academicJournals

Vol. 9(7), pp. 154-173, 16 April, 2014
DOI: 10.5897/IJPS2014.4121
Article Number:075DCA344159
ISSN 1992 - 1950

http://www.academicjournals.org/IJPS

International Journal of Physical Sciences

Full Length Research Paper

Theoretical spin assignment and study of the A~100 – 140 superdeformed mass region by using ab formula

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Received 31 January, 2013; Accepted 2 April, 2014

Using an empirical formula of rotational spectra which consists of two parameters a and b (known as ab formula) and derived from the Bohr-Hamiltonian, we extract the spins of fifteen superdeformed bands observed in the A~ 100-140 mass region. The two parameters "a" and "b" were determined by using a search program to fit the proposed transition energies with their observed values. The calculated transition energies depend sensitively on the prescribed spins. The agreement between the calculated and observed transition energies is incredibly well when a correct spin assignment is made. The quality of the fit is generally good in the proposed nuclei, namely: ^{104}Pd(b1), ^{132}Ce(b1), ^{134}Nd(b1), ^{136}Nd(b2), ^{142}Sm(b1), ^{148}Eu(b1, b2), ^{131}Ce(b1, b2), ^{133}Ce(b1, b2), ^{133}Pr(b1, b3), ^{137}Sm(b1), ^{143}Eu(b1). Also, a good agreement is achieved between the extracted spin values in this paper with other theoretical models and also with the available experimental data.

Key words: Superdeformed nuclei, Even-A and Odd-A Nuclei of the superdeformed Mass Region 100-140, ab formula.

INTRODUCTION

The topic of Superdeformation has been at the forefront of nuclear structure physics since the observation of the first superdeformed band in ^{152}Dy (Twin et al., 1986; Bently et al., 1987). Experimental data on SD rotational bands are now available in different parts of the periodic table, namely; in the A~ 190, ~ 150, ~ 130, and ~ 80 mass regions (Hu and Zeng, 1997; Sevensson, 1999; Singh et al., 1996; Afanasjev et al., 1996).

Specially, the superdeformed mass region A~ 100-140 is of particular interest (Szymanski , 1996; Laird, 2002; Leoni et al., 2001; Khazov et al., 2005; Khazov et al., 2006) because of limited number of particles in these nuclei. Moreover, the poor study of these SD nuclei theoretically. The vast majority of the SD bands in this mass region show similar behavior of their dynamic moment of inertia, $\theta^{(2)}$, with the rotational frequency, ħω, in that, they exhibit a smooth decrease as ħω increases.

For the SD bands, gamma ray energies are unfortunately the only spectroscopic information universally available. Because of the non-observation of the discrete linking transitions between the SD states and the low lying states at normal deformation (ND), the

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experimental data for the spin of the rotational bands is poor and the only way to obtain the value of the spin is doing theoretically. Several approaches for assigning spins to SD states have been proposed (Draper et al., 1990; Stephens, 1990; Becker et al., 1990; Wu et al., 1992; Hegazi et al., 1999; Khalaf et al., 2002; Shalaby, 2004). These approaches involve direct and indirect methods for assigning spin to the states in SD bands.

In the direct method like Bohr-Mottelson I (I+1) expansion (Bohr and Mottelson, 1975), ab formula (Wu et al., 1989; Holmberg and Lipas, 1968) and the abc formula (Wu et al., 1989), the energies of the states of rotational bands are expressed as a function of spin based on two or more parameters formula. Assuming various values for the spin I_0 of the lowest state in the SD bands, the two or more parameters can be adjusted to obtain a minimum root mean square deviation of the calculated and measured energies. On the other hand, the indirect methods rely mainly on the fitting of the experimental dynamical moment of inertia values with Harris formula (Stephens, 1990; Becker et al., 1990a; Hegazi et al., 1999; Shalaby, 2004; Becker et al., 1990b; Harris, 1964). The parameters obtained from the fit are then used to calculate the spin. In such a parameterization, the spin may be expressed as an expansion in the rotational frequency ħω. Such available approaches are usually referred to the best-fit method (BFM). There are two main problems within this method. On one hand, neither dynamic moment of inertia ($\theta^{(2)}$) nor rotational frequency (ħω) is quantities directly measured in experiment. One can estimate their values. On the other hand, this procedure contains an integration constant, which is in some way an additional parameter.

The method used in the present paper is a direct method based on a formula connecting directly spin and energy of a same level. To our opinion, this method has two advantages than the Harris three-parameter approach (Harris, 1964) done in our previous work (Shalaby, 2004) and others (Hegazi et al., 1999; Khalaf, 2002), where we study in our previous work (Shalaby, 2004) the SD mass region 60-90. First, it is rather general and is obtained within different approaches. Second, it is not necessary to introduce quantities, which are not directly measured in experiment. Spin assignment can be done by using only the transition energies. Unlike the rotational frequency employed in our previous work (Shalaby, 2004), the angular momentum is a directly measured quantity, hence the parameters exist in the present formula connecting the energy and spin, can be determined very easily from the observed rotational spectrum. A theory used to determine the spins of the considered superdeformed rotational bands in the A~ 100-140 mass region with the present ab approach was given. All the data on the fifteen SD bands observed in the A~ 100-140 region are analyzed by making use of this approach. The spins of all these SD bands are determined and the results seem reasonable. With the spin values thus assigned, the energy spectra of these SD bands were calculated and the results turned out unexpectedly well.

THEORY

The simple and effective method used to determine the spin values of the SD rotational bands is the ab-method (also called ab fitting) (Wu et al., 1992). In this method, the two parameter empirical expression for the rotational energy is given as:-

$$E(I) = a\left[\sqrt{1 + bI(I+1)} - 1\right] \quad (1)$$

Which may be derived from the Bohr Hamiltonian for a well deformed nucleus with small axial symmetry. According to Equation (1), the transition energy from levels I to I-2 is:-

$$\begin{aligned} E_\gamma(I) &= E_\gamma(I \to I-2) \\ &= a\left[\sqrt{1 + bI(I+1)} - \sqrt{1 + b(I-2)(I-1)}\right] \end{aligned} \quad (2)$$

For an SD cascade:

$$I_0 + 2n \to I_0 + 2n - 2 \to ...... \to I_0 + 4 \to I_0 + 2 \to I_0, \quad (3)$$

The observed transition energies:

$$E_\gamma(I_0 + 2n), E_\gamma(I_0 + 2n - 2), ... \; E_\gamma(I_0 + 4) \text{ and } E_\gamma(I_0 + 2)$$

Can be least-squares fit by Equation (2) (ab fit) with fitting parameters a and b. It is found that, in general, the agreement between the calculated and observed transition energies depends sensitively on the prescribed level spins. When a correct "I_0" value is assigned, the calculated transition energies were found to coincide with the observed ones relatively well. However, if I_0 is shifted away from the correct ones even merely by ±1, the root mean square (rms) deviation:

$$\sigma = \left[\frac{1}{n}\sum_{i=1}^{n}\left|\frac{E_\gamma^{cal}(I_i) - E_\gamma^{\exp}(I_i)}{E_\gamma^{\exp}(I_i)}\right|^2\right]^{1/2} \quad (4)$$

n which is the number of transitions involved in the fitting will increase drastically. Therefore, the spin value of the lowest (proposed) spin and hence all the spin values of the SD band levels can be determined. This quantity "σ" used in Wu et al. (1992) was taken to fit because the experimental error bars in determination of the level energies are used not reported. As the spin value of the lowest spin I_o is known, all the spin values of the SD band levels can be determined, it is clear that this fitting procedure is quite easy and straight forward and is much simpler than that using the Harris ω^2 - expansion (Hegazi et al., 1999; Khalaf, 2002; Shalaby, 2004).

We now remind the reader that a three-parameter Harris expansion of the cranking model (Harris, 1964) gives rise to the expression of energy:

$$E(\omega) = \alpha\omega^2 + \beta\omega^4 + \gamma\omega^6 \quad (5)$$

Which leads to the dynamic moment of inertia (Shalaby, 2004)

$$\theta^{(2)}(\omega) = 2\alpha + 4\beta\omega^2 + 6\gamma\omega^4 \quad (\hbar^2\ \text{MeV}^{-1}) \quad (6)$$

Table 1. involves the three fitting parameters and the bandhead spin resulting from our present calculations, other works and the available experimental data for the assumed even-A and odd-A SD bands.

SD band	$E_\gamma(I_0+2\rightarrow I_0)$ (KeV)	Parameter			Band head spin or proposed spin (I_0)		
		ax10^5 (KeV)	b x 10^{-4}	$J_0 = \hbar^2/ab$ ($\hbar^2$/MeV^{-1})	Present work	Previous works [Ref.]: [1](Singh et al., 1996), [2](Singh et al., 2002), [3](Shalaby et al., 2012)	Exp.data[Ref.] (Singh et al., 1996)
^{104}Pd(b1)	1263.0	1.93	2.01	25.78	15	22[2], 21[3]	-
^{132}Ce(b1)	808.0	1.61	1.42	43.74	15	17[2], 18[1], 28[3]	-
^{134}Nd(b1)	663.9±0.5	0.72	3.21	43.57	13	15[2,3]	14
^{136}Nd(b2)	888.0±1.0	5.57	0.325	55.24	23	30[3]	23
^{142}Sm(b1)	680.0	1.14	1.75	50.13	15	23[2], 22[3]	-
^{148}Eu(b1)	748.0	1.24	1.40	57.60	20	30[2], 31[3]	-
^{148}Eu(b2)	844.0	1.27	1.38	57.06	23	26[2], 35[3]	-
^{131}Ce(b1)	591.0	1.61	1.37	45.34	11.5	12.5[2], 16.5[3]	-
^{131}Ce(b2)	847.0	1.43	1.54	45.41	17.5	21.5[2], 26.5[3]	-
^{133}Ce(b1)	748.0	1.44	1.50	46.30	15.5	19.5[2], 21.5[1], 24.5[3]	-
^{133}Ce(b2)	720.3	1.49	1.46	45.97	14.5	16.5[2], 18.5[1], 21.5[3]	-
^{133}Pr(b1)	871.0	1.81	1.16	47.62	19.5	25.5[1,2], 22.5[3]	-
^{133}Pr(b3)	821.0	1.51	1.51	43.86	16.5	25.5[2], 24.5[3]	-
^{137}Sm(b1)	379.0	1.99	0.98	51.43	9.5	8.5[1], 6.52, 8.5[3]	-
^{143}Eu(b1)	483.0	2.31	0.73	59.14	12.5	14.5[2,3]	15.5

This equation can be rewritten as: -

$$\theta^{(2)} = A + B\omega^2 + C\omega^4 \quad (\hbar^2\ \mathrm{MeV}^{-1}) \tag{7}$$

The spin can be predicted by integrating $\theta^{(2)}$ (Equation 7) with respect to ω

$$\hbar\hat{I} = A\omega + \left(B/3\right)\omega^3 + \left(C/5\right)\omega^5 + i_0 \quad (\hbar) \tag{8}$$

where the constant of integration (i_0) is called the aligned spin (resulting from the alignment of a pair of high-j particles) and it is always equal to zero.

RESULTS AND DISCUSSION

All the transition energies in the fifteen SD bands observed in the A~ 100-140 mass region have been least-squares fit by Equation (2) and the results are encouraging. We have obtained the values of the expansion coefficients a, b by using the Levenberg-Marquardt method (Flannery et al., 1992), to fit the proposed transition energies with their observed values. These fitting were done for the SD bands ^{104}Pd(b1), ^{132}Ce(b1), ^{134}Nd(b1), ^{136}Nd(b2), ^{142}Sm(b1), ^{148}Eu(b1, b2), ^{131}Ce(b1, b2), ^{133}Ce(b1, b2), ^{133}Pr(b1, b3), ^{137}Sm(b1), ^{143}Eu(b1) in the A~ 100-140 mass region, where b1, b2 and b3 refer to band 1, 2 and 3, respectively.

