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The modified simple equation method for solving nonlinear diffusive predator-prey system and Bogoyavlenskii equations

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The modified simple equation method (The modified SEM) is employed to find the exact traveling wave solutions involving parameters for two nonlinear evolution equations, namely, the diffusive predator-prey system and the Bogoyavlenskii equations. When these parameters are taken to be special values, the solitary wave solutions are derived from the exact traveling wave solutions. It is shown that the modified SEM provides an effective and a more powerful mathematical tool for solving nonlinear partial differential equations in mathematical physics. Our results in this paper are new and different from those obtained in the literature.

Key words: A diffusive predator-prey system, the Bogoyavlenskii equation, the modified SEM, traveling wave solutions, solitary wave solutions.

INTRODUCTION

The investigation of the traveling wave solutions for nonlinear partial differential equations plays an important role in the study of nonlinear physical phenomena. Nonlinear wave phenomena appears in various scientific and engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics and geochemistry. Nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction and convection are very important in nonlinear wave equations. In the past several decades, new exact solutions may help to find new phenomena. A variety of powerful methods such as the tanh- sech method (Malfliet, 1992; Malfliet and Hereman, 1996; Wazwaz, 2004a), the Sine - cosine method (Wazwaz, 2004b; Wazwaz, 2005; Yan, 1996), the homogeneous balance method (Fan and Zhang, 1998; Wang 1996), the Jacobi elliptic function method (Dai and Zhang, 2006; Fan and Zhang, 2002; Liu et al., 2001; Zhao et al., 2006), the F-expansion method (Abdou, 2007; Ren and Zhang, 2006; Zhang et al., 2006), the exp-function method (He and Wu, 2006; Aminikhad et al., 2009), the trigonometric function series method (Zhang, 2008), the (G'/G)-expansion method (Alam and Akbar, 2014a, b, c, d; Alam et al., 2014a, b, c; Hafez et al., 2014; Wang et al. 2008; Zayed, 2009; Zayed and Gepreel, 2009; Zhang et al.,

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2008), the extended tanh- function method (El-Wakil and Abdou, 2007; Fan, 2000; Wazwaz, 2007), the modified SEM (Jawad et al., 2010; Zayed and Hoda, 2012, 2013, 2014), the local fractional differential equations (Yang, 2011, 2012), the local fractional variation iteration method (Yang and Baleanu, 2012), the local fractional Fourier series method (Hu et al., 2012), the cantor-type cylindrical coordinate method (Yang et al., 2013), the Yang-Fourier and Yang-Laplace transforms (He, 2012).

The objective of this article is to apply the modified SEM for finding the exact traveling wave solution of the following diffusive predator-prey system of equations (Petrovskii et al., 1990) and the Bogoyavlenskii equations (Bogoyavlenskii, 1990):

(i) We consider the following system of two coupled nonlinear partial differential equations describing the dynamics of the predator- prey system of equations (Petrovskii et al., 1990):

$$\begin{cases} u_t = u_{xx} - \beta u + (1+\beta)u^2 - u^3 - uv, \\ v_t = v_{xx} + kuv - mv - \delta v^3, \end{cases} \tag{1}$$

Where k, δ, m and β are positive parameters. The solutions of predator-prey system of equations have been studied in various aspects (Petrovskii et al., 1990; Kraenkl et al., 2013; Dehghan and Sabouri, 2013). With reference to the article (Petrovskii et al., 1990), the dynamics of the system (1) is assumed the following relations between the parameters, namely $m = \beta$ and $k + \frac{1}{\sqrt{\delta}} = \beta + 1.$ Under these assumptions, Equation (1) can be rewritten in the form:

$$\begin{cases} u_t = u_{xx} - \beta u + \left(k + \frac{1}{\sqrt{\delta}}\right)u^2 - u^3 - uv, \\ v_t = v_{xx} + kuv - \beta v - \delta v^3. \end{cases} \tag{2}$$

(ii) We consider the Bogoyavlenskii equations (Bogoyavlenskii, 1990) in the form

$$\begin{cases} 4u_t + u_{xxy} - 4u^2 u_y - 4u_x v = 0, \\ uu_y = v_x. \end{cases} \tag{3}$$

Equation (3) were derived by Kudryashov and Pickering (1998) as a member of the (2+1) Schwarzian breaking soliton hierarchy. The above equations also appeared in Clarkson and Gordoa (1996) as one of the equations associated to nonisospectral scattering problems. Estevez and Prada (2004); showed that Equation (3) possess the Painleve property. Equation (3) were the modified version of the breaking soliton equation:

$$4u_{xt} + 8u_x u_{xy} + 4u_y u_{xx} + u_{xxxy} = 0, \tag{4}$$

Which describes the (2 + 1)-dimensional interaction of a Riemann wave propagation along the y-axis with a long wave through the x-axis.

