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Full Length Research Paper

Adaptive control of the generalized Korteweg-de Vries-Burgers equation

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In this paper, the adaptive and non-adaptive stabilization of the generalized Korteweg-de Vries (KdV)-Burgers equation by nonlinear boundary control are analyzed. This is motivated by the method proposed by Smaoui (2004). We use this method to resolve the adaptive control of the generalized KdV-Burgers equation. For the non-adaptive case, using Lyapunov method, we show that the controlled system is exponentially stable in L^2. As for the adaptive case, we show the L^2 regulation of the solution of the generalized KdV-Burgers equation.

Key words: Adaptive control, KdV-Burgers equation, stabilization.

INTRODUCTION

In recent year, extensive attention has been paid to the problems of control and stabilization for the KdV equation, Burgers equation and KdV-Burgers equation, where most of these studies involved non-adaptive control (Balogh and Krstic, 2000; Burns and Kang, 1992; Byrnes et al., 1998; Krstic, 1999; Lixin and Xiaoyan, 2003; Rosier, 1997; Russell and Bingyu, 1996). Adaptive control was also used to investigate different distributed parameter systems (Smaoui, 2004; Weijiu and Krstic, 2001; Xiaoyan et al., 2009). However, adaptive control methods have been developed only for a few of the classes of partial differential equations for which non-adaptive controllers exist. The KdV equation and KdV-Burgers equation are nonlinear mathematical models that incorporate effects of both dispersion and dissipation. They serve as models of long waves in shallow water and some other physical phenomena.

The main difference between adaptive control and non-adaptive control is that in adaptive control, good control performance can be directly achieved even in the presence of undesirable or unpredictable disturbances. Here, we focus on the adaptive control. Nonlinear boundary control laws that achieve global asymptotic stability were derived by Krstic (1999) for both the viscous and inviscid Burgers equation, and in 2001, adaptive control of Burgers equation with unknown viscosity was investigated by Weijiu and Krstic (2001) to regulate the solution of the closed-loop system to zero in L^2 sense using an extension to Barbalat's lemma. In 2004, adaptive control of the generalized Burgers equation was studied by Smaoui (2004). The adaptive control of the generalized KdV-Burgers equation seems not yet to have been discussed. By Lyapunov method and applying the proved lemmas 1 to 3, more elegant than the Barbalat's lemma, we get the adaptive control of the generalized KdV-Burgers equation.

In this paper, we consider the adaptive and non-adaptive control of the generalized Korteweg-de Vries-Burgers (KdVB) equation:

$$u_t - \varepsilon u_{xx} + \delta u_{xxx} + uu_x - mu = 0 \qquad x \in (0, 2\pi), \quad t > 0, \tag{1}$$

subject to:

$$au_x(0,t) + bu(0,t) = \omega_1(t), \quad cu_x(2\pi,t) + du(2\pi,t) = \omega_2(t) \tag{2}$$

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$$\delta u(0,t)u_{xx}(0,t)-\frac{\delta}{2}u_x^2(0,t)=0,$$

$$\delta u(2\pi,t)u_{xx}(2\pi,t)-\frac{\delta}{2}u_x^2(2\pi,t)=0. \quad (3)$$

where ε is a positive constant, m is a small positive constant, a, b, c and d are all constants, $\omega_1(t)$ and $\omega_2(t)$ are inputs and $u(0,t)$ and $u(2\pi,t)$ are outputs. This paper is organized as follows: the global exponential stability in L^2 for the system (1)-(3) when non-adaptive control is used was shown (that is, when ε, a,b,c and d are known), L^2 regulation of the solution of the generalized KdV-Burgers system was established in the case of adaptive control (that is, where ε, a,b,c and d are unknown) using an approach that used in Smaoui (2004), and finally, conclusion was presented.

