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Full Length Research Paper

Decompositions of fuzzy continuity by fuzzy strongly pre-I-continuous function

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Recently, El-Naschie has shown that the notion of fuzzy topology may be relevant to quantum physics in connection with string theory and E-infinity space time theory. In this paper, we introduce fuzzy strongly pre-I-open sets and fuzzy strongly pre-I-continuous function in fuzzy ideal topological spaces, and obtain several characterizations of fuzzy continuous functions by this function.

Key words: Fuzzy topology, decomposition, fuzzy continuity, fuzzy strongly pre-I-open sets.

INTRODUCTION

The notion of continuity is an important concept in general topology (Shakeel, 2010) and fuzzy topology (Liu and Luo, 1997; Palaniappan, 2005) as well as in all branches of mathematics and quantum physics (El-Naschie, 2004a, b; 2005; 2006). Also, its weak and strong forms are important, too. Stakhov and Rozin (2005; 2007) introduced the Fibonacci and Lucas functions, which are a class of elementary continuous functions for Fibonacci and Lucas numbers. Moreover, Stakhov and Rozin (2006) studied the continuous functions for the Fibonacci and Lucas p-numbers. These functions play an important role in mathematics and theoretical physics (Stakhov and Rozin, 2005).

After Zadeh (1965) introduced the concept of fuzzy sets, it is natural to extend the concept of general topology to fuzzy sets, resulting in a theory of fuzzy topology. The topic of fuzzy ideal topological spaces was studied by several authors. Mahmoud (1997) and Sarkar (1997) independently presented some of the ideal concepts in the fuzzy trend and studied many of their properties. Abd El-Monsef et al. (2001) defined quasi-α-cover of fuzzy set and fuzzy weak α-compatibility of fuzzy topologies via fuzzy ideals. Nasef et al. (2002) presented one operator utilizing the fuzzy ideal *L*, denoted and given as *F*. ψ(L): $I^X \to \tau$, where *I* denotes the closed unit interval [0, 1]. Hatir and Jafari (2007), Nasef and Hatir (2009) and Yuksel et al. (2009) studied fuzzy ideal topological spaces by defining fuzzy semi-I-open sets, fuzzy pre- I-open sets and fuzzy α-I-open sets.

In this paper, we introduce fuzzy strongly pre-I-open sets and fuzzy strongly pre-I-continuous function in fuzzy ideal topological spaces, and obtain several characterizations of fuzzy continuous function by this function. Since professor El-Naschie (2000) has shown that the notion of fuzzy topology may be relevant to quantum physics particularly in connection with string theory and e^{∞} theory, professor El-Naschie (2006) used the fuzzy topological properties to give information about the elementary particles content of the standard model of high energy physics and Balachndran et al. (2007) pointed out that “Fuzzy topology and fuzzy manifold have already invaded quantum field theory”, the notions and results given in this paper will turn out to be useful in quantum physics.

PRELIMINARIES

By a space, we always mean a fuzzy topological space (X,τ) with no separation properties assumed. If $A \subset X$, $cl(A)$ and $\text{int}(A)$ will, respectively, denote the fuzzy closure and fuzzy interior of *A* in (X,τ). An ideal I on a fuzzy topological space (X,τ) is a nonempty collection of subsets of X which satisfies (i) $A \in I$ and $B \subset A$ implies $B \in I$ and (ii) $A \in I$ and $B \in I$ implies $A \cup B \in I$. Given a fuzzy topological space (X,τ) with an ideal I on X and if $\wp(X)$ is the

