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Matrices of the thermal and spectral variations for the fabrication materials based arrayed waveguide grating (awg) devices

Abd El-Nasar A. Mohammed*, Abd El-Fattah A. Saad and Ahmed Nabih Zaki Rashed

Electronics and Electrical Communication Engineering Department, Faculty of Electronic Engineering, Menouf 32951, Menoufia University, Egypt.

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In the present paper, we have investigated the basic forms of the matrices of the thermal and spectral variations of the refractive-index (RI) for three different fabrication materials such as lithium niobate, polymethyl metha acrylate (PMMA) and silica-doped that are based arrayed waveguide grating (AWG) device depends on curve fitting program. Then, we are able to study the transmission spectrum characteristics of arrayed waveguide grating (AWG) device in the mathematical matrix form through the wavelength and temperature ranges.

Key words: Arrayed waveguide gratings (AWGs), integrated optics, modeling, optical planar waveguide.

INTRODUCTION

The use of waveguide division multiplexing systems is increasing rapidly and in these systems (Hibino, 2002), arrayed-waveguide gratings (AWGs) play important roles as multiplexer/demultiplexers (Hida, 2004). They offer compactness, high stability, excellent optical characteristics and mass producibility (Noguchi et al., 2004). Until now, AWGs have been developed for telecommunication applications, so their wavelength range has been limited to 1.3 - 1.6 μm (Takada et al., 1999) However, for novel applications such as sensors (Sano et al., 2003), we need AWGs with a shorter wavelength range including the visible wavelength range (Komai, et al., 2004). This is because many materials and analytes have specific characteristics at these wavelengths. Until now, only theoretical consideration has been given to AWGs operating in the visible wavelength range (Przyrembel and Kuhlow, 2004; Nakajima et al., 2005). One of the key advantages of AWGs is their ability to provide the fine wavelength resolution required for optical spectroscopic sensors designed to identify materials and analyses (Okamoto, 2006). In the present study, we have analyzed the matrices of the thermal and spectral variations of the refractive

index (RI) for three different fabrication materials such as lithium niobate, polymethyl metha acrylate (PMMA) and silica-doped that are based arrayed waveguide grating (AWG) device depends on fitting program model.

THEORETICAL MODEL AND GOVERNING EQUATIONS ANALYSIS

Lithium niobate (LiNbO₃) material

The investigation of both the thermal and spectral variations of the waveguide refractive index (n) requires Sellmeier equation. The set of parameters required to completely characterize the temperature dependence of the refractive-index (n) is given below, Sellmeier equation is under the form (Jundt, 1997):

$$n^2 = A_1 + A_2H + \frac{A_3 + A_4H}{\lambda^2 - (A_5 + A_6H)^2} + \frac{A_7 + A_8H}{\lambda^2 - A_9^2} - A_{10}\lambda^2 \quad (1)$$

Where λ is the optical wavelength in μm and $H = T^2 - T_0^2$. T is the temperature of the material in K and T_0 is the reference temperature and is considered as 300 K. The set of parameters of Sellmeier equation coefficients (LiNbO₃) are recast as below:

$A_1=5.35583$, $A_2=4.629 \times 10^{-7}$, $A_3=0.100473$, $A_4=3.862 \times 10^{-8}$, $A_5=0.20692$, $A_6= -0.89 \times 10^{-8}$, $A_7=100$, $A_8=2.657 \times 10^{-5}$, $A_9=11.34927$, and $A_{10}=0.01533$.

*Corresponding author. E-mail: abd_elnaser6@yahoo.com.