DESCRIPTION OF THE MODIFIED SEM

Consider the following nonlinear evolution equation:

$$F(u, u_t, u_x, u_y, u_{tt}, u_{xx}, u_{yy}, ...) = 0, \tag{5}$$

Where F is a polynomial in $u(x, y, t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method (Jawad et al., 2010; Zayed and Hoda, 2012, 2013,2014):

Step 1. We use the wave transformation

$$u(x, y, t) = u(\xi), \quad \xi = x + y - ct, \tag{6}$$

where c is a nonzero constant, to reduce Equation (5) to the following ODE:

$$P(u, u', u'', u''', ...) = 0, \tag{7}$$

Where P is a polynomial in $u(\xi)$ and its total derivatives, while $' = \frac{d}{d\xi}.$

Step 2. Suppose that the solution of Equation (7) has the formal solution

$$u(\xi) = \sum_{k=0}^{N} A_k \left(\frac{\psi'(\xi)}{\psi(\xi)}\right)^k, \tag{8}$$

Where A_k are arbitrary constants to be determined, such that $A_N \neq 0,$ while the function $\psi(\xi)$ is an unknown function to be determined later, such that $\psi' \neq 0.$

Step 3. Determined the positive integer N in Equation (8) by considering the homogenous balance between the highest order derivatives and the nonlinear terms in Equation (7).

Step 4. Substitute Equation (8) into Equation (7), we calculate all the necessary derivative $u', u'', \ldots$ of the function $u(\xi)$ and we account the function $\psi(\xi)$. As a result of this substitution, we get a polynomial of $\psi^{-j}\,(j=0,1,2,\ldots,N)$. In this polynomial, we gather all terms of the same power of $\psi^{-j}\,(j=0,1,2,\ldots,N)$, and we equate with zero all coefficient of this polynomial. This operation yields a system of equations which can be solved to find A_k and $\psi(\xi)$. Consequently, we can get the exact solution of Equation (5).

APPLICATIONS

Here, we will apply the modified SEM described in description of the modified SEM, to find the exact traveling wave solutions and then the solitary wave solutions for the following nonlinear systems of evolution equations.

Example 1: A nonlinear diffusive predator-prey system

Here, we determine the exact solutions and the solitary wave solutions of Equation (2). To this end, we use the wave transformation (6) to reduce Equation (2) to the following nonlinear system of ordinary differential equations:

$$\begin{cases} u''+cu'-\beta u+\left(k+\dfrac{1}{\sqrt{\delta}}\right)u^2-u^3-uv=0, \\ v''+cv'+kuv-\beta v-\delta v^3=0. \end{cases} \tag{9}$$

In order to solve Equation (9) let us consider the following transformation

$$v=\frac{1}{\sqrt{\delta}}u. \tag{10}$$

Substituting the transformation (10) into Equation (9), we get

$$u''+cu'-\beta u+ku^2-u^3=0. \tag{11}$$

Balancing u'' with u^3 in Equation (11), we get $N=1$. Consequently, we get the formal solution

$$u(\xi)=A_0+A_1\left(\frac{\psi'}{\psi}\right), \tag{12}$$

Where A_0 and A_1 are constants to be determined, such that $A_1 \neq 0$. It is easy to see that

$$u'=A_1\left(\frac{\psi''}{\psi}-\left(\frac{\psi'}{\psi}\right)^2\right), \tag{13}$$

$$u''=A_1\left(\frac{\psi'''}{\psi}-\frac{3\psi'\psi''}{\psi^2}+2\left(\frac{\psi'}{\psi}\right)^3\right). \tag{14}$$

Substituting Equation (12) to (14) into Equation (11) and equating the coefficients of $\psi^{-3},\psi^{-2},\psi^{-1},\psi^{0}$ to zero, we respectively obtain

$$\psi^{-3}:\psi'^3A_1\left(2-A_1^2\right)=0, \tag{15}$$

$$\psi^{-2}:\psi'A_1\left(-3\psi''-(c-kA_1+3A_0A_1)\psi'\right)=0, \tag{16}$$

$$\psi^{-1}:A_1\left(\psi'''+c\psi''-(\beta-2kA_0+3A_0^2)\psi'\right)=0, \tag{17}$$

$$\psi^{0}:A_0\left(-\beta+kA_0-A_0^2\right)=0. \tag{18}$$

From Equations (15) and (18), we deduce that

$$A_1=\pm\sqrt{2},\ A_0=0,\ A_0=\frac{-k\pm\sqrt{k^2-4\beta}}{-2},$$

Where $k^2>4\beta.$ Let us discuss the following cases.