set of all subsets of X, a set operator $(.)^*:\wp(X)\to\wp(X)$, called a local function of *A* with respect to τ and I, is denoted as follows: for $A\subset X$, $A^*(I,\tau)=\{x\in X|U\cap A\notin I$ for every $U\in\tau(x)\}$ where $\tau(x)=\{U\in\tau|x\in U\}$.
We will make use of the basic facts concerning local functions without mentioning it explicitly. A Kuratowski closure operator $cl^*()$, for a fuzzy topology $\tau^*(I,\tau)$, is denoted by $cl^*(A)=A\bigcup A^*(I,\tau)$. When there is no chance for confusion, we will simply write A^* for $A^*(I,\tau)$ and τ^* or $\tau^*(I)$ for $\tau^*(I,\tau)$. If I is an ideal on X, then (X,τ,I) is called a fuzzy ideal topological space. Given a fuzzy ideal space (X,τ,I), *I* is said to be codense if $\tau\bigcap I=\{\phi\}$. A subset *A* of (X,τ,I) is said to be fuzzy $\alpha-I-$open if $A\subset\text{int}(\ cl^*(\ \text{int}(A)))$. We will denote the fuzzy $\alpha-I-$interior of a subset *A* of X by $\alpha I\ \text{int}(A)$. A subset *A* of (X,τ,I) is said to be fuzzy pre-I-open if $A\subset\text{int}(\ cl^*(A))$. The family of all fuzzy pre-I-open sets in (X,τ,I) is denoted by $PIO(X,\tau)$ or simply $PIO(X)$. Clearly, $\tau\subset PIO(X)$. The largest fuzzy pre-I-open set contained in *A*, denoted by pIint (*A*), is called the fuzzy pre-I-interior of *A*. A subset *A* of (X,τ,I) is said to be fuzzy semi-I-open if $A\subset cl^*(\ \text{int}(A))$. A subset *A* of (X,τ,I) is said to be a fuzzy C^I-set if $A=U\bigcap V$ where *U* is open and $\text{int}(\ cl^*(\ \text{int}(V)))=\text{int}(V)$. Clearly, every fuzzy open set is a fuzzy C^I-set. A subset *A* of (X,τ,I) is said to be a fuzzy *A*^{I}-set if $A=U\bigcap V$ where *U* is fuzzy open and $V=(\ \text{int}(V))^*$. A subset *A* of (X,τ,I) is said to be fuzzy I-locally closed if $A=U\cap V$ where *U* is fuzzy open and $V=V^*$. Clearly, *A* is fuzzy I-locally closed if and only if $A=G\cap A^*$ for some fuzzy open set *G*. A subset *A* of (X,τ,I) is said to be fuzzy $\delta-I-$open set if $\text{int}(\ cl^*(A))\subset cl^*(\ \text{int}(A))$. Clearly, every fuzzy open set is $\delta-I-$open set. For more details about fuzzy ideal topological space, one can refer to Mahmoud(1997) and Sarkar (1997). The following lemmas will be useful in the sequel.

Lemma 1

If *A* is any subset of a fuzzy ideal topological space, then the following holds:

(a) $pI\ \text{int}(A)=A\cap\text{int}(\ cl^*(A))$.
(b) $\alpha I\ \text{int}(A)=A\cap\text{int}(\ cl^*(\ \text{int}(A)))$.

Proof

(a)
$\because A\cap\text{int}(cl^*(A))\subset\text{int}(cl^*(A))=\text{int}(\text{int}(cl^*(A)))$
$=\text{int}(cl^*(A)\cap\text{int}(cl^*(A)))\subset\text{int}(cl^*(A\cap\text{int}(cl^*(A))))$,
$A\cap\ \text{int}(cl\ ^*(A))$ is a fuzzy pre-I-open set contained in *A* and so $A\cap\ \text{int}(cl\ ^*(A))\subset pI\ \text{int}(\ A)$ since $pI\ \text{int}(\ A)$ is fuzzy pre-I-open, $pI\ \text{int}(\ A)\subset\text{int}(\ cl\ ^*(\ pI\ \text{int}(\ A)))\subset\text{int}(\ cl\ ^*(A))$. And so $pI\ \text{int}(\ A)\subset A\cap\text{int}(\ cl\ ^*(A))$. Hence, $pI\ \text{int}(\ A)=A\cap\text{int}(\ cl\ ^*(A))$ which proves (a).