Equation (1) can be simplified as the following:

$$n^2 = A_{12} + \frac{A_{34}}{\lambda^2 - A_{56}^2} + \frac{A_{78}}{\lambda^2 - A_9^2} - A_{10}\lambda^2 \quad (2)$$

Where:

$$A_{12} = A_1 + A_2H, A_{34} = A_3 + A_4H, A_{56} = A_5 + A_6H, \text{ and } A_{78} = A_7 + A_8H$$

Then, the differentiation w. r. t λ gives:

$$\frac{dn}{d\lambda} = \left(\frac{-\lambda}{n} \right) \left[\frac{A_{34}}{(\lambda^2 - A_{56}^2)^2} + \frac{A_{78}}{(\lambda^2 - A_9^2)^2} + A_{10} \right] \quad (3)$$

Also, the differentiation w. r. t T gives:

$$\frac{dn}{dT} = \left(\frac{T}{n} \right) \left[A_2 + \frac{(\lambda^2 - A_{56}^2)A_4 + 2A_6A_{56}A_{34}}{(\lambda^2 - A_{56}^2)^2} + \frac{A_8}{(\lambda^2 - A_9^2)} \right] \quad (4)$$

Polymethyl metha acrylate (PMMA) polymer material

The refractive index of this fabrication material is cast as (Ishigure et al., 1996):

$$n^2 = 1 + \frac{C_1\lambda^2}{\lambda^2 - C_2^2} + \frac{C_3\lambda^2}{\lambda^2 - C_4^2} + \frac{C_5\lambda^2}{\lambda^2 - C_6^2} \quad (5)$$

The set of parameters of Sellmeier equation coefficients (PMMA) are recast as below:

$$C_1 = 0.4963, C_2 = 0.0718, C_3 = 0.6965, C_4 = 0.1174, C_5 = 0.3223, \text{ and } C_6 = 0.9237.$$

The differentiation of Eq. (5) w. r. t λ gives:

$$\frac{dn}{d\lambda} = -(\lambda/n)$$

$$\left[\frac{C_1C_2^2}{(\lambda^2 - C_2^2)^2} + \frac{C_3C_4^2}{(\lambda^2 - C_4^2)^2} + \frac{C_5C_6^2}{(\lambda^2 - C_6^2)^2} \right] \quad (6)$$

Also, the differentiation w. r. t T yields:

$$\frac{dn}{dT} = \left(-\lambda^2/n \right) \left[\frac{C_1C_2}{(\lambda^2 - C_2^2)^2} \frac{\partial C_2}{\partial T} + \frac{C_3C_4}{(\lambda^2 - C_4^2)^2} \frac{\partial C_4}{\partial T} + \frac{C_5C_6}{(\lambda^2 - C_6^2)^2} \frac{\partial C_6}{\partial T} \right] \quad (7)$$

Germania doped Silica (GeO₂(x) + SiO₂(1-x)) material

The refractive index of this fabrication material is cast as (Fleming, 1985):

$$n^2 = 1 + \frac{B_1\lambda^2}{\lambda^2 - B_2^2} + \frac{B_3\lambda^2}{\lambda^2 - B_4^2} + \frac{B_5\lambda^2}{\lambda^2 - B_6^2} \quad (8)$$

The Sellemier coefficients as a function of temperature, and germania mole fraction (x) as:

$$\begin{aligned} B_1 &= 0.691663 + 0.1107001 * x, \\ B_2 &= (0.0684043 + 0.000568306 * x)^2 * (T/T_0)^2 \\ B_3 &= 0.4079426 + 0.31021588 * x, \\ B_4 &= (0.1162414 + 0.03772465 * x)^2 * (T/T_0)^2 \\ B_5 &= 0.8974749 - 0.043311091 * x, \\ B_6 &= (9.896161 + 1.94577 * x)^2 * (T/T_0)^2 \end{aligned}$$

The differentiation of Eq. (8) w. r. t λ gives:

$$\frac{dn}{d\lambda} = -(\lambda/n) \left[\frac{B_1B_2^2}{(\lambda^2 - B_2^2)^2} + \frac{B_3B_4^2}{(\lambda^2 - B_4^2)^2} + \frac{B_5B_6^2}{(\lambda^2 - B_6^2)^2} \right] \quad (9)$$

Also, the differentiation w. r. t T yields:

$$\frac{dn}{dT} = \left(-\lambda^2/n \right) \left[\frac{B_1B_2}{(\lambda^2 - B_2^2)^2} \frac{\partial B_2}{\partial T} + \frac{B_3B_4}{(\lambda^2 - B_4^2)^2} \frac{\partial B_4}{\partial T} + \frac{B_5B_6}{(\lambda^2 - B_6^2)^2} \frac{\partial B_6}{\partial T} \right] \quad (10)$$

RESULTS

As shown in Figures 1, 2 and 3, we can fit the thermal variations of the refractive-index for three different fabrication materials based on fitting program model (Tatian, 1984).