The spin assignments for each SD bands and also the corresponding fitting parameters are given in Table 1. For most of the fifteen SD bands, the spin assignments relatively coincide with the results of another theoretical results (Singh et al., 2002), our previous work (Shalaby et al., 2012), and also with the available experimental data (Singh et al., 1996).

We have two types of superdeformed nuclei

Even-A nuclei (7 nuclei)

As a first illustrative example, the results of the least-squares fitting of ^{104}Pd(b1) are listed in Table 2. It is seen that the E_2-transition energies can be reproduced relatively well when I_0 = 15, that is, E(17>15) = 1263.0 MeV. The deviations between the calculated and observed E(I) values are mostly less than 30 MeV. However, if I_0 is assumed to be 14 or 16, the rms deviation immediately increases (Figures 1 to 7). Therefore, the assignments of I_0 = 14 or 16 are completely unacceptable.

Similarly, we have obtained the results of the least-squares fitting of the other 6 even – A nuclei and these are listed in Tables 3 to 8.

Odd-A nuclei (8 nuclei)

As a first illustrative example, the results of the least-squares fitting of ^{131}Ce(b1) are listed in Table 9. It is seen

Table 2. Spin determination for the SD band ^{104}Pd(b1). I_0 is the spin value prescribed to the lowest level observed. $E_\gamma(I) = E_\gamma(I \rightarrow I-2)$ is the transition energy from level I to I-2. $\delta = E_\gamma^{exp.}(I) - E_\gamma^{cal.}(I)$. σ is the rms deviation defined by Equation 4.

Observed[a] E_γ(I) values (KeV)	Calculated E_γ(I) values (KeV)								
	I_0=14[b]			I_0=15[c]			I_0=16[d]		
	I	E_γ(I)	δ	I	E_γ(I)	δ	I	E_γ(I)	δ
2079.0	28	2034.6	44.4	29	2050.1	28.9	30	1897.2	181.8
1919.0	26	1914.3	4.7	27	1924.6	-5.6	28	1794.5	124.5
1763.0	24	1788.7	-25.7	25	1795.5	-32.5	26	1687.3	75.7
1638.0	22	1657.9	-19.9	23	1663.0	-25.0	24	1575.6	62.4
1511.0	20	1522.2	-11.2	21	1527.2	-16.2	22	1459.6	51.4
1381.0	18	1381.5	-0.51	19	1388.3	-7.3	20	1339.3	41.7
1263.0	16	1236.2	26.8	17	1246.4	16.6	18	1214.9	48.1
σ		1.38×10^{-2}			1.19×10^{-2}			5.16×10^{-2}	

[a]Reference (Singh et al., 2002): [b]a = 1.22×10^5 KeV, b = 3.40×10^{-4}; [c]a = 1.93×10^5 KeV, b = 2.01×10^{-4}; [d]a = 1.13×10^5 KeV, b = 3.22×10^{-4}.

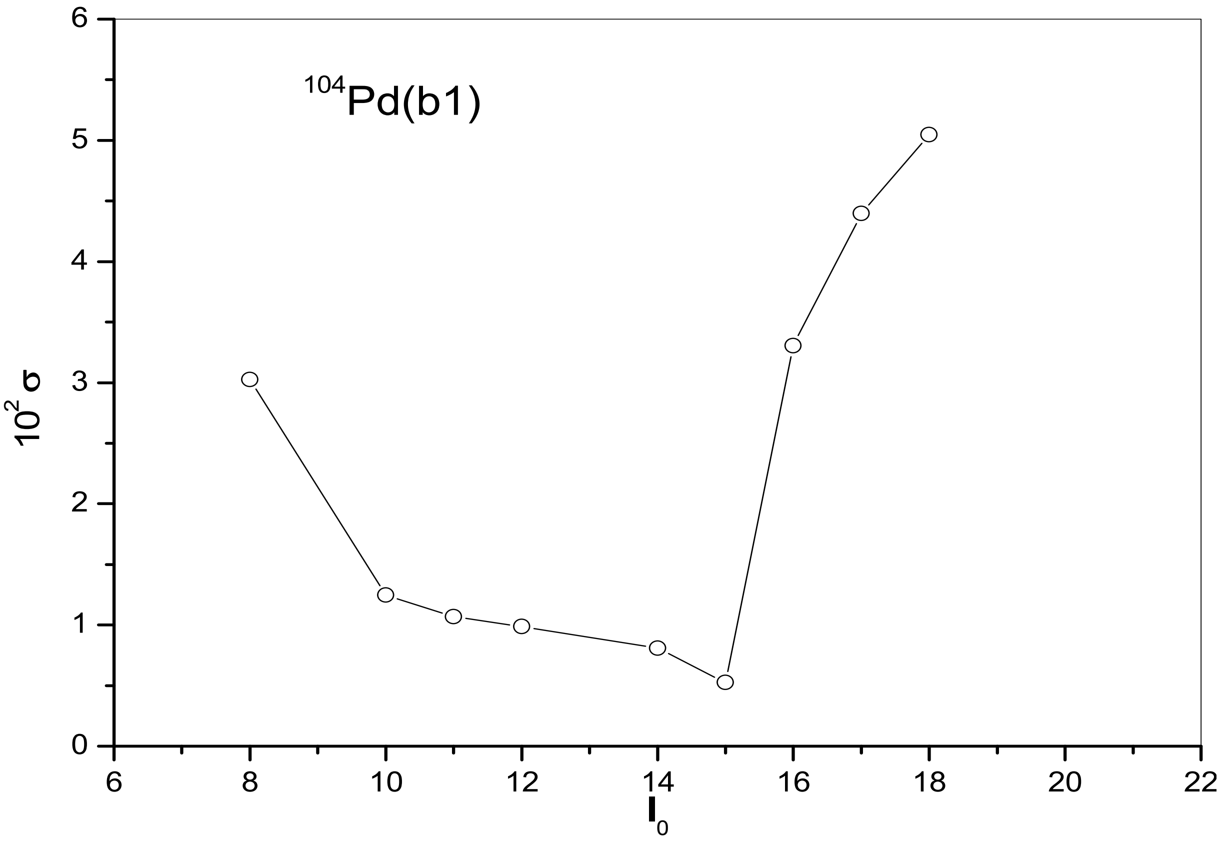

Figure 1. The rms deviation for various spin assignments in ^{104}Pd(b1). I_0 is the value prescribed to the lowest level observed [E $\gamma(I_0 + 2 ==> I_0)$ = 1263 KeV].

that the E_2-transition energies can be reproduced relatively well when I_0 = 11.5, that is, E(13.5→11.5) = 591.0 MeV. The deviations between the calculated and observed E(I) values are mostly less than 81 MeV. However, if I_0 is assumed to be 10.5 or 12.5, the rms deviation immediately increases (Figures 8 to 15). Therefore, the assignments of I_0 = 10.5 or 12.5 are completely unacceptable.

Similarly, we have obtained the results of the least-squares fitting of the other 7 odd – nuclei and these are

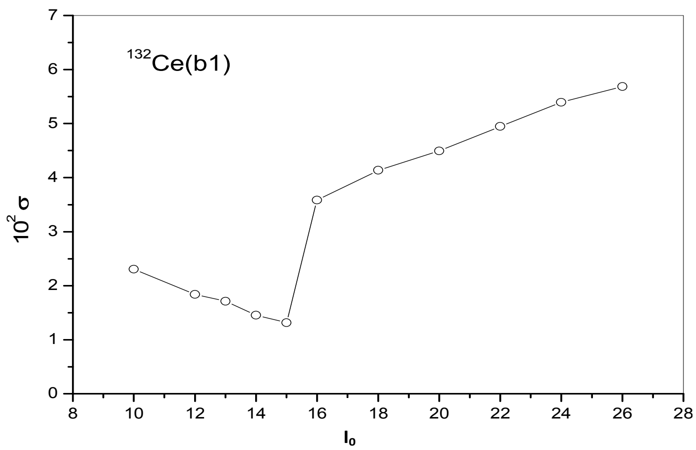

Figure 2. The rms deviation for various spin assignments in ^{132}Ce(b1). I_0 is the value prescribed to the lowest level observed [E $\gamma(I_0 + 2 ==> I_0)$ = 808 KeV].

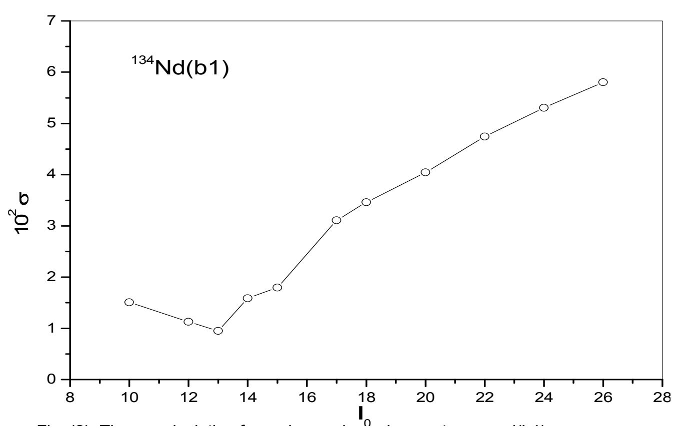

Figure 3. The rms deviation for various spin assignments in ^{134}Nd(b1). I_0 is the value prescribed to the lowest level observed [E $\gamma(I_0 + 2 ==> I_0)$ = 663.9 KeV].

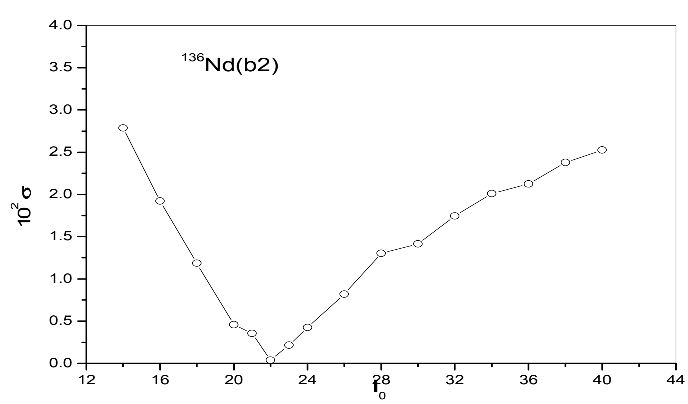

Figure 4. The rms deviation for various spin assignments in ^{136}Nd(b2). I_0 is the value prescribed to the lowest level observed [E $\gamma(I_0 + 2 ==> I_0)$ = 888 KeV].

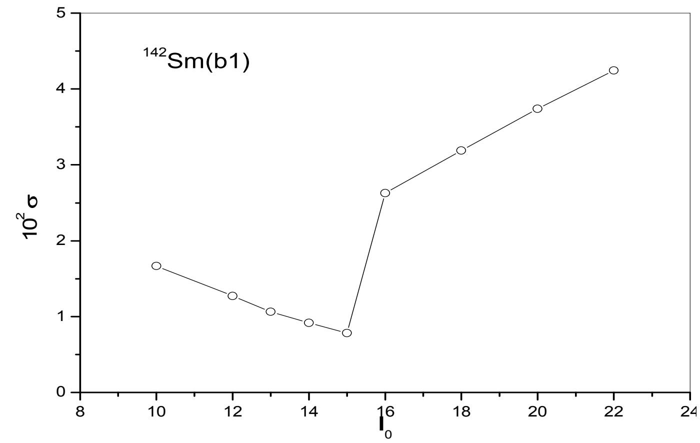

Figure 5. The rms deviation for various spin assignments in ^{142}Sm(b1). I_0 is the value prescribed to the lowest level observed [E $\gamma(I_0 + 2 ==> I_0)$ = 680 KeV].