(b)
$\because A\cap\text{int}(cl^*(\text{int}(A)))\subset\text{int}(cl^*(\text{int}(A)))=$
$\text{int}(\text{int}((cl^*(\text{int}(A))))=$
$\text{int}(cl^*(\text{int}(A))\cap\text{int}(cl^*(\text{int}(A))))$
$\subset\text{int}(cl^*(\text{int}(A)\cap\text{int}(cl^*(\text{int}(A)))))$
$=\text{int}(cl^*(\text{int}(A\cap\text{int}(cl^*(\text{int}(A))))))$,

$A\cap\ \text{int}(\ cl\ ^*(\text{int}(\ A)))$ is a fuzzy $\alpha-I-$open set contained in *A* and so $A\cap\ \text{int}(\ cl\ ^*(\text{int}(\ A)))\subset\alpha I\ \text{int}(\ A)$. Since $\alpha I\ \text{int}(\ A)$ is $\alpha-I-$open, $\alpha I\ \text{int}(A)\subset\text{int}(cl^*(\text{int}(\alpha I\ \text{int}(A))))\subset\text{int}(cl^*(\text{int}(A)))$ and so $\alpha I\ \text{int}(A)\subset A\cap\text{int}(cl^*(\text{int}(A)))$. Hence $\alpha I\ \text{int}(\ A)=A\cap\text{int}(\ cl\ ^*(\text{int}(\ A)))$ which proves (b).

Lemma 2

If *A* is a C^I-set of (X,τ,I), then $\alpha I\ \text{int}(A)=\text{int}(A)$.

Proof

By the definition of fuzzy $\alpha-I-$interior of the subset *A*, we have $\alpha I\,\text{int}(A) \supset \text{int}(A)$. Since A is a C^{I}-set, $A = U \cap V$ where U is open and $\text{int}(cl^{*}(\text{int}(V))) = \text{int}(V)$.

Now, $A \subset V \Rightarrow \text{int}(cl^{*}(\text{int}(A))) \subset \text{int}(cl^{*}(\text{int}(V))) = \text{int}(V)$.

So, by Lemma 1(b), $\alpha I\,\text{int}(A) = A \cap \text{int}(cl^{*}(\text{int}(A))) \subset A \cap \text{int}(V)$ $= U \cap \text{int}(V) = \text{int}(U \cap V) = \text{int}(A)$.

Thus, we have $\alpha I\,\text{int}(A) \subset \text{int}(A)$, and so $\alpha I\,\text{int}(A) = \text{int}(A)$.

Lemma 3

If A is a $\delta - I -$open set of (X, τ, I), then $\alpha I\,\text{int}(A) = pI\,\text{int}(A)$.

Proof

Since every $\alpha - I -$open set is a pre-I-open set, $\alpha I\,\text{int}(A) \subset pI\,\text{int}(A)$. By Lemma 1(b), $\alpha I\,\text{int}(A) = A \cap \text{int}(cl^{*}(\text{int}(A)))$. Because A is $\delta - I -$open, $\alpha I\,\text{int}(A) \supset$ $A \cap \text{int}(\text{int}(cl^{*}(A))) = A \cap \text{int}(cl^{*}(A)) = pI\,\text{int}(A)$ and so $\alpha I\,\text{int}(A) \supset pI\,\text{int}(A)$. Thus, we have $\alpha I\,\text{int}(A) = pI\,\text{int}(A)$.

Lemma 4

If (X, τ, I) is a fuzzy ideal space, then the following holds:

(a) Every fuzzy A^{I}-set is a fuzzy semi-I-open set.

(b) Every fuzzy semi-I-open set is a fuzzy $\delta - I -$open set.

Proof

(a) If A is a fuzzy A^{I}-set, then $A = G \cap V$ where G is fuzzy open and $V = (\text{int}(V))^{*}$. So we have $A = G \cap V =$ $G \cap (\text{int}(V))^{*} \subset (G \cap \text{int}(V))^{*} = (\text{int}(G \cap V))^{*}$ $= (\text{int}(A))^{*} \subset cl^{*}(\text{int}(A))$ and A is fuzzy semi-I-open.