Lithium niobate (LiNbO₃) material

The variation of the refractive-index w. r. t temperature can be expressed as:

$$\frac{dn}{dT} = \sum_{i=0}^2 a_i T^i \quad (11)$$

Where;

$$a_i = \sum_{j=0}^2 b_j \lambda^j \quad (12)$$

Equation (11) can be expressed in the form of three terms as:

$$\frac{dn}{dT} = a_0 + a_1T + a_2T^2 \quad (13)$$

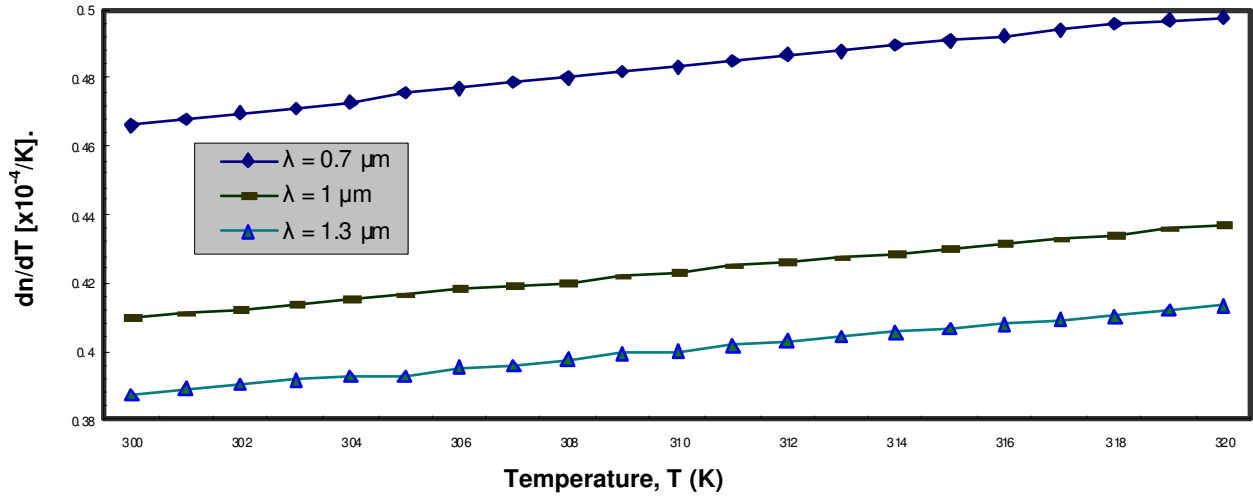


Figure 1. Variation of dn/dT versus temperature for LiNbO₃ material.