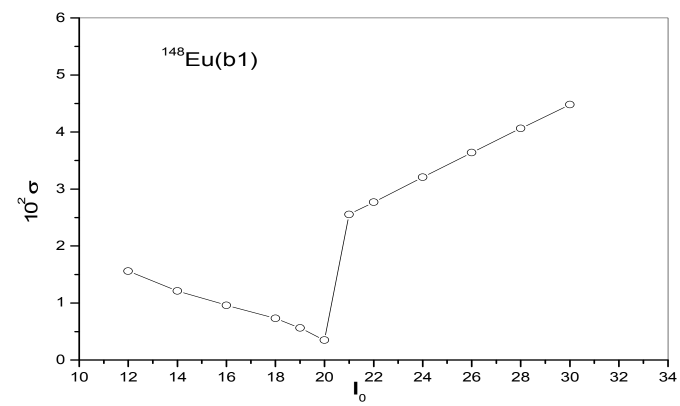

Figure 6. The rms deviation for various spin assignments in ^{148}Eu(b1). I_0 is the value prescribed to the lowest level observed [E $\gamma(I_0 + 2 ==> I_0)$ = 748 KeV].

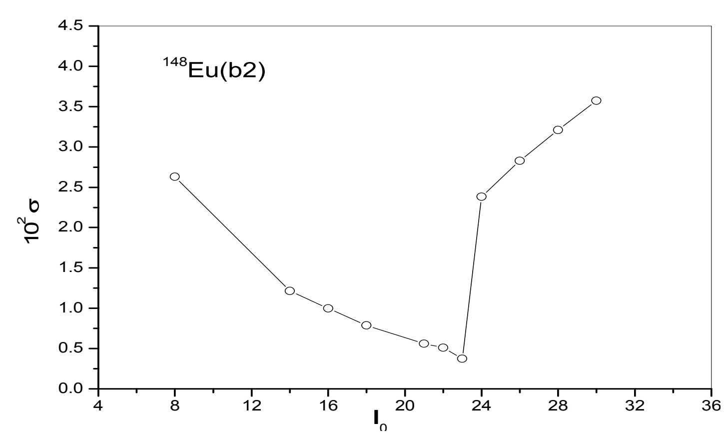

Figure 7. The rms deviation for various spin assignments in ^{148}Eu(b2). I_0 is the value prescribed to the lowest level observed [E $\gamma(I_0 + 2 ==> I_0)$ = 884 KeV].

Table 3. Spin determination for the SD band ^{132}Ce(b1). I_0 is the spin value prescribed to the lowest level observed. $E_\gamma(I) = E_\gamma(I \rightarrow I\text{-}2)$ is the transition energy from level I to I-2. $\delta = E_\gamma^{exp.}(I) - E_\gamma^{cal.}(I)$. σ is the rms deviation defined by Equation 4.

Observed[a] E_γ(I) values (KeV)	**Calculated E_γ(I) values (KeV)**								
	I_0=14[b]			**I_0=15[c]**			**I_0=16[d]**		
	I	**E_γ(I)**	**δ**	**I**	**E_γ(I)**	**δ**	**I**	**E_γ(I)**	**δ**
2030.0	48	1908.3	121.7	49	1920.0	110.0	50	1730.1	299.9
1930.0	46	1851.9	78.1	47	1859.7	70.3	48	1690.2	239.8
1836.0	44	1793.2	42.8	45	1797.6	38.4	46	1648.1	187.9
1739.0	42	1732.2	6.8	43	1733.6	5.4	44	1603.7	135.3
1650.0	40	1669.0	-19.0	41	1667.7	-17.7	42	1556.8	93.2
1565.0	38	1603.4	-38.4	39	1599.9	-35.0	40	1507.5	57.5
1484.0	36	1535.5	-51.5	37	1530.4	-46.4	38	1455.6	28.4
1406.0	34	1465.2	-59.2	35	1458.9	-52.9	36	1401.0	5.0
1333.0	32	1392.6	-59.6	33	1385.7	-52.7	34	1343.7	-10.7
1262.0	30	1317.7	-55.7	31	1310.6	-48.6	32	1283.5	-21.5
1193.0	28	1240.4	-47.4	29	1233.9	-40.9	30	1220.5	-27.5
1126.0	26	1160.9	-34.9	27	1155.4	-29.4	28	1154.6	-28.6
1059.0	24	1079.3	-20.3	25	1075.3	-16.3	26	1085.8	-26.8
993.0	22	995.5	-2.5	23	993.7	-0.65	24	1014.1	-21.1
928.0	20	909.8	18.2	21	910.5	17.5	22	939.5	-11.5
864.0	18	822.1	41.9	19	826.0	38.0	20	862.2	1.8
808.0	16	732.8	75.3	17	740.2	67.8	18	782.2	25.8
σ		4.04×10^{-2}			3.61×10^{-2}			6.06×10^{-2}	

[a]Reference (Singh et al., 2002): [b]$a = 1.21 \times 10^5$ KeV, $b = 2.00 \times 10^{-4}$; [c]$a = 1.61 \times 10^5$ KeV, $b = 1.42 \times 10^{-4}$; [d]$a = 7.19 \times 10^4$ KeV, $b = 3.26 \times 10^{-4}$.

Table 4. Spin determination for the SD band ^{134}Nd(b1). I_0 is the spin value prescribed to the lowest level observed. $E_\gamma(I) = E_\gamma(I \rightarrow I\text{-}2)$ is the transition energy from level I to I-2. $\delta = E_\gamma^{exp.}(I) - E_\gamma^{cal.}(I)$. σ is the rms deviation defined by Equation 4.

Observed[a] E_γ(I) values (KeV)	**Calculated E_γ(I) values (KeV)**								
	I_0=12[b]			I_0=13[c]			I_0=14[d]		
	I	**E_γ(I)**	**δ**	**I**	**E_γ(I)**	**δ**	**I**	**E_γ(I)**	**δ**
1450.0	36	1393.3	56.7	37	1402.2	47.8	38	1370.2	79.8
1367.3	34	1341.8	25.5	35	1347.0	20.3	36	1318.4	48.9
1289.9	32	1287.1	2.8	33	1289.1	0.76	34	1264.0	25.9
1216.0	30	1229.0	-13.0	31	1228.5	-12.5	32	1207.0	9.0
1143.8	28	1167.5	-23.7	29	1165.1	-21.3	30	1147.4	-3.6
1074.8	26	1102.5	-27.7	27	1098.8	-24.0	28	1085.1	-10.3
1007.4	24	1033.9	-26.5	25	1029.7	-22.3	26	1020.1	-12.7
942.2	22	961.7	-19.5	23	957.8	-15.6	24	952.4	-10.2
876.5	20	886.0	-9.4	21	883.2	-6.7	22	882.1	-5.6
807.8	18	806.7	1.1	19	806.0	1.8	20	809.3	-1.5
736.7	16	724.1	12.6	17	726.3	10.4	18	734.0	2.7
663.9	14	638.2	25.7	15	644.1	19.8	16	656.4	7.5
σ		2.26×10^{-2}			1.86×10^{-3}			2.11×10^{-2}	

[a]Reference (Singh et al., 2002): [b]$a = 5.43 \times 10^4$ KeV, $b = 4.53 \times 10^{-4}$; [c]$a = 7.15 \times 10^4$ KeV, $b = 3.21 \times 10^{-4}$; [d]$a = 6.93 \times 10^4$ KeV, $b = 3.17 \times 10^{-4}$

listed in Tables 10 to 16. The results for the transition energies in fifteen superdeformed bands observed in A~ 100-140 mass region are given in Tables 17 to 24, where the experimental data for the transition energies (labeled Exp[a]) are taken from Ref. (Singh et al., 2002) and the calculated transition energies (labeled Cal[b]) are done at

Table 6. Spin determination for the SD band ^{142}Sm(b1). I_0 is the spin value prescribed to the lowest level observed. $E_\gamma(I) = E_\gamma(I \rightarrow I-2)$ is the transition energy from level I to I-2. $\delta = E_\gamma^{exp.}(I) - E_\gamma^{cal.}(I)$. σ is the rms deviation defined by Equation 4.

Observed[a] $E_\gamma(I)$ values (KeV)	Calculated $E_\gamma(I)$ values (KeV)								
	$I_0=14$[b]			$I_0=15$[c]			$I_0=16$[d]		
	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ
1733.0	50	1665.5	67.5	51	1675.4	57.6	52	1539.8	193.2
1668.0	48	1621.7	46.3	49	1628.7	39.3	50	1506.1	161.9
1603.0	46	1576.0	27.0	47	1580.3	22.8	48	1470.6	132.4
1538.0	44	1528.2	9.8	45	1530.1	7.9	46	1433.2	104.8
1475.0	42	1478.4	-3.4	43	1478.1	-3.1	44	1393.8	81.2
1411.0	40	1426.5	-15.5	41	1424.3	-13.3	42	1352.4	58.6
1348.0	38	1372.5	-24.5	39	1368.8	-20.8	40	1308.8	39.2
1286.0	36	1316.2	-30.2	37	1311.4	-25.4	38	1263.0	23.0
1224.0	34	1257.8	-33.8	35	1252.2	-28.2	36	1215.0	9.0
1163.0	32	1197.2	-34.2	33	1191.3	-28.3	34	1164.6	-1.6
1102.0	30	1134.3	-32.3	31	1128.5	-26.5	32	1111.8	-9.8
1041.0	28	1069.3	-28.3	29	1064.0	-23.0	30	1056.6	-15.6
981.0	26	1002.1	-21.1	27	997.8	-16.8	28	998.9	-18.0
920.0	24	932.8	-12.8	25	929.9	-9.9	26	938.9	-18.9
860.0	22	861.5	-1.5	23	860.4	-0.40	24	876.4	-16.4
800.0	20	788.1	11.9	21	789.4	10.6	22	811.5	-11.5
739.0	18	713.0	26.0	19	716.9	22.1	20	744.3	-5.3
680.0	16	636.1	43.9	17	643.2	36.8	18	675.0	5.0
σ		2.70×10^{-2}			2.26×10^{-2}			4.77×10^{-2}	

[a]Reference (Singh et al., 2002): [b]$a = 9.05 \times 10^4$ KeV, $b = 2.33 \times 10^{-4}$; [c]$a = 1.14 \times 10^5$ KeV, $b = 1.75 \times 10^{-4}$; [d]$a = 6.51 \times 10^4$ KeV, $b = 3.10 \times 10^{-4}$.

Table 7. Spin determination for the SD band ^{148}Eu(b1). I_0 is the spin value prescribed to the lowest level observed. $E_\gamma(I) = E_\gamma(I \rightarrow I-2)$ is the transition energy from level I to I-2. $\delta = E_\gamma^{exp.}(I) - E_\gamma^{cal.}(I)$. σ is the rms deviation defined by Equation 4.