THE NON-ADAPTIVE CASE

Theorem 1

The generalized KdV-Burgers equation given in the system of Equations (1)-(3) with $\varepsilon > 8m\pi^2$ is globally exponentially stable in $L^2(0,2\pi)$ under the following control laws:

$$\omega_1(t)=ak_1u(0,t)+(b+\frac{a}{2\pi})u(0,t)+\frac{a}{3\varepsilon}u^2(0,t),\quad k_1\geq 0$$

$$\omega_2(t)=-ck_2u(2\pi,t)+du(2\pi,t)+\frac{c}{3\varepsilon}u^2(2\pi,t),\quad k_2\geq 0. \quad (4)$$

Proof

We start our analysis from the Lyapunov function:

$$V(t)=\frac{1}{2}\int_0^{2\pi}u^2(x,t)dx.$$

Taking the time derivative of $V(t)$, we get:

$$\dot{V}(t)=\frac{1}{2}\int_0^{2\pi}2u(x,t)u_t(x,t)dx=\int_0^{2\pi}u(x,t)\{\varepsilon u_{xx}(x,t)-\delta u_{xxx}(x,t)-u(x,t)u_x(x,t)+mu(x,t)\}dx$$
$$=\varepsilon\int_0^{2\pi}u(x,t)u_{xx}(x,t)dx-\int_0^{2\pi}\left(\frac{1}{3}u^3(x,t)\right)_x dx+m\int_0^{2\pi}u^2(x,t)dx$$
$$-\delta\int_0^{2\pi}u(x,t)u_{xxx}(x,t)dx. \quad (5)$$

Using integration by parts, we get:

$$\dot{V}(t)=\varepsilon u(2\pi,t)u_x(2\pi,t)-\varepsilon u(0,t)u_x(0,t)-\frac{1}{3}u^3(2\pi,t)+\frac{1}{3}u^3(0,t)+m\int_o^{2\pi}u^2(x,t)dx$$
$$-\varepsilon\int_0^{2\pi}u_x^2(x,t)dx-\delta u(2\pi,t)u_{xx}(2\pi,t)+\delta u(0,t)u_{xx}(0,t)+\frac{\delta}{2}u_x^2(x,t)-\frac{\delta}{2}u_x^2(0,t). \quad (6)$$

Using Poincare inequality (Adams, 1975) on the term $\varepsilon\int_0^{2\pi}u_x^2(x,t)dx$,

$$\dot{V}(t)\leq\varepsilon u(2\pi,t)u_x(2\pi,t)-\varepsilon u(0,t)u_x(0,t)-\frac{1}{3}u^3(2\pi,t)+\frac{1}{3}u^3(0,t)+(m-\frac{\varepsilon}{8\pi^2})\int_o^{2\pi}u^2(x,t)dx$$
$$+\frac{\varepsilon}{2\pi}u^2(0,t)-\delta u(2\pi,t)u_{xx}(2\pi,t)+\delta u(0,t)u_{xx}(0,t)+\frac{\delta}{2}u_x^2(x,t)-\frac{\delta}{2}u_x^2(0,t). \quad (7)$$

Now, using the boundary condition stated in Equations (2)-(3), Equation (7) becomes:

$$\dot V(t) \le (m - \frac{\varepsilon}{8\pi^2})\int_0^{2\pi} u^2(x,t)dx - \varepsilon u(0,t)\left[\frac{1}{a}\omega_1(t) - \left(\frac{b}{a}+\frac{1}{2\pi}\right)u(0,t) - \frac{1}{3\varepsilon}u^2(0,t)\right]$$
$$+ \varepsilon u(2\pi,t)\left[\frac{1}{c}\omega_2(t) - \frac{d}{c}u(2\pi,t) - \frac{1}{3\varepsilon}u^2(2\pi,t)\right]. \quad (8)$$

If we apply the following control law:

$$\omega_1(t) = ak_1u(0,t) + \left(b+\frac{a}{2\pi}\right)u(0,t) + \frac{a}{3\varepsilon}u^2(0,t) \qquad k_1 \ge 0$$

$$\omega_2(t) = -ck_2u(2\pi,t) + du(2\pi,t) + \frac{c}{3\varepsilon}u^2(2\pi,t) \qquad k_2 \ge 0 \quad (9)$$

Then, Equation (8) can be rewritten as:

$$\dot V(t) \le (m - \frac{\varepsilon}{8\pi^2})\int_0^{2\pi} u^2(x,t)dx - \varepsilon\left[k_1u^2(0,t) + k_2u^2(2\pi,t)\right], \quad (10)$$

which implies that:

$$\dot V(t) \le (m - \frac{\varepsilon}{8\pi^2})\int_0^{2\pi} u^2(x,t)dx. \quad (11)$$

Letting $\alpha = m - \frac{\varepsilon}{8\pi^2}$, then $\dot V(t) \le 2\alpha V(t)$, that is, $V(t) \le const \cdot e^{2\alpha t}$. Therefore, if $\alpha < 0$ or $\varepsilon > 8m\pi^2$, then $V(t)$ converges to zero exponentially as t tends to ∞.