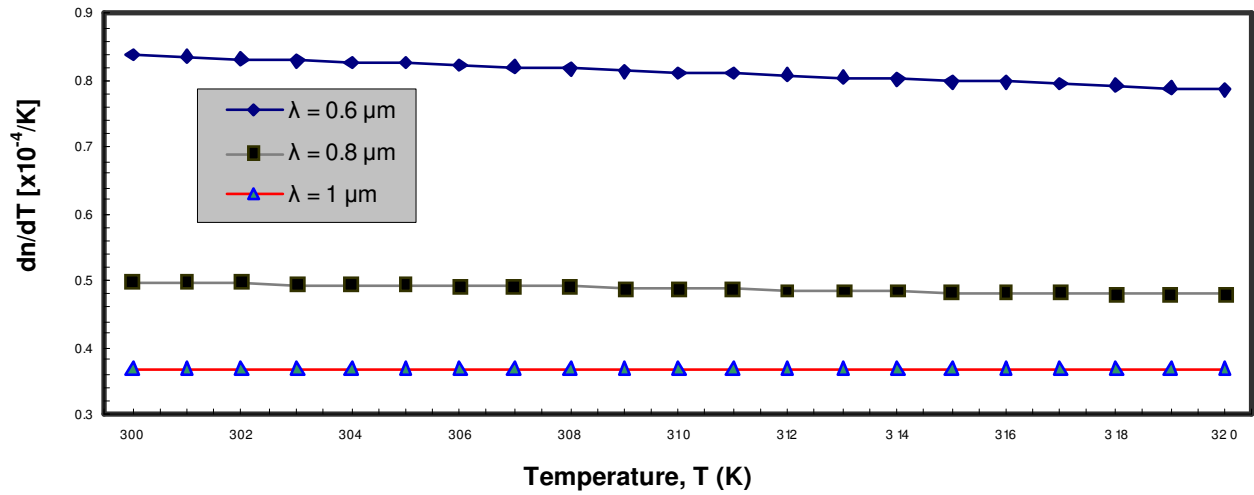


Figure 2. Variation of dn/dT versus temperature for PMMA Polymer material.

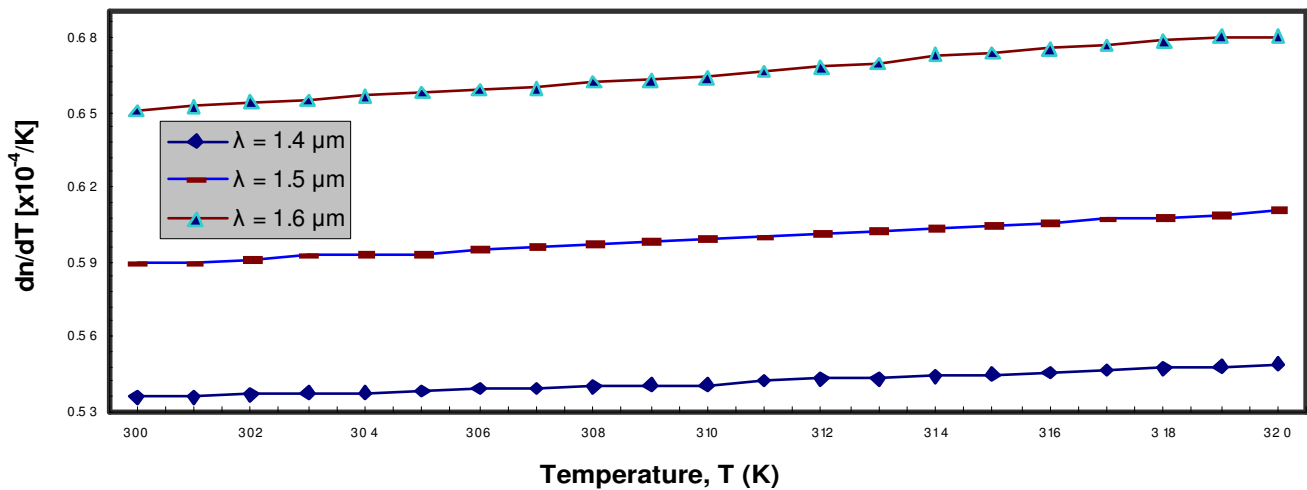


Figure 3. Variation of dn/dT versus temperature for silica-doped material.