Observed[a] $E_\gamma(I)$ values (KeV)	Calculated $E_\gamma(I)$ values (KeV)								
	$I_0=19$[b]			$I_0=20$[c]			$I_0=21$[d]		
	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ
1555.0	51	1520.09	34.91	52	1526.89	28.11	53	1396.12	158.88
1499.0	49	1478.05	20.95	50	1482.96	16.04	51	1366.33	132.67
1443.0	47	1434.44	8.56	48	1437.65	5.35	49	1334.95	108.05
1388.0	45	1389.21	-1.21	46	1390.95	-2.95	47	1301.93	86.08
1331.0	43	1342.36	-11.36	44	1342.84	-11.84	45	1267.17	63.83
1276.0	41	1293.86	-17.86	42	1293.33	-17.33	43	1230.62	45.38
1220.0	39	1243.69	-23.69	40	1242.40	-22.40	41	1192.20	27.80
1165.0	37	1191.85	-26.85	38	1190.06	-25.06	39	1151.84	13.16
1111.0	35	1138.34	-27.34	36	1136.34	-25.34	37	1109.50	1.50
1057.0	33	1083.16	-26.16	34	1081.23	-24.23	35	1065.12	-8.12
1004.0	31	1026.33	-22.33	32	1024.77	-20.77	33	1018.64	-14.64
951.0	29	967.87	-16.87	30	966.99	-15.99	31	970.05	-19.05
900.0	27	907.82	-7.82	28	907.91	-7.91	29	919.31	-19.31
848.0	25	846.23	1.77	26	847.58	0.42	27	866.43	-18.43
798.0	23	783.15	14.85	24	786.06	11.94	25	811.41	-13.41
748.0	21	718.65	29.347	22	723.41	24.59	23	754.29	-6.29
σ		1.92×10^{-2}			1.71×10^{-2}			4.60×10^{-2}	

[a]Reference (Singh et al., 2002): [b]$a = 1.01 \times 10^5$ KeV, $b = 1.80 \times 10^{-4}$; [c]$a = 1.24 \times 10^5$ KeV, $b = 1.40 \times 10^{-4}$; [d]$a = 5.96 \times 10^4$, KeV, $b = 3.02 \times 10^{-4}$.

Table 8. Spin determination for the SD band ^{148}Eu(b2). I_0 is the spin value prescribed to the lowest level observed. $E_\gamma(I) = E_\gamma(I\rightarrow I\text{-}2)$ is the transition energy from level I to I-2. $\Delta = E_\gamma^{exp.}(I) - E_\gamma^{cal.}(I)$. σ is the rms deviation defined by Equation 4.

Observed[a] $E_\gamma(I)$ values (KeV)	Calculated $E_\gamma(I)$ values (KeV)								
	I_0=22[b]			I_0=23[c]			I_0=24[d]		
	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ
1544.0	50	1514.4	29.6	51	1522.4	21.6	52	1406.3	137.7
1489.0	48	1470.8	18.3	49	1477.1	11.9	50	1376.1	112.9
1434.0	46	1425.5	8.5	47	1430.4	3.6	48	1344.2	89.8
1378.0	44	1378.6	-0.60	45	1382.3	-4.3	46	1310.6	67.4
1322.0	42	1330.1	-8.1	43	1332.8	-10.8	44	1275.1	46.9
1269.0	40	1279.9	-11.0	41	1281.9	-12.9	42	1237.8	31.2
1212.0	38	1228.2	-16.1	39	1229.6	-17.6	40	1198.4	13.6
1158.0	36	1174.7	-16.7	37	1175.8	-17.8	38	1157.0	0.96
1104.0	34	1119.6	-15.6	35	1120.7	-16.7	36	1113.5	-9.5
1051.0	32	1062.9	-11.9	33	1064.2	-13.2	34	1067.8	-16.8
998.0	30	1004.5	-6.5	31	1006.4	-8.4	32	1019.9	-21.9
946.0	28	944.6	1.4	29	947.3	-1.3	30	969.7	-23.7
895.0	26	883.1	11.9	27	886.9	8.1	28	917.3	-22.3
844.0	24	820.1	23.9	25	825.2	18.8	26	862.5	-18.5
σ		1.31×10^{-2}			1.17×10^{-2}			4.19×10^{-2}	

[a]Reference (Singh et al., 2002): [b]$a = 1.05\times10^5$ KeV, $b = 1.74\times10^{-4}$; [c]$a = 1.27\times10^5$ KeV, $b = 1.38\times10^{-4}$; [d]$a = 5.76\times10^4$ KeV, $b = 3.23\times10^{-4}$.

Table 9. Spin determination for the SD band ^{131}Ce(b1). I_0 is the spin value prescribed to the lowest level observed. $E_\gamma(I)= E_\gamma(I\rightarrow I\text{-}2)$ is the transition energy from level I to I-2. $\delta= E_\gamma^{exp.}(I) - E_\gamma^{cal.}(I)$. σ is the rms deviation defined by Equation 4.

Observed[a] $E_\gamma(I)$ values (KeV)	Calculated $E_\gamma(I)$ values (KeV)								
	I_0=10.5[b]			I_0=11.5[c]			I_0=12.5[d]		
	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ
1822.0	44.5	1727.5	94.5	45.5	1741.0	81.0	46.5	1565.5	256.5
1732.0	42.5	1670.3	61.7	43.5	1678.6	53.4	44.5	1522.7	209.3
1640.0	40.5	1610.8	29.2	41.5	1614.4	25.6	42.5	1477.7	162.3
1550.0	38.5	1549.0	0.99	39.5	1548.4	1.6	40.5	1430.4	119.6
1464.0	36.5	1484.8	-20.8	37.5	1480.6	-16.6	38.5	1380.6	83.4
1381.0	34.5	1418.1	-37.1	35.5	1411.2	-30.2	36.5	1328.3	52.7
1301.0	32.5	1349.1	-48.1	33.5	1340.0	-39.0	34.5	1273.4	27.6
1225.0	30.5	1277.6	-52.6	31.5	1267.1	-42.1	32.5	1215.9	9.1
1151.0	28.5	1203.8	-52.8	29.5	1192.6	-41.6	30.5	1155.8	-4.8
1080.0	26.5	1127.6	-47.6	27.5	1116.5	-36.5	28.5	1092.9	-12.9
1011.0	24.5	1049.2	-38.2	25.5	1038.9	-27.9	26.5	1027.4	-16.4
943.0	22.5	968.5	-25.5	23.5	959.8	-16.8	24.5	959.1	-16.1
874.0	20.5	885.7	-11.7	21.5	879.3	-5.3	22.5	888.3	-14.3
804.0	18.5	800.9	3.1	19.5	797.6	6.4	20.5	814.9	-10.9
733.0	16.5	714.3	18.7	17.5	714.6	18.4	18.5	739.1	-6.1
662.0	14.5	625.9	36.1	15.5	630.6	31.4	16.5	660.9	1.1
591.0	12.5	558.7	32.3	13.5	566.9	24.1	14.5	600.8	-9.8
σ		3.55×10^{-2}			2.87×10^{-2}			5.78×10^{-2}	

[a]Reference (Singh et al., 2002): [b]$a = 1.07\times10^5$ KeV, $b = 2.21\times10^{-4}$; [c]$a = 1.61\times10^5$ KeV, $b = 1.37\times10^{-4}$; [d]$a = 7.02\times10^4$ KeV, $b = 3.15\times10^{-4}$

the two fitting parameters (a and b) given in Table 1. These tables give successively the calculations for even-mass nuclei and even spin (Tables 17 to 20), odd-mass nuclei with odd spin (Tables 21 to 24). The fact that the

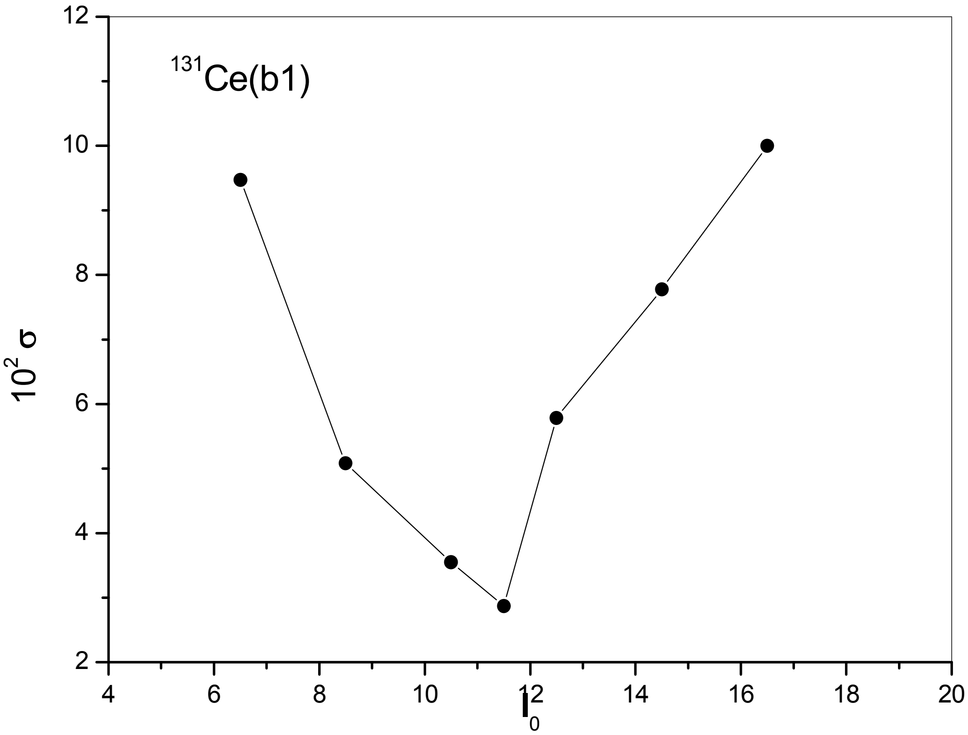

Figure 8. The rms deviation for various spin assignments in $^{131}Ce(b1)$. I_0 is the value prescribed to the lowest level observed [E $\gamma(I_0 + 2 ==> I_0)$ = 591 KeV].

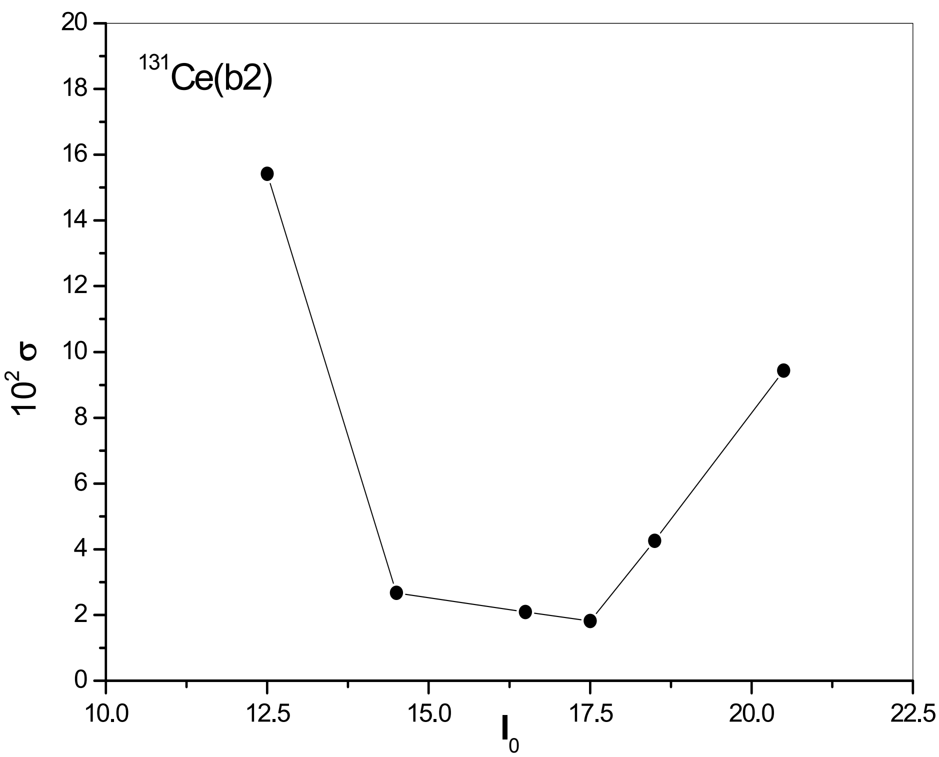

Figure 9. The rms deviation for various spin assignments in $^{131}Ce(b2)$. I_0 is the value prescribed to the lowest level observed [E $\gamma(I_0 + 2 ==> I_0)$ = 847 KeV].