(b) If A is fuzzy semi-I-open, then $A \subset cl^{*}(\text{int}(A))^{*}$. Now $\text{int}(cl^{*}(A)) \quad \subset \text{int}(cl^{*}(cl^{*}(\text{int}(A)))) = \text{int}(cl^{*}(\text{int}(A)))$ $\subset cl^{*}(\text{int}(A))$ which implies that A is a fuzzy $\delta - I -$open set.

Lemma 5

Let (X, τ, I) be a fuzzy ideal space and A be a subset of X such that $A \subset A^{*}$. Then $A^{*} = cl^{*}(A) = cl(A)$ (Renuka et al., 2005).

Lemma 6

Let (X, τ, I) be a fuzzy ideal space. Then I is codense if and only if $G \subset G^{*}$ for every open set G (Jankovic and Hamlett, 1990). For definitions and results not explained in this paper, one can refer to Abd El-Monsef et al. (2001), Nasef et al. (2002) and Yuksel et al. (2009) assuming them to be well known.

DECOMPOSITIONS OF FUZZY CONTINUITY

Definition 1

A subset A of a fuzzy ideal topological space (X, τ, I) is said to be fuzzy strongly pre-I-open, if A is fuzzy pre-I-open as well as a fuzzy C^{I}-set. The family of all fuzzy strongly pre-I-open sets in (X, τ, I) is denoted by SPIO(X, τ, I) or simply SPIO(X). Clearly, $\tau \subset SPIO\ (X) \subset PIO\ (X)$.

The following theorem 1 gives a characterization of fuzzy open sets in terms of fuzzy strongly pre-I-open sets and fuzzy $\delta - I -$open sets. Theorem 2 gives characterizations of fuzzy open sets in terms of different kinds of subsets in fuzzy ideal spaces, if the ideal is codense.

Theorem 1

If (X, τ, I) is any fuzzy ideal space and $A \subset X$, then the following are equivalent.

(a) A is fuzzy open.

(b) A is both fuzzy strongly pre-I-open and fuzzy $\delta - I -$open.

Proof

(a) $\Rightarrow$ (b) is clear.

(b) $\Rightarrow$ (a).

Suppose *A* is fuzzy strongly pre-I-open and also a fuzzy $\delta - I -$open set. By Lemma 3, since *A* is fuzzy pre-I-open, $\alpha I \operatorname{int}(A) = pI \operatorname{int}(A) = A$. By Lemma 2, $\alpha I \operatorname{int}(A) = \operatorname{int}(A)$. Therefore, $A = \operatorname{int}(A)$ which implies that *A* is fuzzy open.

Theorem 2

If (X, τ, I) is any fuzzy ideal topological space where I is codense and $A \subset X$, then the following are equivalent.

(a) *A* is fuzzy open.

(b) *A* is a fuzzy $\alpha - I -$open and fuzzy I-locally closed set.

(c) *A* is a fuzzy pre-I-open and fuzzy I-locally closed set.

(d) *A* is a fuzzy pre-I-open set and a fuzzy A^I-set.

(e) *A* is a fuzzy strongly pre-I-open and a fuzzy A^I-set.

(f) *A* is a fuzzy strongly pre-I-open and fuzzy semi-I-open set.

(g) *A* is a fuzzy strongly pre-I-open and fuzzy $\delta - I -$open set.

Proof

(a) $\Rightarrow$ (b). If *A* is fuzzy open, then *A* is fuzzy $\alpha - I -$open. Since I is codense, by Lemma 6, $A \subset A^*$, and so $A = A \cap A^*$. Therefore, A is fuzzy I-locally closed.

(b) $\Rightarrow$ (c). Follows from the fact that every fuzzy $\alpha - I -$open set is fuzzy pre-I-open.

