\pi\tau}}e^{-\left(\frac{\zeta}{\sqrt{2\tau}}-\sqrt{(b^2+i)\tau}\right)^2}\right]$$
$$+e^{\sqrt{2}\left(b+\sqrt{b^2+i}\right)\zeta}\left[\frac{\sqrt{2}}{2}\left(b+\sqrt{b^2+i}\right)\operatorname{erfc}\left(\frac{\zeta}{\sqrt{2\tau}} + \sqrt{(b^2+i)\tau}\right)\right.$$
$$\left.-\frac{1}{\sqrt{2\pi\tau}}e^{-\left(\frac{\zeta}{\sqrt{2\tau}}+\sqrt{(b^2+i)\tau}\right)^2}\right], \tag{22}$$

where

$$\widehat{T}_{xz} = \frac{T_{xz}}{\sqrt{\mu\rho\Omega^3/2\,\ell}}, \quad \widehat{T}_{yz} = \frac{T_{yz}}{\sqrt{\mu\rho\Omega^3/2\,\ell}}. \tag{23}$$

Finally, we have the dimensionless horizontal components of shear stress on the disk in the following form:

$$\left(\widehat{T}_{xz} + i\widehat{T}_{yz}\right)_{\zeta=0} = -\sqrt{2}\,(b-\sqrt{b^2+i}) - \sqrt{\frac{2}{\pi\tau}}e^{-(b^2+i)\tau}$$
$$+\frac{\sqrt{2}}{2}(b-\sqrt{b^2+i})\operatorname{erfc}\left(-\sqrt{(b^2+i)\tau}\right)$$
$$+\frac{\sqrt{2}}{2}(b+\sqrt{b^2+i})\operatorname{erfc}\left(\sqrt{(b^2+i)\tau}\right). \tag{24}$$

RESULTS AND DISCUSSION

As opposed to the aforementioned studies, this paper is concerned with the transition from eccentric rotation to concentric rotation. The initial condition is a special solution obtained by Erdoğan (1976). The results are obtained from Figures 1 to 4.

Figures 1 and 2 show how $f/\Omega\ell$ and $g/\Omega\ell$ change with the time for both the suction and blowing. Figures 1 through 3 reveal that a suction velocity through the porous disk gives rise to a boundary layer thinning, whereas a blowing velocity thickens the boundary layer.