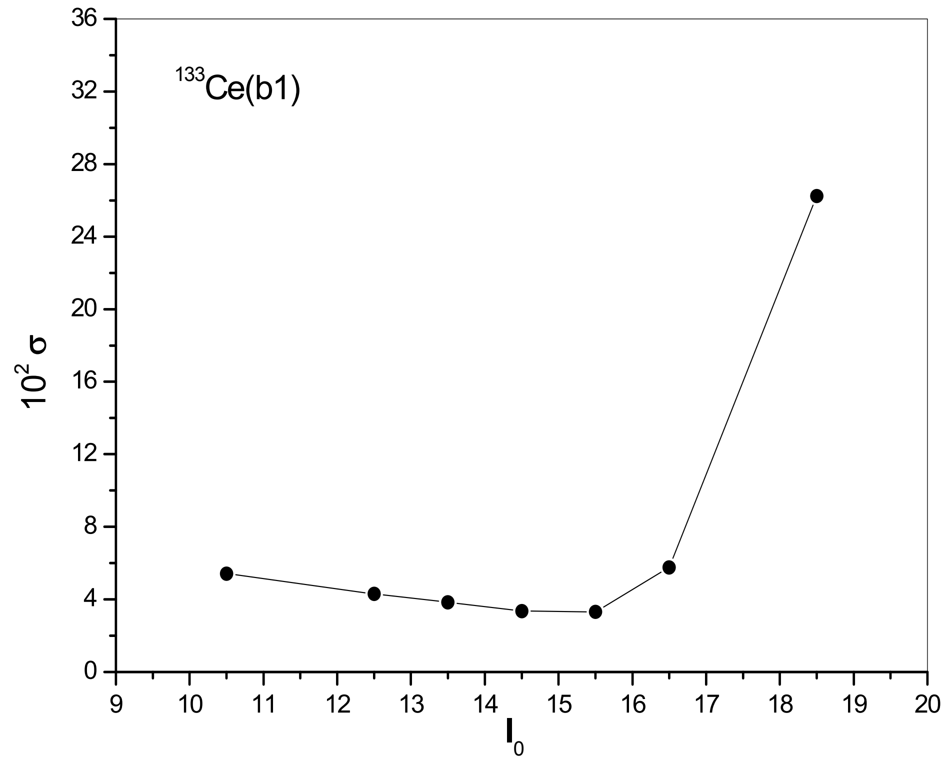

Figure 10. The rms deviation for various spin assignments in $^{133}Ce(b1)$. I_0 is the value prescribed to the lowest level observed [E $\gamma(I_0 + 2\ \ I_0)$ = 748 KeV].

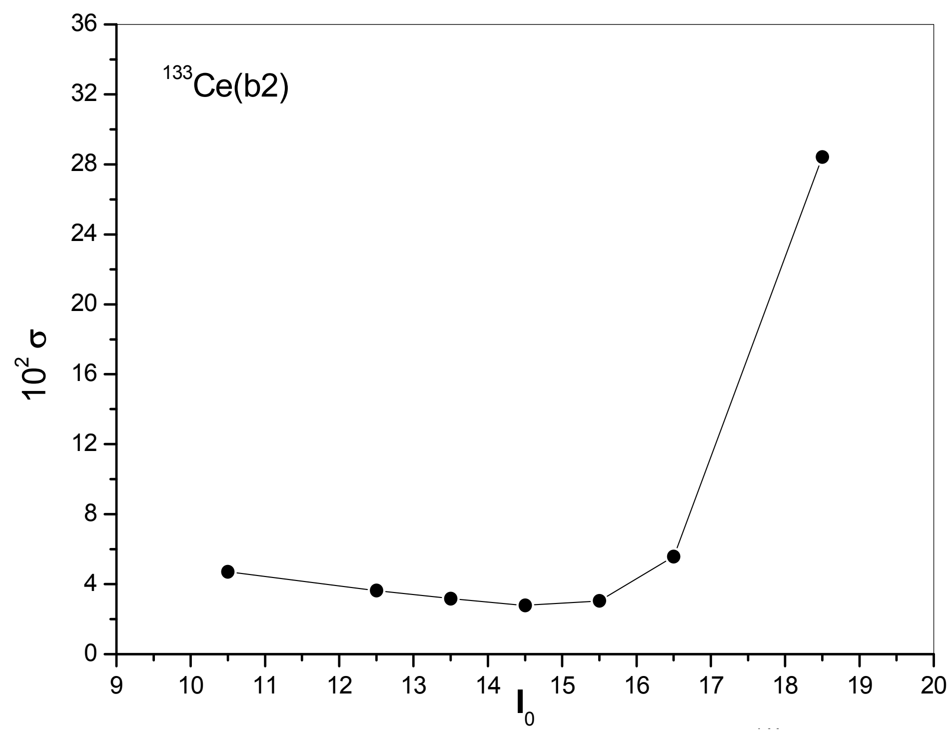

Figure 11. The rms deviation for various spin assignments in $^{133}Ce(b2)$. I_0 is the value prescribed to the lowest level observed [E $\gamma(I_0 + 2\ \ I_0)$ = 720.3 KeV].

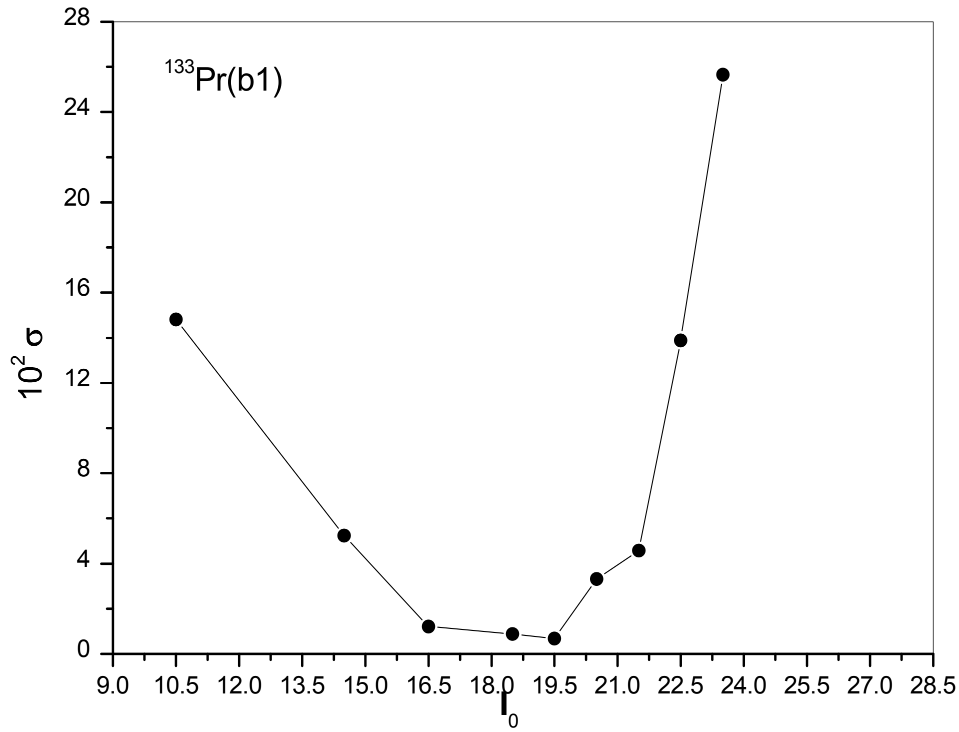

Figure 12. The rms deviation for various spin assignments in ^{133}Pr(b1). I_0 is the value prescribed to the lowest level observed [E $\gamma(I_0 + 2 \;\; I_0)$ = 871.0 KeV].

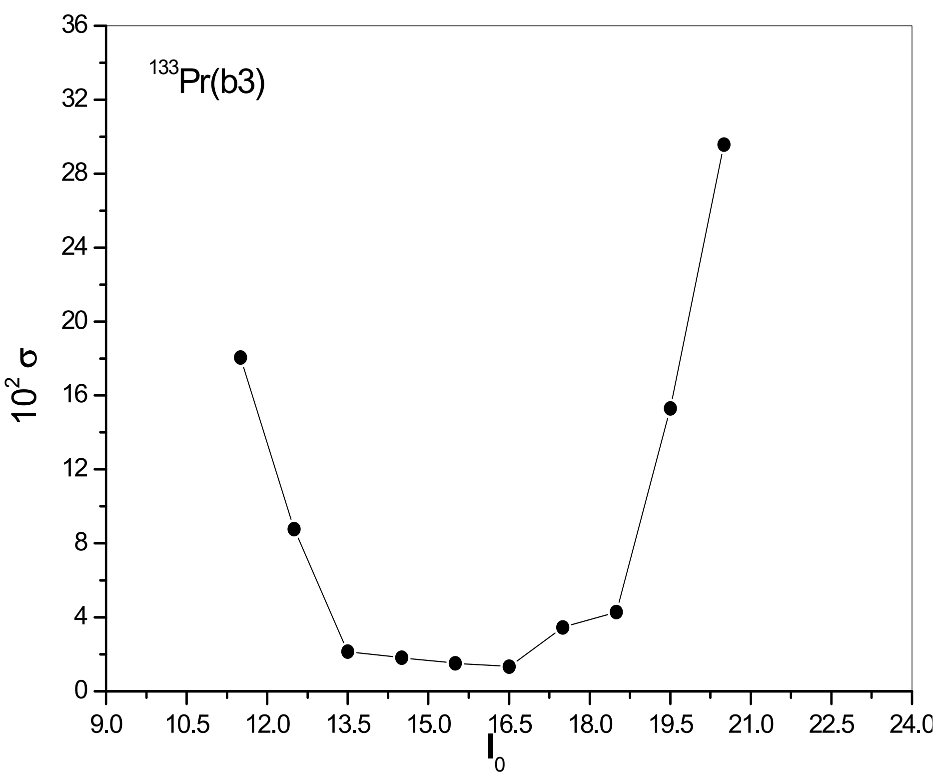

Figure 13. The rms deviation for various spin assignments in ^{133}Pr(b3). I_0 is the value prescribed to the lowest level observed [E $\gamma(I_0 + 2 \;\; I_0)$ = 821.0 KeV].