The three thermal coefficients (a_0, a_1, a_2) of LiNbO₃ material can be expressed as a function of spectral wavelength:

$$a_0 = b_{01} + b_{11}\lambda + b_{21}\lambda^2 \quad (\text{at 300 K}) \quad (14)$$

$$a_1 = b_{02} + b_{12}\lambda + b_{22}\lambda^2 \quad (\text{at 310 K}) \quad (15)$$

$$a_2 = b_{03} + b_{13}\lambda + b_{23}\lambda^2 \quad (\text{at 320 K}) \quad (16)$$

Then we can put the thermal coefficients in the matrix form as the following:

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} b_{01} & b_{11} & b_{21} \\ b_{02} & b_{12} & b_{22} \\ b_{03} & b_{13} & b_{23} \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix} \quad (17)$$

Then the coefficients at the different temperatures of LiNbO₃ material are obtained based on curve fitting program (Tatian, 1984):

$$\begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 0.95 \times 10^{-4} & -0.9 \times 10^{-4} & 0.34 \times 10^{-4} \\ 0.99 \times 10^{-4} & -0.95 \times 10^{-4} & 0.37 \times 10^{-4} \\ 1.02 \times 10^{-3} & -0.98 \times 10^{-4} & 0.38 \times 10^{-4} \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix} \quad (18)$$

Polymethyl metha acrylate (PMMA) polymer material

In the same way, the three thermal coefficients (c_0, c_1, c_2) of PMMA material can be expressed as a function of spectral wavelength based on fitting program (Tatian, 1984):

$$c_0 = d_{01} + d_{11}\lambda + d_{21}\lambda^2 \quad (\text{at 300 K}) \quad (19)$$

$$c_1 = d_{02} + d_{12}\lambda + d_{22}\lambda^2 \quad (\text{at 310 K}) \quad (20)$$

$$c_2 = d_{03} + d_{13}\lambda + d_{23}\lambda^2 \quad (\text{at 320 K}) \quad (21)$$

Then we can put the thermal coefficients in the matrix form as the following:

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} d_{01} & d_{11} & d_{21} \\ d_{02} & d_{12} & d_{22} \\ d_{03} & d_{13} & d_{23} \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix} \quad (22)$$

Then the coefficients at the different temperatures of PMMA material are obtained based on fitting program model (Tatian, 1984):

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0.42 \times 10^{-3} & -0.78 \times 10^{-3} & 0.39 \times 10^{-3} \\ 0.40 \times 10^{-3} & -0.75 \times 10^{-3} & 0.37 \times 10^{-3} \\ 0.39 \times 10^{-3} & -0.72 \times 10^{-3} & 0.36 \times 10^{-3} \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix} \quad (23)$$

Silica-doped material

Also in the same way, the three thermal coefficients (h_0, h_1, h_2) of silica-doped material can be expressed as a function of spectral wavelength based on fitting program (Tatian, 1984):

$$h_0 = z_{01} + z_{11}\lambda + z_{21}\lambda^2 \quad (\text{at 300 K}) \quad (24)$$

$$h_1 = z_{02} + z_{12}\lambda + z_{22}\lambda^2 \quad (\text{at 310 K}) \quad (25)$$

$$h_2 = z_{03} + z_{13}\lambda + z_{23}\lambda^2 \quad (\text{at 320 K}) \quad (26)$$

Then we can put the thermal coefficients in the matrix form as the following:

$$\begin{bmatrix} h_0 \\ h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} z_{01} & z_{11} & z_{21} \\ z_{02} & z_{12} & z_{22} \\ z_{03} & z_{13} & z_{23} \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix} \quad (27)$$

Then the coefficients at the different temperatures of silica-doped material are obtained based on fitting program model (Tatian, 1984):

$$\begin{bmatrix} h_0 \\ h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} 0.66 \times 10^{-4} & -0.67 \times 10^{-4} & 0.41 \times 10^{-4} \\ 0.58 \times 10^{-4} & -0.58 \times 10^{-4} & 0.39 \times 10^{-4} \\ 0.57 \times 10^{-4} & -0.57 \times 10^{-4} & 0.38 \times 10^{-4} \end{bmatrix} \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix} \quad (28)$$

As shown in Figures 4, 5 and 6, we can fit the spectral variations of the refractive-index for three different fabrication materials based on fitting program model (Tatian, 1984).