THE ADAPTIVE CASE

Here, an adaptive regulator design is constructed for the generalized KdV-Burgers equation given in the system of Equations (1)-(3), where ε, a, b, c and d are unknowns. The L^2 regulation of Burgers equation was proved by Weijiu and Krstic (2001) using an extension to Barbalat's lemma, and in Smaoui (2004) applied a novel approach that seems to be more elegant to show the L^2 regulation of the generalized Burgers equation. Here, we use the approach used in Smaoui (2004) to show the L^2 regulation of the generalized KdV-Burgers equation, we first state the following lemmas:

Lemma 1

Let $\alpha < 0$, if $u(x,t) \in L^2(0,\infty)$, then:

$$\int_0^t e^{\alpha(t-\tau)}u^2(0,\tau)d\tau \to 0 \quad \text{as } t \to \infty. \quad (12)$$

Proof

$$\int_0^t e^{\alpha(t-\tau)}u^2(0,\tau)d\tau \le \int_0^{\frac{t}{2}} e^{\alpha(t-\tau)}u^2(0,\tau)d\tau + \int_{\frac{t}{2}}^t e^{\alpha(t-\tau)}u^2(0,\tau)d\tau \quad (13)$$

Setting $s = t - \tau$ on the right-hand side of the first integral term of Equation (13), we get:

$$\int_0^t e^{\alpha(t-\tau)}u^2(0,\tau)d\tau \le \int_{\frac{t}{2}}^t e^{\alpha s}u^2(0,t-s)ds + \int_{\frac{t}{2}}^t e^{\alpha(t-\tau)}u^2(0,\tau)d\tau \quad (14)$$

Consequently, the first term on the right-hand side of Equation (14) can be estimated by the following:

$$\int_{\frac{t}{2}}^t e^{\alpha s}u^2(0,t-s)ds \le \max_{\frac{t}{2}\le s\le t}[e^{\alpha s}] \cdot \int_{\frac{t}{2}}^t u^2(0,t-s)ds \quad (15)$$

Since $\alpha < 0$, then:

$$\max_{\frac{t}{2}\le\tau\le t}[e^{\alpha\tau}] = e^{\frac{\alpha t}{2}}. \quad (16)$$

Hence, the first term on the right-hand side of Equation (14) can be written as:

$$\int_{\frac{t}{2}}^t e^{\alpha\tau}u^2(0,t-\tau)d\tau \le e^{\frac{\alpha t}{2}} \cdot \int_{\frac{t}{2}}^t u^2(t-\tau)d\tau \quad (17)$$

Similarly, we can estimate the second term on the right-hand side of Equation (14) by the following:

$$\int_{\frac{t}{2}}^t e^{\alpha(t-\tau)}u^2(0,\tau)d\tau \le \max_{\frac{t}{2}\le\tau\le t}[e^{\alpha(t-\tau)}] \cdot \int_{\frac{t}{2}}^t u^2(0,\tau)d\tau \le \int_{\frac{t}{2}}^t u^2(0,\tau)d\tau \quad (18)$$

Hence,

$$\int_0^t e^{\alpha(t-\tau)}u^2(0,\tau)d\tau \le e^{\frac{\alpha t}{2}}\int_{\frac{t}{2}}^t u^2(0,t-\tau)d\tau + \int_{\frac{t}{2}}^t u^2(0,\tau)d\tau \quad (19)$$

Now, letting $v = t - \tau$, then Equation 19 becomes:

$$\int_0^t e^{\alpha(t-\tau)}u^2(0,\tau)d\tau \le e^{\frac{\alpha t}{2}}\int_0^{\frac{t}{2}} u^2(0,v)dv + \int_{\frac{t}{2}}^t u^2(0,\tau)d\tau \le e^{\frac{\alpha t}{2}} \cdot \int_0^\infty u^2(0,\tau)d\tau + \int_{\frac{t}{2}}^\infty u^2(0,\tau)d\tau \quad (20)$$