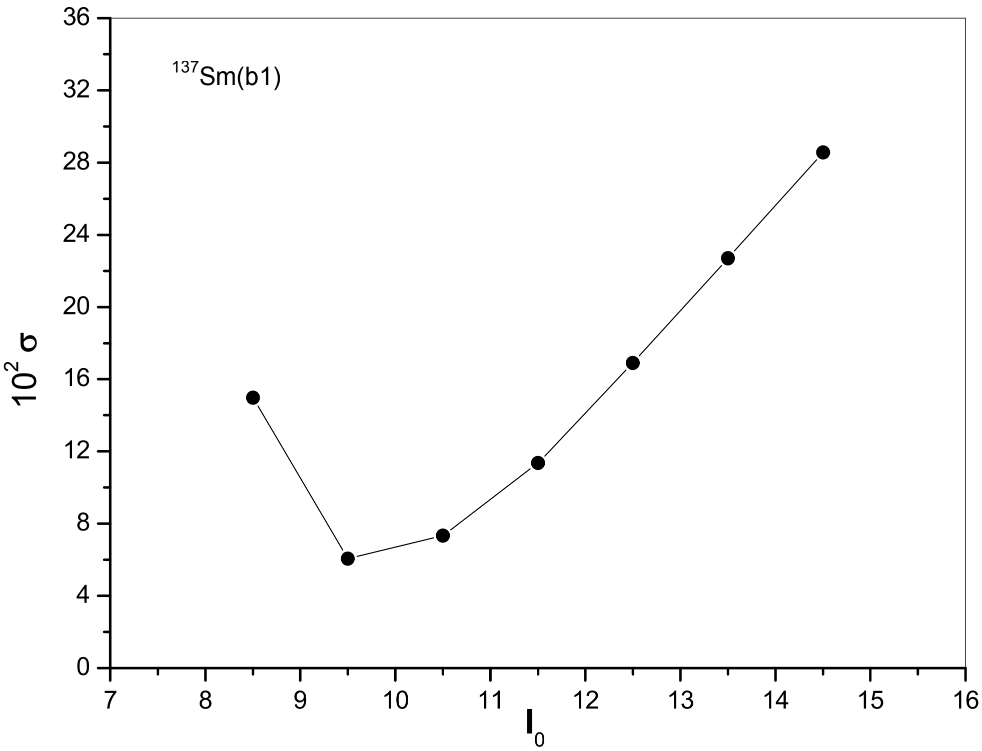

Figure 14. The rms deviation for various spin assignments in ^{137}Sm(b1). I_0 is the value prescribed to the lowest level observed [E γ(I_0 + 2 I_0) = 379.0 KeV].

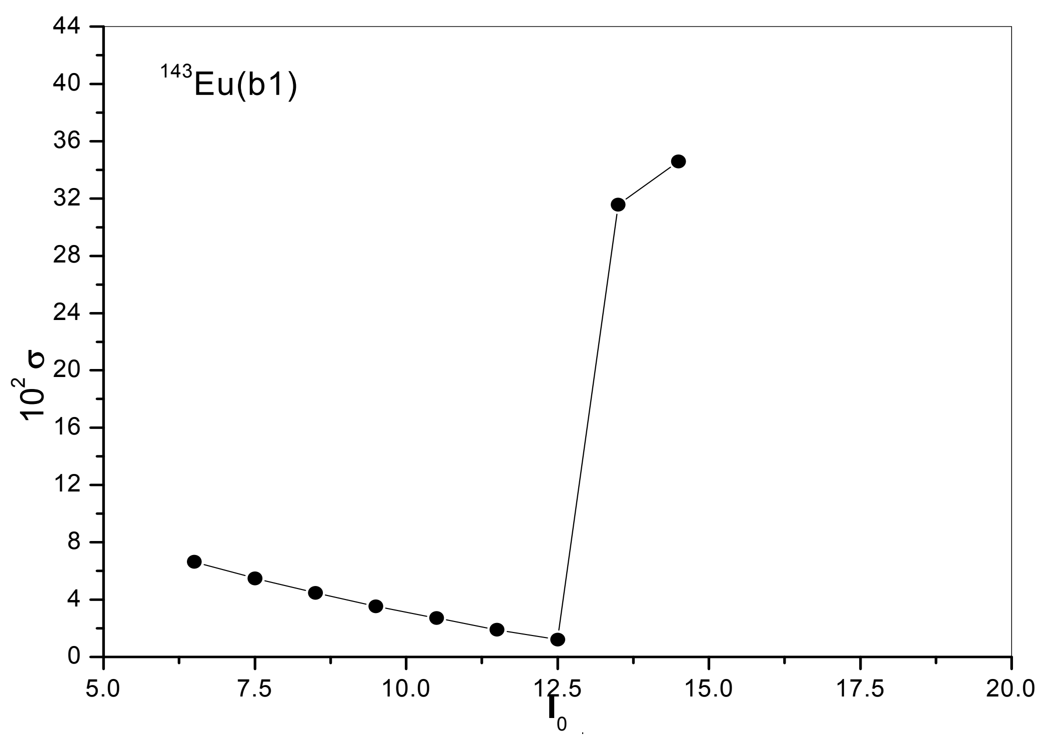

Figure 15. The rms deviation for various spin assignments in ^{143}Eu(b1). I_0 is the value prescribed to the lowest level observed [E γ(I_0 + 2 I_0) = 483.0 KeV].

Table 10. Spin determination for the SD band ^{131}Ce(b2). I_0 is the spin value prescribed to the lowest level observed. $E_\gamma(I) = E_\gamma(I \to I-2)$ is the transition energy from level I to I-2. $\delta = E_\gamma^{exp.}(I) - E_\gamma^{cal.}(I)$. σ is the rms deviation defined by Equation 4.

Observed[a] $E_\gamma(I)$ values (KeV)	Calculated $E_\gamma(I)$ values (KeV)								
	I_0=16.5[b]			I_0=17.5[c]			I_0=18.5[d]		
	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ
1723.0	42.5	1660.7	62.3	43.5	1672.1	50.9	44.5	1678.5	44.5
1635.0	40.5	1602.3	32.7	41.5	1611.9	23.1	42.5	1615.6	19.4
1552.0	38.5	1541.5	10.5	39.5	1549.6	2.4	40.5	1550.9	1.1
1471.0	36.5	1478.4	-7.4	37.5	1485.3	-14.3	38.5	1484.7	-13.7
1396.0	34.5	1413.0	-17.0	35.5	1418.8	-22.8	36.5	1416.7	-20.7
1322.0	32.5	1345.3	-23.3	33.5	1350.2	-28.2	34.5	1347.2	-25.2
1251.0	30.5	1275.2	-24.2	31.5	1279.5	-28.5	32.5	1276.1	-25.1
1181.0	28.5	1202.8	-21.8	29.5	1206.8	-25.8	30.5	1203.4	-22.4
1112.0	26.5	1128.2	-16.2	27.5	1132.1	-20.1	28.5	1129.3	-17.3
1043.0	24.5	1051.3	-9.3	25.5	1055.4	-12.4	26.5	1053.7	-10.7
976.0	22.5	972.3	3.7	23.5	976.8	-0.84	24.5	976.7	-0.72
908.0	20.5	891.2	16.8	21.5	896.5	11.5	22.5	898.4	9.6
847.0	18.5	808.2	38.8	19.5	814.4	32.6	20.5	818.9	28.1
σ		$2.09x10^{-2}$			$1.82x10^{-2}$			$4.26x10^{-2}$	

[a]Reference (Singh et al., 2002): [b]a = $1.08x10^5$ KeV, b = $2.15x10^{-4}$; [c]a = $1.43x10^5$ KeV, b = $1.54x10^{-4}$; [d]a = $1.46x10^5$ KeV, b = $1.52x10^{-4}$.

Table 11. Spin determination for the SD band ^{133}Ce(b1). I_0 is the spin value prescribed to the lowest level observed. $E_\gamma(I) = E_\gamma(I \to I-2)$ is the transition energy from level I to I-2. $\Delta = E_\gamma^{exp.}(I) - E_\gamma^{cal.}(I)$. σ is the rms deviation defined by Equation 4.

Observed[a] $E_\gamma(I)$ values (KeV)	Calculated $E_\gamma(I)$ values (KeV)								
	I_0=14.5[b]			I_0=15.5[c]			I_0=16.5[d]		
	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ
1928.0	48.5	1828.1	99.9	49.5	1815.0	113.0	50.5	1707.0	221.1
1833.0	46.5	1770.4	62.6	47.5	1759.6	73.4	48.5	1653.7	179.2
1743.0	44.5	1710.8	32.2	45.5	1702.5	40.5	46.5	1599.1	144.0
1655.0	42.5	1649.5	5.5	43.5	1643.6	11.4	44.5	1543.1	112.0
1570.0	40.5	1586.3	-16.3	41.5	1582.8	-12.8	42.5	1485.5	84.5
1489.0	38.5	1521.2	-32.2	39.5	1520.2	-31.2	40.5	1426.5	62.5
1411.0	36.5	1454.3	-43.3	37.5	1455.8	-44.8	38.5	1366.0	45.0
1337.0	34.5	1385.6	-48.6	35.5	1389.7	-52.7	36.5	1304.1	33.0
1267.0	32.5	1315.1	-48.1	33.5	1321.7	-54.7	34.5	1240.7	26.3
1198.0	30.5	1242.9	-44.9	31.5	1251.9	-53.9	32.5	1176.0	22.0
1132.0	28.5	1168.9	-36.9	29.5	1180.5	-48.5	30.5	1110.0	22.1
1068.0	26.5	1093.3	-25.3	27.5	1107.4	-39.4	28.5	1042.6	25.4
1003.0	24.5	1016.0	-13.0	25.5	1032.6	-29.6	26.5	974.0	29.1
937.0	22.5	937.3	-0.26	23.5	956.3	-19.3	24.5	904.1	33.0
873.0	20.5	857.0	16.0	21.5	878.6	-5.6	22.5	833.1	40.0
809.0	18.5	775.5	33.5	19.5	799.4	9.6	20.5	761.0	48.0
748.0	16.5	692.6	55.4	17.5	718.9	29.1	18.5	687.9	60.1
σ		$3.36x10^{-2}$			$3.32x10^{-2}$			$5.75x10^{-2}$	

[a]Reference (Singh et al., 2002), [b]a = $1.49x10^5$ KeV, b = $1.48x10^{-4}$; [c]a = $1.44x10^5$ KeV, b = $1.50x10^{-4}$; [d]a = $1.61x10^5$ KeV, b = $1.21x10^{-4}$.

calculated transition energies coincide with the observed results extremely well implies that the rotational spectrum in each SD band can be precisely described by Equation 1.

Conclusion

Fifteen Superdeformed bands observed in the nuclei of

Table 12. Spin determination for the SD band ^{133}Ce(b2). I_0 is the spin value prescribed to the lowest level observed. $E_\gamma(I)= E_\gamma(I\rightarrow I\text{-}2)$ is the transition energy from level I to I-2. $\delta= E_\gamma^{exp.}(I)- E_\gamma^{cal.}(I)$. σ is the rms deviation defined by Equation 4.