Lithium niobate (LiNbO₃) material

The variation of the refractive-index w. r. t wavelength can be expressed as:

$$\frac{dn}{d\lambda} = \sum_{i=0}^2 m_i \lambda^i \quad (29)$$

$$\text{Where } m_i = \sum_{j=0}^2 q_j T^j \quad (30)$$

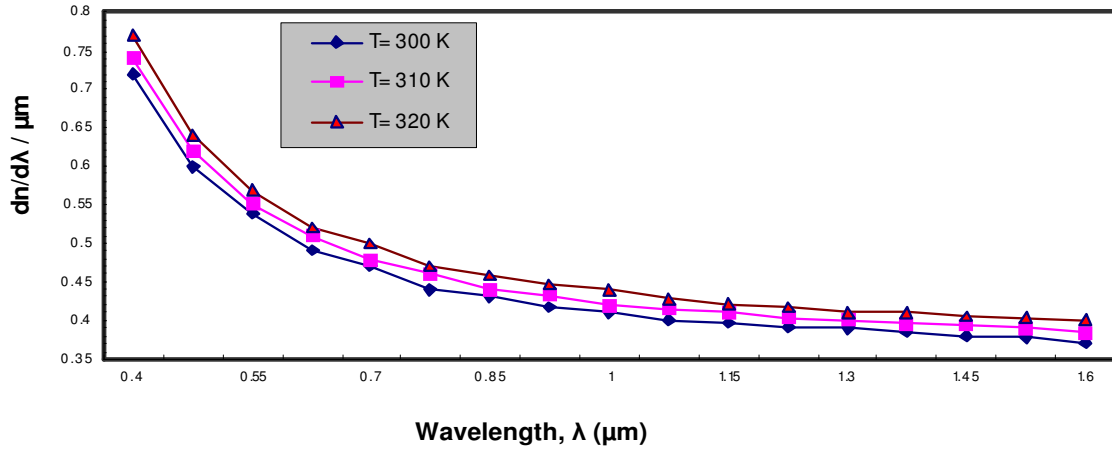


Figure 4. Variation of $dn/d\lambda$ versus wavelength for LiNbO_3 material.

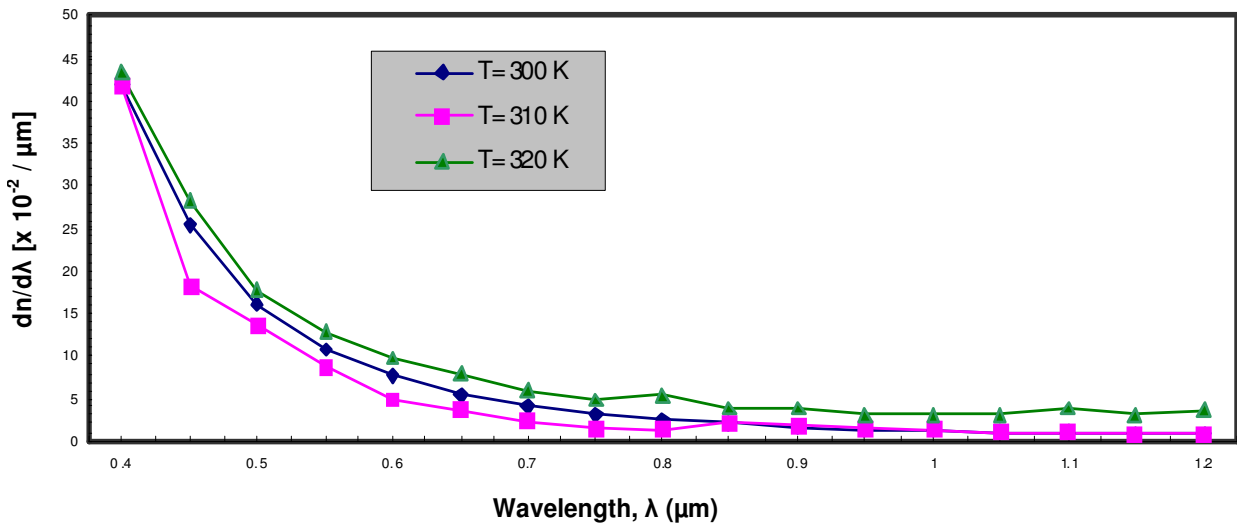


Figure 5. Variation of $dn/d\lambda$ versus wavelength for PMMA Polymer material.