Hence, when $u(0,t) \in L^2(0,\infty)$,

$$\int_0^t e^{\alpha(t-\tau)}u^2(0,\tau)d\tau \to 0 \text{ as } t \to \infty \quad (21)$$

Lemma 2

Let $\alpha < 0$, if $u(x,t) \in L^4(0,\infty)$, then:

$$\int_0^t e^{\alpha(t-\tau)}u^4(0,\tau)d\tau \to 0 \text{ as } t \to \infty \quad (22)$$

Proof

The proof is similar to the one illustrated in Lemma 2.

Lemma 3

Let $\alpha < 0$, for any $u(x,t) \in L^2(0,\infty) \cap L^4(0,\infty)$:

$$\int_0^t e^{\alpha(t-\tau)}\left|u^3(0,\tau)\right|d\tau \to 0 \text{ as } t \to \infty \quad (23)$$

Proof

We use Cauchy-Schwartz inequality to prove this lemma:

$$\int_0^t e^{\alpha(t-\tau)}\left|u^3(0,\tau)\right|d\tau = \int_0^t e^{\alpha(t-\tau)}\left|u^2(0,\tau)\right|\left|u(0,\tau)\right|d\tau$$

$$= \int_0^t (e^{\frac{\alpha}{2}(t-\tau)}\left|u^2(0,\tau)\right|)\cdot(e^{\frac{\alpha}{2}(t-\tau)}\left|u(0,\tau)\right|)d\tau \le \left(\int_0^t e^{\alpha(t-\tau)}u^4(0,\tau)d\right. \quad (24)$$

Using the results of Lemmas 1 and 2, we can conclude that:

$$\int_0^t e^{\alpha(t-\tau)}\left|u^3(0,\tau)\right|d\tau \to 0 \text{ as } t \to \infty \quad (25)$$

Theorem 4

Let $\alpha = 2(m - \frac{\varepsilon}{8\pi^2}) < 0$. The solution u of the closed-loop system of the generalized KdV-Burgers Equations (1)-(3) with unknown parameters is regulated to zero in L^2 sense under the following control law:

$$\omega_1(t) = k_1(t)u^3(0,t) + k_2(t)u^2(0,t) + k_3(t)u(0,t)$$

$$\omega_2(t) = k_4(t)u^3(2\pi,t) + k_5(t)u^2(2\pi,t) + k_6(t)u(2\pi,t) \quad (26)$$

where $k_n(t), n = 1,\cdots,6,$ are bounded for any $t \ge 0$ with:

$$\dot{k}_1(t) = r_1 u^4(0,t), \quad r_1 > 0$$

$$\dot{k}_2(t) = r_2 u^3(0,t), \quad r_2 > 0$$

$$\dot{k}_3(t) = r_3 u^2(0,t), \quad r_3 > 0$$

$$\dot{k}_4(t) = -r_4 u^4(2\pi,t), \quad r_4 > 0$$

$$\dot{k}_5(t) = -r_5 u^3(2\pi,t), \quad r_5 > 0$$

$$\dot{k}_6(t) = -r_6 u^2(2\pi,t), \quad r_6 > 0 \quad (27)$$

Proof

Let $V(t) = \frac{1}{2}\int_0^{2\pi} u^2(x,t)dx$, and applying the boundary conditions from Equations (2)-(3), then from Equation (8), the time derivative of $V(t)$ is:

$$\dot{V}(t) \le (m - \frac{\varepsilon}{8\pi^2})\int_0^{2\pi} u^2(x,t)dx - \varepsilon u(0,t)\left[\frac{1}{a}\omega_1(t) - \left(\frac{b}{a} + \frac{1}{2\pi}\right)u(0,t) - \frac{1}{3\varepsilon}u^2(0,t)\right]$$
$$+ \varepsilon u(2\pi,t)\left[\frac{1}{c}\omega_2(t) - \frac{d}{c}u(2\pi,t) - \frac{1}{3\varepsilon}u^2(2\pi,t)\right] \quad (28)$$