(c) $\Rightarrow$ (d). If *A* is fuzzy I-locally closed, then $A = G \cap A$ for some fuzzy open set *G.* Since $A \subset A^*$, by Lemma 5, $A^* = cl^*(A)$. Since *A* is fuzzy pre-I-open, $A \subset \operatorname{int}(cl^*(A)) = \operatorname{int}(A^*)$ and so $A^* \subset (\operatorname{int}(A^*))^* \subset (A^*)^* \subset A^*$. Therefore, $A^* = (\operatorname{int}(A^*))^*$ which implies that *A* is a fuzzy A^I-set.

(d) $\Rightarrow$ (e). If *A* is a fuzzy A^I-set, then $A = G \cap V$ where *G* is fuzzy open and $V = (\operatorname{int}(V))^*$. Now $\operatorname{int}(cl^*(\operatorname{int}(V))) = \operatorname{int}(\operatorname{int}(V)) \cup (\operatorname{int}(V))^*)$ $= \operatorname{int}(\operatorname{int}(V) \cup V) = \operatorname{int}(V)$. It follows that *A* is fuzzy strongly pre-I-open.

(e) $\Rightarrow$ (f). This follows from Lemma 4(a).
(f) $\Rightarrow$ (g). This follows from Lemma 4(b).
(g) $\Rightarrow$ (a). This follows from Theorem 1.

Definition 2

A mapping $f:(X,\tau) \to (Y,\sigma)$ is said to be fuzzy strongly pre-I-continuous (resp. fuzzy $\delta - I -$continuous, fuzzy $\alpha - I -$continuous, fuzzy pre-I-continuous, fuzzy A^I-continuous, fuzzy semi-I-continuous, fuzzy ILC-continuous) if for every $V \in \sigma, f^{-1}(V)$ is fuzzy strongly pre-I-open (resp. fuzzy $\delta - I -$open, fuzzy $\alpha - I -$open, fuzzy pre-I-open, fuzzy A^I-set, fuzzy semi-I-open, fuzzy I-locally closed).

The following Theorem 3 gives a decomposition of fuzzy continuity in any fuzzy ideal topological space and Theorem 4 gives a decomposition of fuzzy continuity in any fuzzy ideal topological space where the ideal is codense.

Theorem 3

If $f:(X,\tau) \to (Y,\sigma)$ is any mapping, then the following are equivalent.

(a) f is fuzzy continuous.

(b) f is fuzzy strongly pre-I-continuous and fuzzy $\delta - I -$continuous.

Proof

This proof follows from Theorem 1.

Theorem 4

If $f:(X,\tau) \to (Y,\sigma)$ is any mapping and I is codense, then the following are equivalent.

(a) f is fuzzy continuous.

(b) f is fuzzy $\alpha - I -$continuous and fuzzy ILC-continuous.

(c) f is fuzzy pre-I-continuous and fuzzy ILC-continuous.

(d) f is fuzzy pre-I-continuous and fuzzy A^I-continuous .

(e) f is fuzzy strongly pre-I-continuous and fuzzy A^I-continuous.

(f) f is fuzzy strongly pre-I-continuous and fuzzy semi-I-continuous.

(g) f is fuzzy strongly pre-I-continuous and fuzzy $\delta - I -$ continuous.

Proof

This proof follows from Theorem 2.

CONCLUSION

The notions of the sets and functions in fuzzy topological spaces are highly developed and are used extensively in many practical and engineering problems, computational topology for geometric design, computer-aided geometric design, engineering design research and mathematical sciences. We defined fuzzy strongly pre-I-open sets and fuzzy strongly pre-I-continuous function in fuzzy ideal topological spaces, and obtain several characterizations of fuzzy continuous function by this function.

Since Bayram (2010) pointed out that “Currently, works on the construction of a fuzzy topology at fuzzy semi-prime ideals is being carried out.” And the consideration of this decomposition theory on fuzzy semiprime ideals will lead to some interesting research from the view point of fuzzy mathematics.

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