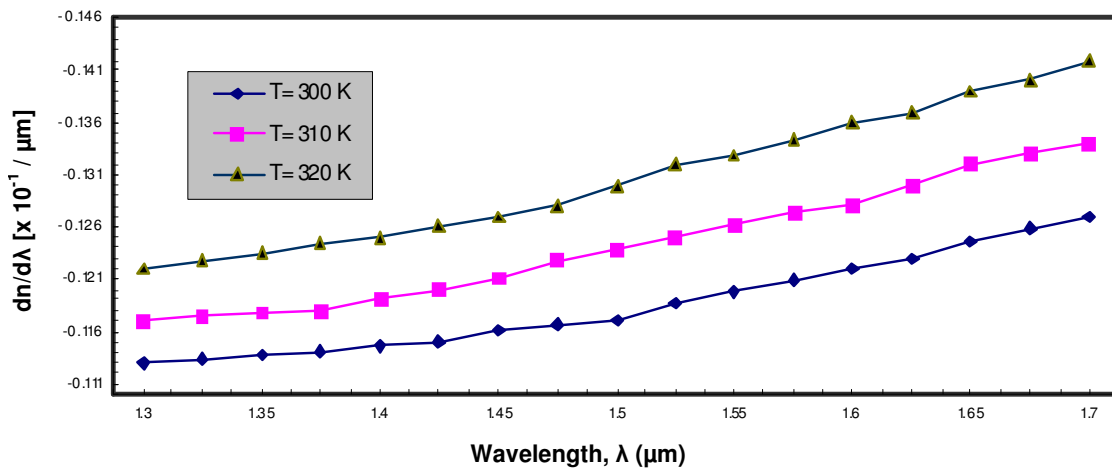


Figure 6. Variation of $dn/d\lambda$ versus wavelength for Silica-doped material.

Equation (29) can be expressed in the form of three terms as:

$$\frac{dn}{d\lambda} = m_0 + m_1\lambda + m_2\lambda^2 \quad (31)$$

The three spectral coefficients (m_0, m_1, m_2) of LiNbO₃ material can be expressed as a function of temperature based on fitting program:

$$m_0 = q_{01} + q_{11}T + q_{21}T^2 \quad (\text{at } \lambda = 0.7 \mu\text{m}) \quad (32)$$

$$m_1 = q_{02} + q_{12}T + q_{22}T^2 \quad (\text{at } \lambda = 1.0 \mu\text{m}) \quad (33)$$

$$m_2 = q_{03} + q_{13}T + q_{23}T^2 \quad (\text{at } \lambda = 1.3 \mu\text{m}) \quad (34)$$

Then we can put the spectral coefficients in the matrix form as the following:

$$\begin{bmatrix} m_0 \\ m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} q_{01} & q_{11} & q_{21} \\ q_{02} & q_{12} & q_{22} \\ q_{03} & q_{13} & q_{23} \end{bmatrix} \begin{bmatrix} 1 \\ T \\ T^2 \end{bmatrix} \quad (35)$$

Then the coefficients at the different wavelengths of LiNbO₃ material are obtained based on curve fitting program (Tatian, 1984):

$$\begin{bmatrix} m_0 \\ m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} 5.8 & -0.45 \times 10^{-1} & 0.73 \times 10^{-4} \\ -0.74 & 0.44 \times 10^{-2} & -0.71 \times 10^{-5} \\ 0.42 & -0.29 \times 10^{-2} & 0.47 \times 10^{-5} \end{bmatrix} \begin{bmatrix} 1 \\ T \\ T^2 \end{bmatrix} \quad (36)$$