Using the control law illustrated in Equation (26), the aforementioned inequality for $\dot{V}(t)$ becomes:

$$\dot{V}(t) \le (m - \frac{\varepsilon}{8\pi^2})\int_0^{2\pi} u^2(x,t)dx + \frac{\varepsilon}{2\pi}u^2(0,t) - \frac{1}{3}u^2(2\pi,t) + \frac{1}{3}u^3(0,t)$$
$$- \varepsilon u(0,t)\left\{\frac{1}{a}\left[k_1(t)u^3(0,t) + k_2(t)u^2(0,t) + k_3(t)u(0,t)\right] - \frac{b}{a}u(0,t)\right\}$$
$$- \varepsilon u(2\pi,t)\left\{\frac{1}{c}\left[k_4(t)u^3(2\pi,t) + k_5(t)u^2(2\pi,t) + k_6(t)u(2\pi,t)\right] - \frac{d}{c}u(2\pi,t)\right\} \quad (29)$$

Therefore, $\dot{V}(t)$ can be rewritten as:

$$\dot{V}(t) \le (m - \frac{\varepsilon}{8\pi^2})\int_0^{2\pi} u^2(x,t)dx - \frac{\varepsilon k_1(t)}{a}u^4(0,t) - \left(\frac{\varepsilon k_2(t)}{a} - \frac{1}{3}\right)u^3(0,t)$$
$$- \left(\frac{\varepsilon k_3(t)}{a} - \frac{\varepsilon}{2\pi} - \frac{\varepsilon b}{a}\right)u^2(0,t) + \frac{\varepsilon}{c}k_4(t)u^4(2\pi,t)$$
$$- \left(\frac{\varepsilon k_5(t)}{c} - \frac{1}{3}\right)u^3(2\pi,t) + \left(\frac{\varepsilon k_6(t)}{c} - \frac{d\varepsilon}{c}\right)u^2(2\pi,t) . \quad (30)$$

Now, let us introduce a non-negative energy function $E(t)$ as follows:

$$E(t) = V(t) + \left(\frac{\varepsilon}{2ar_1}\right)(k_1(t))^2 + \left(\frac{a}{2\varepsilon r_2}\right)\left(\frac{\varepsilon k_2(t)}{a} - \frac{1}{3}\right)^2 + \left(\frac{a}{2\varepsilon r_3}\right)\left(\frac{\varepsilon k_3(t)}{a} - \frac{\varepsilon}{2\pi} - \frac{\varepsilon b}{a}\right)^2$$
$$+ \left(\frac{\varepsilon}{2cr_4}\right)(k_4(t))^2 + \left(\frac{c}{2\varepsilon r_5}\right)\left(\frac{\varepsilon k_5(t)}{c} - \frac{1}{3}\right)^2 + \left(\frac{c}{2\varepsilon r_6}\right)\left(\frac{\varepsilon k_6(t)}{c} - \frac{\varepsilon d}{c}\right)^2 . \quad (31)$$

If we evaluate the time derivative of the energy function illustrated, above:

$$\dot{E}(t) = \dot{V}(t) + \left(\frac{\varepsilon}{ar_1}\right)k_1(t)\dot{k}_1(t) + \frac{1}{r_2}\left(\frac{\varepsilon k_2(t)}{a} - \frac{1}{3}\right)\dot{k}_2(t) + \frac{1}{r_3}\left(\frac{\varepsilon k_3(t)}{a} - \frac{\varepsilon}{2\pi} - \frac{\varepsilon b}{a}\right)\dot{k}_3(t)$$
$$+\left(\frac{\varepsilon}{cr_4}\right)k_4(t)\dot{k}_4(t) + \frac{1}{r_5}\left(\frac{\varepsilon k_5(t)}{c} - \frac{1}{3}\right)\dot{k}_5(t) + \frac{1}{r_6}\left(\frac{\varepsilon k_6(t)}{c} - \frac{\varepsilon d}{c}\right)\dot{k}_6(t). \quad (32)$$

Now, substituting $\dot{V}(t)$ from Equation (30) and $k_n(t), n = 1,\cdots,6,$ from Equation (27) into Equation (32), we get:

$$\dot{E}(t) \le (m - \frac{\varepsilon}{8\pi^2})\int_0^{2\pi} u^2(x,t)dx. \quad (33)$$

This implies that if $\alpha = 2(m - \frac{\varepsilon}{8\pi^2}) < 0$, then $E(t) \le E(0).$ Thus, one can conclude that $k_n(t), n = 1,\cdots,6,$ are bounded for any $t \ge 0$. Therefore, $u(0,t) \in L^2(0,\infty) \bigcap L^4(0,\infty)$ and $u(2\pi,t) \in L^2(0,\infty) \bigcap L^4(0,\infty)$.