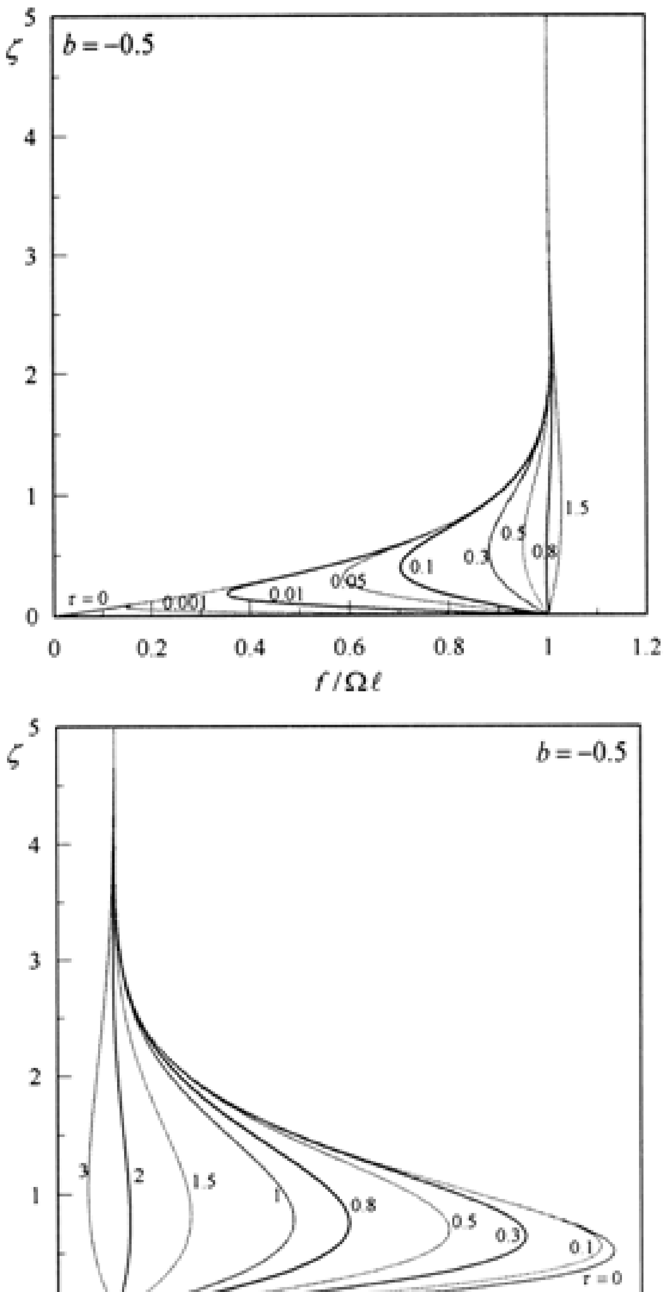

Figure 1. Variation of $f/\Omega\ell$ and $g/\Omega\ell$ with time ($b = -0.5$).

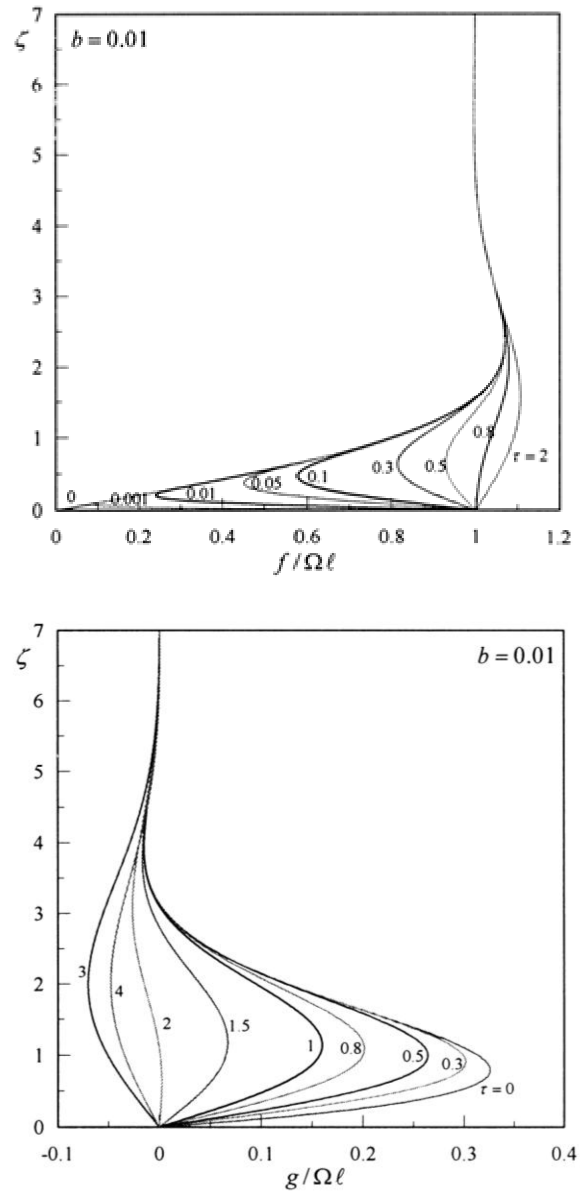

Figure 2. Variation of $f/\Omega\ell$ and $g/\Omega\ell$ with time ($b = 0.01$).

Figure 3 also indicates that the flow attains the steady-state more quickly in the suction case. Figure 4 shows the time-variations of $(\hat{T}_{xz})_{\zeta=0}$ and $(\hat{T}_{yz})_{\zeta=0}$ that are related to the x- and y-components of the force per unit area exerted by the fluid on the porous disk, respectively.

In general, the forces for suction are larger than those for blowing. At small times, the components become negative and positive in the x- and y-directions, respectively; but the component in the x-direction is absolutely larger than that in the y-direction. Conversely, the x-component goes to zero at small times while the y-component becomes zero after a longer time. The fluid does not exert the horizontal force on the porous disk in the steady-state for both the suction and blowing. Furthermore, it is a remarkable result that as the velocity varies from the blowing to suction, the horizontal components of the force increase absolutely.