Observed[a] $E_\gamma(I)$ values (KeV)	Calculated $E_\gamma(I)$ values (KeV)								
	I_0=13.5[b]			I_0=14.5[c]			I_0=15.5[d]		
	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ
1731.3	45.5	1732.5	89.1	46.5	1749.3	72.3	47.5	1740.8	80.8
1731.3	43.5	1677.5	53.8	44.5	1690.3	41.0	45.5	1684.0	47.3
1643.0	41.5	1620.3	22.7	42.5	1629.5	13.5	43.5	1625.4	17.6
1557.3	39.5	1560.8	-3.5	40.5	1566.8	-9.5	41.5	1565.1	-7.8
1475.5	37.5	1499.1	-23.6	38.5	1502.4	-27.0	39.5	1503.0	-27.5
1397.9	35.5	1435.0	-37.1	36.5	1436.2	-38.3	37.5	1439.1	-41.2
1323.9	33.5	1368.7	-44.8	34.5	1368.3	-44.3	35.5	1373.4	-49.5
1253.2	31.5	1300.1	-47.0	32.5	1298.5	-45.3	33.5	1306.1	-52.9
1184.3	29.5	1229.3	-45.0	30.5	1227.1	-42.8	31.5	1237.0	-52.7
1118.2	27.5	1156.1	-38.0	28.5	1153.9	-35.7	29.5	1166.2	-48.0
1052.2	25.5	1080.8	-28.6	26.5	1079.2	-27.0	27.5	1093.8	-41.6
986.9	23.5	1003.4	-16.5	24.5	1002.8	-15.9	25.5	1019.9	-33.0
920.3	21.5	923.9	-3.6	22.5	925.0	-4.7	23.5	944.4	-24.1
854.2	19.5	842.4	11.8	20.5	845.8	8.4	21.5	867.5	-13.3
785.4	17.5	759.1	26.3	18.5	765.2	20.2	19.5	789.3	-3.9
720.3	15.5	674.1	46.2	16.5	683.4	36.9	17.5	709.7	10.6
σ		3.17×10^{-2}			2.78×10^{-2}			3.05×10^{-2}	

[a]Reference (Singh et al., 2002): [b]$a = 1.09\times10^5$ KeV, $b = 2.11\times10^{-4}$; [c]$a = 1.49\times10^5$ KeV, $b= 1.46\times10^{-4}$; [d]$a = 1.45\times10^5$ KeV, $b = 1.47\times10^{-4}$.

Table 13. Spin determination for the SD band ^{133}Pr(b1). I_0 is the spin value prescribed to the lowest level observed. $E_\gamma(I) = E_\gamma(I\rightarrow I\text{-}2)$ is the transition energy from level I to I-2. $\Delta = E_\gamma^{exp.}(I)- E_\gamma^{cal.}(I)$. σ is the rms deviation defined by Equation 4.

Observed[a] $E_\gamma(I)$ values (KeV)	Calculated $E_\gamma(I)$ values (KeV)								
	I_0=18.5[b]			I_0=19.5[c]			I_0=20.5[d]		
	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ
1530.0	38.5	1513.2	16.8	39.5	1513.2	16.8	40.5	1540.9	-10.9
1450.0	36.5	1448.5	1.5	37.5	1446.5	3.5	38.5	1476.2	-26.2
1380.0	34.5	1381.8	-1.8	35.5	1378.2	1.8	36.5	1409.9	-29.9
1299.0	32.5	1313.2	-14.2	33.5	1308.6	-9.6	34.5	1342.0	-43.0
1228.0	30.5	1242.5	-14.5	31.5	1237.4	-9.4	32.5	1272.5	-44.5
1156.0	28.5	1169.9	-14.0	29.5	1164.9	-8.9	30.5	1201.5	-45.5
1085.0	26.5	1095.4	-10.4	27.5	1091.0	-6.0	28.5	1129.0	-44.0
1013.0	24.5	1019.1	-6.1	25.5	1015.9	-2.9	26.5	1055.0	-42.0
943.0	22.5	941.1	1.9	23.5	939.5	3.5	24.5	979.7	-36.7
871.0	20.5	861.4	9.6	21.5	862.0	9.0	22.5	903.1	-32.1
σ		8.85×10^{-3}			6.79×10^{-3}			3.32×10^{-2}	

[a]Reference (Singh et al., 2002): [b]$a = 1.28\times10^5$ KeV, $b = 1.74\times10^{-4}$; [c]$a = 1.81\times10^5$ KeV, $b = 1.16\times10^{-4}$; [d]$a = 1.64\times10^5$ KeV, $b = 1.29\times10^{-4}$.

the mass region A~ 100-140 have been analyzed in terms of the expression for the γ-transition energies as a function of spin (ab formula). The use of direct expression of energy as a function of spin avoids necessary assumptions done in (Shalaby et al., 2012) about the values of quantities not directly measured in experiment like rotational frequency and dynamic moment of inertia. It is quite unexpected that so large a number of SD bands can be reproduced incredibly well by the simple two-parameter closed expression. It was found that the argument between the calculated and observed transition energies depends sensitively on the prescribed level

Table 14. Spin determination for the SD band $^{133}Pr(b2)$. I_0 is the spin value prescribed to the lowest level observed. $E_\gamma(I) = E_\gamma(I \rightarrow I-2)$ is the transition energy from level I to I-2. $\Delta = E_\gamma^{exp.}(I) - E_\gamma^{cal.}(I)$. σ is the rms deviation defined by Equation 4.

Observed[a] $E_\gamma(I)$ values (KeV)	Calculated $E_\gamma(I)$ values (KeV)								
	I_0=15.5[b]			I_0=16.5[c]			I_0=17.5[d]		
	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ
1656.0	39.5	1625.4	30.6	40.5	1637.0	19.0	41.5	1663.4	-7.4
1576.0	37.5	1561.8	14.2	38.5	1570.1	6.0	39.5	1597.1	-21.1
1500.0	35.5	1495.8	4.2	36.5	1501.3	-1.3	37.5	1529.0	-29.0
1420.0	33.5	1427.3	-7.3	34.5	1430.6	-10.6	35.5	1459.0	-39.0
1342.0	31.5	1356.3	-14.3	32.5	1358.0	-16.0	33.5	1387.3	-45.3
1264.0	29.5	1282.9	-18.9	30.5	1283.6	-19.6	31.5	1313.7	-49.7
1184.0	27.5	1207.1	-23.1	28.5	1207.3	-23.3	29.5	1238.4	-54.4
1110.0	25.5	1128.9	-19.0	26.5	1129.4	-19.4	27.5	1161.3	-51.3
1035.0	23.5	1048.4	-13.4	24.5	1049.7	-14.7	25.5	1082.7	-47.7
961.0	21.5	965.7	-4.7	22.5	968.4	-7.4	23.5	1002.4	-41.4
885.0	19.5	880.8	4.1	20.5	885.6	-0.64	21.5	920.7	-35.7
821.0	17.5	794.0	27.0	18.5	801.4	19.6	19.5	837.6	-16.6
σ		1.52×10^{-2}			1.33×10^{-2}			3.45×10^{-2}	

[a]Reference (Singh et al., 2002): [b]$a = 1.09 \times 10^5$ KeV, $b = 2.21 \times 10^{-4}$; [c]$a = 1.51 \times 10^5$ KeV, $b = 1.51 \times 10^{-4}$; [d]$a = 1.57 \times 10^5$ KeV, $b = 1.44 \times 10^{-4}$.

Table 15. Spin determination for the SD band $^{137}Sm(b1)$. I_0 is the spin value prescribed to the lowest level observed. $E_\gamma(I) = E_\gamma(I \rightarrow I-2)$ is the transition energy from level I to I-2. $\delta = E_\gamma^{exp.}(I) - E_\gamma^{cal.}(I)$. σ is the rms deviation defined by Equation (4).

Observed[a] $E_\gamma(I)$ values (KeV)	Calculated $E_\gamma(I)$ values (KeV)								
	I_0=8.5[b]			I_0=9.5[c]			I_0=10.5[d]		
	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ
1115.0	28.5	936.2	178.8	29.5	1084.0	31.0	30.5	1012.4	102.6
1045.0	26.5	870.4	174.6	27.5	1014.4	30.6	28.5	949.1	95.9
970.0	24.5	804.4	165.6	25.5	943.7	26.3	26.5	885.0	85.0
890.0	22.5	738.2	151.8	23.5	872.1	18.0	24.5	820.1	69.9
806.3	20.5	671.7	134.6	21.5	799.5	6.8	22.5	754.5	51.8
719.7	18.5	605.1	114.6	19.5	726.1	-6.4	20.5	688.2	31.5
629.1	16.5	538.3	90.8	17.5	651.9	-22.8	18.5	621.2	7.9
538.5	14.5	471.4	67.1	15.5	576.9	-38.4	16.5	553.7	-15.2
451.0	12.5	404.2	46.8	13.5	501.4	-50.4	14.5	485.7	-34.7
379.0	10.5	337.1	42.0	11.5	425.2	-46.2	12.5	417.2	-38.2
σ		1.50×10^{-1}			6.06×10^{-2}			7.33×10^{-2}	

[a]Reference (Singh et al., 2002): [b]$a = 7.06 \times 10^5$ KeV, $b = 2.39 \times 10^{-5}$; [c]$a = 1.99 \times 10^5$ KeV, $b = 9.77 \times 10^{-5}$; [d]$a = 2.14 \times 10^5$ KeV, $b = 8.17 \times 10^{-5}$.

Table 16. Spin determination for the SD band $^{143}Eu(b1)$. I_0 is the spin value prescribed to the lowest level observed. $E_\gamma(I) = E_\gamma(I \rightarrow I-2)$ is the transition energy from level I to I-2. $\delta = E_\gamma^{exp.}(I) - E_\gamma^{cal.}(I)$. σ is the rms deviation defined by Equation 4.

Observed[a] $E_\gamma(I)$ values (KeV)	Calculated $E_\gamma(I)$ values (KeV)								
	I_0=11.5[b]			I_0=12.5[c]			I_0=13.5[d]		
	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ	I	$E_\gamma(I)$	δ
1743.0	55.5	1690.6	52.4	56.5	1707.9	35.1		2078.7	-335.7
1684.0	53.5	1644.5	39.5	54.5	1657.8	26.2		2030.9	-346.9
1623.5	51.5	1597.0	26.5	52.5	1606.7	16.8		1981.4	-357.8
1562.3	49.5	1548.2	14.1	50.5	1554.6	7.7		1929.9	-367.6
1502.2	47.5	1498.2	4.0	48.5	1501.6	0.06	49.5	1876.6	-374.4

Table 16. contd.

1442.7	45.5	1446.8	-4.1	46.5	1447.5	-4.8	47.5	1821.3	-378.6
1383.2	43.5	1394.0	-10.8	44.5	1392.6	-9.4	45.5	1764.0	-380.8
1324.1	41.5	1340.0	-15.9	42.5	1336.7	-12.6	43.5	1704.8	-380.7
1265.1	39.5	1284.6	-19.5	40.5	1279.8	-14.7	41.5	1643.5	-378.4
1206.5	37.5	1228.0	-21.5	38.5	1222.1	-15.6	39.5	1580.3	-373.8
1148.1	35.5	1170.1	-22.0	36.5	1163.5	-15.4	37.5	1514.9	-366.8
1089.9	33.5	1110.9	-21.0	34.5	1104.0	-14.1	35.5	1447.5	-357.6
1031.1	31.5	1050.5	-19.4	32.5	1043.8	-12.7	33.5	1378.1	-347.0
972.2	29.5	989.0	-16.8	30.5	982.7	-10.5	31.5	1306.7	-334.5
912.8	27.5	926.2	-13.4	28.5	920.9	-8.0	29.5	1233.3	-320.5
853.5	25.5	862.5	-9.0	26.5	858.3	-4.8	27.5	1158.0	-304.5
793.4	23.5	797.6	-4.2	24.5	795.0	-1.6	25.5	1080.8	-287.4
732.6	21.5	731.8	0.82	22.5	731.1	1.5	23.5	1001.8	-269.2
671.2	19.5	665.0	6.2	20.5	666.7	4.5	21.5	921.1	-249.9
609.0	17.5	597.4	11.6	18.5	601.6	7.4	19.5	838.7	-229.7
546.5	15.5	529.0	17.5	16.5	536.1	10.4	17.5	754.8	-208.3
483.0	13.5	460.0	23.0	14.5	470.1	12.9	15.5	669.5	-186.5
σ		1.91×10^{-2}			1.21×10^{-2}			3.16×10^{-1}	

[a]Reference (Singh et al., 2002): [b]a = 1.54×10^{5} KeV, b = 1.16×10^{-4}; [c]a = 2.31×10^{5} KeV, b = 7.32×10^{-5}; [d]a = 1.33×10^{5} KeV, b = 1.71×10^{-4}.