Polymethyl metha acrylate (PMMA) polymer material

In the same way, the three spectral coefficients (w_0, w_1, w_2) of PMMA material can be expressed as a function of temperature based on fitting program:

$$w_0 = s_{01} + s_{11}T + s_{21}T^2 \quad (\text{at } \lambda = 0.6 \mu\text{m}) \quad (37)$$

$$w_1 = s_{02} + s_{12}T + s_{22}T^2 \quad (\text{at } \lambda = 0.8 \mu\text{m}) \quad (38)$$

$$w_2 = s_{03} + s_{13}T + s_{23}T^2 \quad (\text{at } \lambda = 1.0 \mu\text{m}) \quad (39)$$

Then we can put the spectral coefficients in the matrix form as the following:

$$\begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} s_{01} & s_{11} & s_{21} \\ s_{02} & s_{12} & s_{22} \\ s_{03} & s_{13} & s_{23} \end{bmatrix} \begin{bmatrix} 1 \\ T \\ T^2 \end{bmatrix} \quad (40)$$

Then the coefficients at the different wavelengths of PMMA material are obtained based on fitting program model (Tatian, 1984):

$$\begin{bmatrix} w_0 \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 34.5 & -0.22 & 0.35 \times 10^{-3} \\ -1.05 & 0.69 \times 10^{-2} & -0.11 \times 10^{-4} \\ 9.24 & -0.59 \times 10^{-1} & 0.95 \times 10^{-4} \end{bmatrix} \begin{bmatrix} 1 \\ T \\ T^2 \end{bmatrix} \quad (41)$$

Silica-doped material

Also in the same way, the three spectral coefficients (p_0, p_1, p_2) of silica-doped material can be expressed as a function of temperature based on fitting program (Tatian, 1984):

$$p_0 = f_{01} + f_{11}T + f_{21}T^2 \quad (\text{at } \lambda = 1.4 \mu\text{m}) \quad (42)$$

$$p_1 = f_{02} + f_{12}T + f_{22}T^2 \quad (\text{at } \lambda = 1.5 \mu\text{m}) \quad (43)$$

$$p_2 = f_{03} + f_{13}T + f_{23}T^2 \quad (\text{at } \lambda = 1.6 \mu\text{m}) \quad (44)$$

Then we can put the spectral coefficients in the matrix form as the following:

$$\begin{bmatrix} p_0 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} f_{01} & f_{11} & f_{21} \\ f_{02} & f_{12} & f_{22} \\ f_{03} & f_{13} & f_{23} \end{bmatrix} \begin{bmatrix} 1 \\ T \\ T^2 \end{bmatrix} \quad (45)$$

Then the coefficients at the different wavelengths of silica-doped material are obtained based on curve fitting program (Tatian, 1984):

$$\begin{bmatrix} p_0 \\ p_1 \\ p_2 \end{bmatrix} = \begin{bmatrix} 0.85 & -0.55 \times 10^{-2} & 0.88 \times 10^{-5} \\ -0.25 & 0.16 \times 10^{-2} & -0.27 \times 10^{-5} \\ 0.71 & -0.46 \times 10^{-2} & 0.73 \times 10^{-5} \end{bmatrix} \begin{bmatrix} 1 \\ T \\ T^2 \end{bmatrix} \quad (46)$$

Conclusion

In summary, both the thermal and spectral variations of refractive index for different fabrication materials based arrayed waveguide grating (AWG) device are deeply and parametrically investigated. Refractive-index (RI) variations w. r. t temperature and spectral wavelength for three different fabrication materials are also investigated in the matrix form based on Sellmeier equation. Moreover positive correlations or negative correlations or both are found in the matrix form. In this study, we are able to study the transmission spectrum behavior of the AWG device in the matrix form within the allowable wavelength (0.6 - 1.6 μm) and temperature ranges from 300 - 320 K.

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