To show the L^2 regulation of the generalized KdV-Burgers equation, we see that from Equation (30):

$$\dot{V}(t) \le (m - \frac{\varepsilon}{8\pi^2})\int_0^{2\pi} u^2(x,t)dx + \varepsilon\left\{\frac{-1}{a}k_1(t)u^4(0,t) + \left(\frac{1}{3\varepsilon} - \frac{k_2(t)}{a}\right)u^3(0,t)\right.$$
$$\left. + \left(\frac{1}{2\pi} - \frac{k_3(t)}{a} + \frac{b}{a}\right)u^2(0,t)\right\} + \varepsilon\left\{\frac{1}{c}k_4(t)u^4(2\pi,t) + \left(\frac{1}{c}k_5(t) - \frac{1}{3\varepsilon}\right)u^3(2\pi,t)\right.$$
$$\left. + \left(\frac{k_6(t)}{c} - \frac{d}{c}\right)u^2(2\pi,t)\right\}. \quad (34)$$

Using Gronwall's inequality, we get:

$$V(t) \le V(0)e^{\alpha t} + \varepsilon\int_0^t\left\{\frac{-k_1(\tau)}{a}u^4(0,\tau) + \left(\frac{1}{3\varepsilon} - \frac{k_2(\tau)}{a}\right)u^3(0,\tau)\right.$$
$$\left. + \left(\frac{1}{2\pi} - \frac{k_3(t)}{a} + \frac{b}{a}\right)u^2(0,\tau)\right\}e^{\alpha(t-\tau)}d\tau + \varepsilon\int_0^t\left\{\frac{k_4(\tau)}{c}u^4(2\pi,\tau)\right.$$
$$\left.\left(\frac{k_5(\tau)}{c} - \frac{1}{3\varepsilon}\right)u^3(2\pi,\tau) + \left(\frac{k_6(\tau)}{c} - \frac{d}{c}\right)u^2(2\pi,\tau)\right\}e^{\alpha(t-\tau)}d\tau \quad (35)$$

or

$$V(t) \le V(0)e^{\alpha t} + \varepsilon C_{\max}\int_0^t e^{\alpha(t-\tau)}[u^4(0,\tau) + \left|u^3(0,\tau)\right| + u^2(0,\tau)]d\tau$$
$$+ \varepsilon C_{\max}\int_0^t e^{\alpha(t-\tau)}[u^4(2\pi,\tau) + \left|u^3(2\pi,\tau)\right| + u^2(2\pi,\tau)]d\tau, \quad (36)$$

where

$$C_{\max} = \max\left\{ \sup\left|\frac{-k_1(\tau)}{a}\right|, \sup\left|\frac{1}{3\varepsilon} - \frac{k_2(\tau)}{a}\right|, \sup\left|\frac{1}{2\pi} + \frac{b}{a} - \frac{k_3(\tau)}{a}\right|, \right.$$

$$\left. \sup\left|\frac{k_4(\tau)}{c}\right|, \sup\left|\frac{k_5(\tau)}{c} - \frac{1}{3\varepsilon}\right|, \sup\left|\frac{k_6(\tau)}{c} - \frac{d}{c}\right| \right\}. \quad (37)$$

Now using Lemmas (1)-(3), one can deduce from Equation 36 that:

$\int_0^{2\pi} u^2(x,t)dx \to 0$ as $t \to \infty$.

Conclusion

We have shown the adaptive and non-adaptive stabilization of the generalized KdV-Burgers equation by nonlinear boundary control. As for the adaptive case, we refer to the new approach proposed by Smaoui (2004) to show the L^2 regulation of the generalized KdV-Burgers equation. This approach seems more elegant than using an extension to Barbalat's lemma. It should be noted that the control laws is a general control law that can be used for the Neumann boundary conditions and the mixed boundary conditions.

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