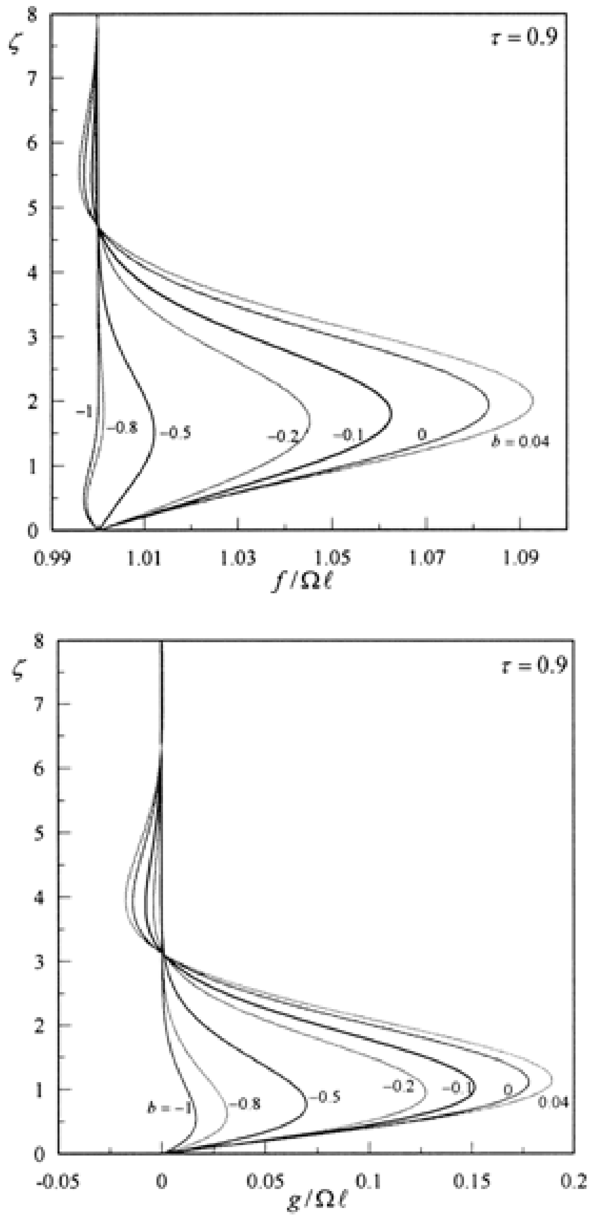

Figure 3. Variation of $f/\Omega\ell$ and $g/\Omega\ell$ for both suction and blowing ($\tau = 0.9$).

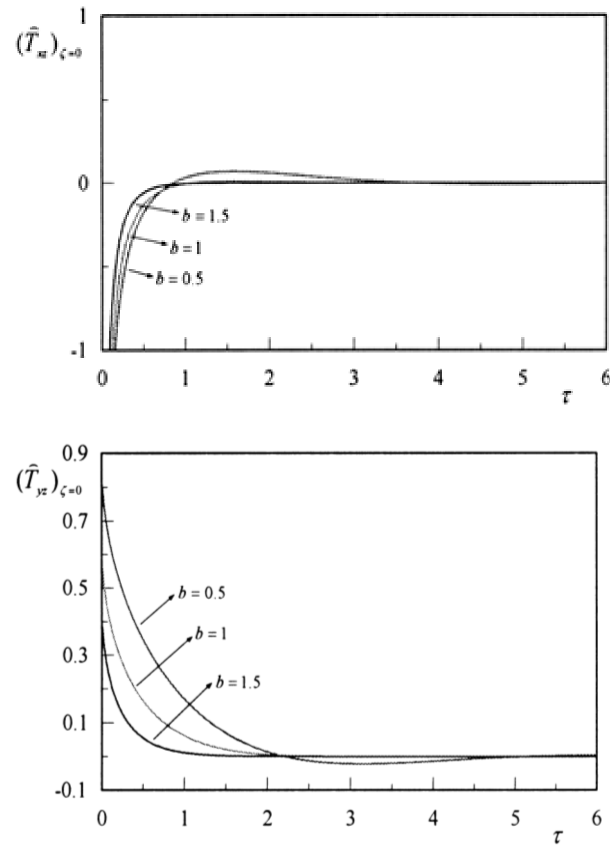

Figure 4. Variation of $(\hat{T}_{xz})_{\zeta=0}$ and $(\hat{T}_{yz})_{\zeta=0}$ with time ($b = 0.5, 1, 1.5$).

Conclusion

In this study, the unsteady flow of a Newtonian fluid produced by the rotation of a porous disk and the fluid at infinity about a common axis is examined while they are initially rotating about non-coaxial axes with the same angular velocity. The velocity field can be considered as the superposition of a helical motion $\{-\Omega y; \Omega x; w\}$ and a translational motion $\{f(z,t); g(z,t); 0\}$. It is shown that the flow tends to a helical motion in the course of time.

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REFERENCES

Asghar S, Hanif K, Hayat T (2007a). The effect of the slip condition on unsteady flow due to non-coaxial rotations of disk and a fluid at infinity. Meccanica, 42: 141-148.