Table 17. Calculation of the transition energies in Nd^{134}(b1) and Eu^{148}(b2).

Nd^{134}(b1)			Eu^{148}(b2)		
$E_\gamma(I)$ (KeV) Exp[a]	Cal[b]	Assigned I	$E_\gamma(I)$ (KeV) Exp[a]	Cal[b]	Assigned I
663.9	644.1	15	844.0	825.2	25
736.7	726.3	17	895.0	886.9	27
807.8	806.0	19	946.0	947.3	29
876.5	883.2	21	998.0	1006.4	31
942.2	957.8	23	1051.0	1064.2	33
1007.4	1029.7	25	1104.0	1120.7	35
1074.4	1098.8	27	1158.0	1175.8	37
1143.8	1165.1	29	1212.0	1229.6	39
1216.0	1228.5	31	1269.0	1281.9	41
1289.9	1289.1	33	1322.0	1332.8	43
1367.3	1347.0	35	1378.0	1382.3	45
1450.0	1402.2	37	1434.0	1430.4	47
			1489.0	1477.1	49
			1544.0	1522.4	51
	σ = 1.86×10^{-3}			σ = 1.17×10^{-2}	

spins. Therefore, the spins of each band levels can be determined.

For all fifteen SD bands, the spin values and also the corresponding parameters a and b are tabulated in Table 1. Of course, the spin assignments made in this paper remain to be confirmed from the future direct measurements.

It is seen from Table 1 that the values of the parameters a and b for all these bands are close to each other:

$$a = (3.11 \pm 2.39) \times 10^{5} \text{ KeV},$$
$$b = (1 : 3) \times 10^{-4}$$

except for ^{136}Nd(b1) and ^{143}Eu(b1), where "b" is smaller than for the other SD bands. It is noted from these 15

Table 18. Calculation of the transition energies in Pd^{104}(b1) and Nd^{136}(b2).

Pd^{104}(b1)			Nd^{136}(b2)		
E_γ(I) (KeV) Exp[a]	Cal[b]	Assigned I	E_γ(I) (KeV) Exp[a]	Cal[b]	Assigned I
1263.0	1246.4	17	888.0	870.8	24
1381.0	1388.3	19	951.0	943.3	26
1511.0	1527.2	21	1017.0	1015.5	28
1638.0	1663.0	23	1081.0	1087.3	30
1763.0	1795.5	25	1151.0	1158.7	32
1919.0	1924.6	27	1221.0	1229.6	34
2079.0	2050.1	29	1290.0	1300.1	36
			1364.0	1370.1	38
			1438.0	1439.6	40
	$\sigma = 1.19x10^{-2}$			$\sigma = 5.36x10^{-3}$	

Table 19. Calculation of the transition energies in Ce^{132}(b1) and Sm^{142}(b1).

Ce^{132}(b1)			Sm^{142}(b1)		
E_γ(I) (KeV) Exp[a]	Cal[b]	Assigned I	E_γ(I) (KeV) Exp[a]	Cal[b]	Assigned I
808.0	740.2	17	680.0	643.2	17
864.0	826.0	19	739.0	716.9	19
928.0	910.5	21	800.0	789.4	21
993.0	993.7	23	860.0	860.4	23
1059.0	1075.3	25	920.0	929.9	25
1126.0	1155.4	27	981.0	997.8	27
1193.0	1233.9	29	1041.0	1074.0	29
1262.0	1310.3	31	1102.0	1128.5	31
1333.0	1385.7	33	1163.0	1191.3	34
1406.0	1458.9	35	1224.0	1252.2	35
1484.0	1530.4	37	1286.0	1311.4	37
1565.0	1599.9	39	1348.0	1368.8	39
1650.0	1667.6	41	1411.0	1424.3	41
1739.0	1733.6	43	1475.0	1478.1	43
1836.0	1797.6	45	1538.0	1530.1	45
1930.0	1859.7	47	1603.0	1580.3	47
2030.0	1920.0	49	1668.0	1628.7	49
			1733.0	1675.4	51
	$\sigma = 3.61x10^{-2}$		$\sigma = 2.26x10^{-2}$		

Table 20. Calculation of the transition energies in Eu^{148}(b1).

Eu^{148}(b1)		
E_γ(I) (KeV) Exp[a]	Cal[b]	Assigned I
748.0	723.41	22
798.0	786.06	24
848.0	947.58	26
900.0	907.91	28
951.0	966.99	30
1004.0	1024.77	32
1057.0	1081.23	34

Table 20. Contd.

1111.0	1136.34	36
1165.0	1190.06	38
1220.0	1242.40	40
1276.0	1293.33	42
1331.0	1342.84	44
1388.0	1390.95	46
1443.0	1437.65	48
1499.0	1482.96	50
1555.0	1526.89	52
	$\sigma = 1.71x10^{-2}$	

Table 21. Calculation of the transition energies in Ce^{131}(b1) and Ce^{133}(b1).

	Ce^{131}(b1)			Ce^{133}(b1)	
E_γ(I) (KeV) Exp[a]	**Cal[b]**	**Assigned I**	**E_γ(I) (KeV) Exp[a]**	**Cal[b]**	**Assigned I**
591.0	566.9	13.5	748.0	692.6	16.5
662.0	630.6	15.5	809.0	775.5	18.5
733.0	714.6	17.5	873.0	857.0	20.5
804.0	797.6	19.5	937.0	937.3	22.5
874.0	879.3	21.5	1003.0	1016.0	24.5
943.0	959.8	23.5	1068.0	1093.3	26.5
1011.0	1038.9	25.5	1132.0	1168.9	28.5
1080.0	1116.5	27.5	1198.0	1242.9	30.5
1151.0	1192.6	29.5	1267.0	1315.1	32.5
1225.0	1267.1	31.5	1337.0	1385.6	34.5
1301.0	1340.0	33.5	1411.0	1454.3	36.5
1381.0	1411.2	35.5	1489.0	1521.2	38.5
1464.0	1480.6	37.5	1570.0	1586.3	40.5
1550.0	1548.4	39.5	1655.0	1649.5	42.5
1640.0	1614.4	39.5	1743.0	1710.8	44.5
1732.0	1678.6	41.5	1833.0	1770.4	46.5
1822.0	1741.0	43.5	1928.0	1828.1	48.5
	$\sigma = 2.87x10^{-2}$			$\sigma = 3.32x10^{-2}$	

Table 22. Calculation of the transition energies in Pr^{133}(b1) and Sm^{137}(b1).

	Pr^{133}(b1)			Sm^{137}(b1)	
E_γ(I) (KeV) Exp[a]	**Cal[b]**	**Assigned I**	**E_γ(I) (KeV) Exp[a]**	**Cal[b]**	**Assigned I**
871.0	903.1	22.5	379.0	425.2	11.5
943.0	979.7	24.5	451.0	501.4	13.5
1013.0	1055.0	26.5	538.5	576.9	15.5
1085.0	1129.0	28.5	629.1	651.1	17.5
1156.0	1201.5	30.5	719.7	726.1	19.5
1228.0	1272.5	32.5	806.3	799.5	21.5
1299.0	1342.0	34.5	890.0	872.1	23.5
1380.0	1409.9	36.5	970.0	943.7	25.5
1450.0	1476.2	38.5	1045.0	1014.4	27.5
1530.0	1540.9	40.5	1115.0	1084.0	29.5
σ = 6.79x10-3				$\sigma = 6.06x10^{-2}$	

Table 23. Calculation of the transition energies in Pr^{133}(b2) and Ce^{131}(b2).

Pr^{133}(b2)			Ce^{131}(b2)		
$E_\gamma(I)$ (KeV) Exp[a]	Cal[b]	Assigned I	$E_\gamma(I)$ (KeV) Exp[a]	Cal[b]	Assigned I
821.0	837.6	19.5	847.0	818.9	20.5
885.0	920.7	21.5	908.0	898.4	22.5
961.0	1002.4	23.5	976.0	976.7	24.5
1035.0	1082.7	25.5	1043.0	1053.7	26.5
1110.0	1161.3	27.5	1112.0	1129.3	28.5
1184.0	1238.4	29.5	1181.0	1203.4	30.5
1264.0	1313.7	31.5	1251.0	1276.1	32.5
1342.0	1387.3	33.5	1322.0	1347.2	34.5
1420.0	1459.0	35.5	1396.0	1416.7	36.5
1500.0	1529.0	37.5	1471.0	1484.7	38.5
1576.0	1597.1	39.5	1552.0	1550.9	40.5
1656.0	1663.4	41.5	1635.0	1615.6	42.5
			1723.5	1678.5	44.5
	$\sigma = 1.33 \times 10^{-2}$			$\sigma = 1.82 \times 10^{-2}$	

Table 24. Calculation of the transition energies in Ce^{133}(b2) and Eu^{143}(b1).

Ce^{133}(b2)			Eu^{143}(b1)		
$E_\gamma(I)$ (KeV) Exp[a]	Cal[b]	Assigned I	$E_\gamma(I)$ (KeV) Exp[a]	Cal[b]	Assigned I
720.3	683.4	16.5	483.0	470.1	14.5
785.4	765.2	18.5	546.5	536.1	16.5
854.2	845.8	20.5	609.0	601.6	18.5
920.3	925.0	22.5	671.2	666.7	20.5
986.9	1002.8	24.5	732.6	731.1	22.5
1052.2	1079.2	26.5	793.4	795.0	24.5
1118.2	1153.9	28.5	853.5	858.3	26.5
1184.3	1227.1	30.5	912.8	920.9	28.5
1253.2	1298.5	32.5	972.2	982.7	30.5
1323.9	1368.3	34.5	1031.1	1043.8	32.5
1397.9	1436.2	36.5	1089.9	1104.0	34.5
1475.5	1502.4	38.5	1148.1	1163.5	36.5
1557.3	1566.8	40.5	1206.5	1222.1	38.5
1643.0	1629.5	42.5	1265.5	1279.8	40.5
1731.3	1690.3	44.5	1324.1	1336.7	42.5
1821.6	1749.3	46.5	1383.2	1392.6	44.5
			1442.7	1447.5	46.5
			1502.2	1501.6	48.5
			1562.3	1554.6	50.5
			1623.5	1606.7	52.5
			1684.0	1657.8	54.5
			1743.0	1707.9	56.5
	$\sigma = 2.78 \times 10^{-2}$			$\sigma = 1.21 \times 10^{-2}$	

bands that the band head moment of inertia, $j_0 = \hbar^2/ab$, are close to each other, $j_0 \sim$ (25-60) $\hbar^2/MeV^{-1}$ and no evidence for odd-even difference is found. Moreover, the smallness of parameter b [~(1 : 3) x 10^{-4}, (Table 1)] implies the super-rigidity of these SD nuclei. The radius of convergence I_r for the I(I+1) expansion of Equation (1) can be estimated as

$I_r \approx 1/\sqrt{b} \approx 58\text{-}100$.

Conflict of Interests

The author(s) have not declared any conflict of interests.

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