Asghar S, Hanif K, Hayat T, Khalique CM (2007b). MHD non-Newtonian flow due to non-coaxial rotations of an accelerated disk and a fluid at infinity. Commun. Nonlinear Sci. Numer. Sim., 12: 465-485.

Erdoğan ME (1976). Flow due to eccentric rotating a porous disk and a fluid at infinity. J. Appl. Mech. - T. ASME, 43E: 203-204.

Erdoğan ME (1997). Unsteady flow of a viscous fluid due to non-coaxial rotations of a disk and a fluid at infinity. Int. J. Nonlinear Mech., 32: 285-290.

Erdoğan ME (2000). Flow induced by non-coaxial rotation of a disk executing non-torsional oscillations and a fluid rotating at infinity. Int. J. Eng. Sci., 38: 175-196.

Ersoy HV (2003). Unsteady viscous flow induced by eccentric-concentric rotation of a disk and the fluid at infinity. Turk. J. Eng. Env. Sci., 27: 115-123.

Ersoy HV (2005). Unsteady flow of a Maxwell fluid induced by non-coaxial rotation of a disk and the fluid at infinity. Int. J. Appl. Mech.

Eng., 10: 159-165.
Ersoy HV (2010). MHD flow of a second order/grade fluid due to non-coaxial rotation of a porous disk and the fluid at infinity. Math. Comp. Appl., 15: 354-363.
Guria M, Das S, Jana RN (2007a). Hall effects on unsteady flow of a viscous fluid due to non-coaxial rotation of a porous disk and a fluid at infinity. Int. J. Nonlinear Mech., 42: 1204-1209.
Guria M, Das BK, Jana RN (2007b). Oscillatory flow due to eccentrically rotating porous disk and a fluid at infinity. Meccanica. 42: 487-493.
Hayat T, Asghar S, Siddiqui AM, Haroon T (2001). Unsteady MHD flow due to non-coaxial rotations of a porous disk and a fluid at infinity. Acta Mech., 151: 127-134.
Hayat T, Haroon T, Asghar S, Siddiqui AM (2003a). MHD flow of a third-grade fluid due to eccentric rotations of a porous disk and a fluid at infinity. Int. J. Nonlinear Mech., 38: 501-511.
Hayat T, Zamurad M, Asghar S, Siddiqui AM (2003b). Magnetohydrodynamic flow due to non-coaxial rotations of a porous oscillating disk and a fluid at infinity. Int. J. Eng. Sci., 41: 1177-1196.
Hayat T, Ellahi R, Asghar S (2004a). Unsteady periodic flows of a magnetohydrodynamic fluid due to noncoaxial rotations of a porous disk and a fluid at infinity. Math. Comput. Model., 40: 173-179.
Hayat T, Ellahi R, Asghar S, Siddiqui AM (2004b). Flow induced by non-coaxial rotation of a porous disk executing non-torsional oscillations and a second grade fluid rotating at infinity. Appl. Math. Model., 28: 591-605.
Hayat T, Ellahi R, Asghar S (2007). Unsteady magnetohydrodynamic non-Newtonian flow due to non-coaxial rotations of disk and a fluid at infinity. Chem. Eng. Commun., 194: 37-49.
Hayat T, Ellahi R, Asghar S (2008). Hall effects on unsteady flow due to non-coaxially rotating disk and a fluid at infinity. Chem. Eng. Commun., 195: 958-976.
Islam S, Haroon T, Elahi M, Ullah M, Siddiqui AM (2011). Steady and unsteady exact inverse solutions for the flow of a viscous fluid. Int. J. Phys. Sci., 6: 6234-6240.
Maji SL, Manna G, Guria M, Rana JL (2010). Unsteady flow due to non-coaxial rotation of a porous disk and a fluid at infinity through porous medium. Chem. Eng. Commun., 197: 791-803.
Oberhettinger F, Badii L (1973). Tables of Laplace Transforms. Springer-Verlag, Berlin, p. 264.
Siddiqui AM, Haroon T, Hayat T, Asghar S (2001). Unsteady MHD flow of a non-Newtonian fluid due to eccentric rotations of a porous disk and a fluid at infinity. Acta Mech., 147: 99-109.
Wang Y, Wu W (2007). Time-dependent magnetohydrodynamic flow induced by non-coaxial rotations of a non-torsionally oscillating porous plate and a third-order fluid at infinity. Math. Comput. Model., 46: 1277-1293.