Case 1. If $A_0=0,\ A_1=\pm\sqrt{2}$

In this case, we deduce from Equations (16) and (17) that

$$\psi'=\frac{-3}{c-kA_1}\psi'', \tag{19}$$

and

$$\psi'''+c\psi''-\beta\psi'=0, \tag{20}$$

Where $c\neq kA_1$. Equations (19) and (20) yield

$$\frac{\psi'''}{\psi''}=E_0, \tag{21}$$

Where $E_0=-\left(c+\dfrac{3\beta}{c-kA_1}\right).$ Integrating Equation

(21) and using Equation (19) we have

$$\psi' = \frac{-3c_1}{c - kA_1}\exp[E_0\xi], \quad (22)$$

$$\psi = \frac{-3c_1}{E_0(c - kA_1)}\exp[E_0\xi] + c_2, \quad (23)$$

Where c_1 and c_2 are arbitrary constants of integration. Substituting Equations (22) and (23) into Equation (12) and from Equation (10) we have the exact solutions:

$$u(\xi) = \pm\sqrt{2}E_0\left(\frac{\exp[E_0\xi]}{\exp[E_0\xi] + c_2}\right), \quad (24)$$

and from Equation (10) we have

$$v(\xi) = \pm\sqrt{\frac{2}{\delta}}E_0\left(\frac{\exp[E_0\xi]}{\exp[E_0\xi] + c_2}\right), \quad (25)$$

Where $c_1 = \frac{E_0(c - kA_1)}{-3}$. If $c_2 = 1$, we have the solitary wave solutions:

$$u(\xi) = \frac{\pm E_0}{\sqrt{2}}\left(1 + \tanh\left[\frac{E_0}{2}\xi\right]\right), \quad (26)$$

and consequently, we get

$$v(\xi) = \frac{\pm E_0}{\sqrt{2\delta}}\left(1 + \tanh\left[\frac{E_0}{2}\xi\right]\right), \quad (27)$$

While, if $c_2 = -1$, we get the solitary wave solutions:

$$u(\xi) = \frac{\pm E_0}{\sqrt{2}}\left(1 + \coth\left[\frac{E_0}{2}\xi\right]\right), \quad (28)$$

and consequently, we get

$$v(\xi) = \frac{\pm E_0}{\sqrt{2\delta}}\left(1 + \coth\left[\frac{E_0}{2}\xi\right]\right). \quad (29)$$

Case 2. If $A_0 \neq 0$, $A_1 = \pm\sqrt{2}$

In this case, we deduce from Equation (16) and (17) that

$$\psi' = \frac{-3}{c - kA_1 + 3A_0A_1}\psi'', \quad (30)$$

and

$$\psi''' + c\psi'' - \left(\beta - 2kA_0 + 3A_0^2\right)\psi' = 0. \quad (31)$$

Substituting Equation (30) into Equation (31) we get

$$\frac{\psi'''}{\psi''} = E_1, \quad (32)$$

Where $E_1 = -\left(c + \frac{3\left(\beta - 2kA_0 + 3A_0^2\right)}{c - kA_1 + 3A_0A_1}\right)$.

Consequently, we have

$$\psi' = \frac{-3c_3}{c - kA_1 + 3A_0A_1}\exp[E_1\xi], \quad (33)$$

$$\psi = \frac{-3c_3}{E_1(c - kA_1 + 3A_0A_1)}\exp[E_1\xi] + c_4, \quad (34)$$

Where c_3 and c_4 are arbitrary constants of integration. Substituting Equation (33) and (34) into Equation (12) and from Equation (10) we have the exact solutions:

$$u(\xi) = \frac{-k \pm\sqrt{k^2 - 4\beta}}{-2} \pm \sqrt{2}E_1\left(\frac{\exp[E_1\xi]}{\exp[E_1\xi] + c_4}\right), \quad (35)$$

$$v(\xi) = \frac{1}{\sqrt{\delta}}\left(\frac{-k \pm\sqrt{k^2 - 4\beta}}{-2} \pm \sqrt{2}E_1\left(\frac{\exp[E_1\xi]}{\exp[E_1\xi] + c_4}\right)\right), \quad (36)$$

Where $c_3 = \frac{E_1(c - kA_1 + 3A_0A_1)}{-3}$.

If $c_4 = 1$, we get the solitary wave solutions:

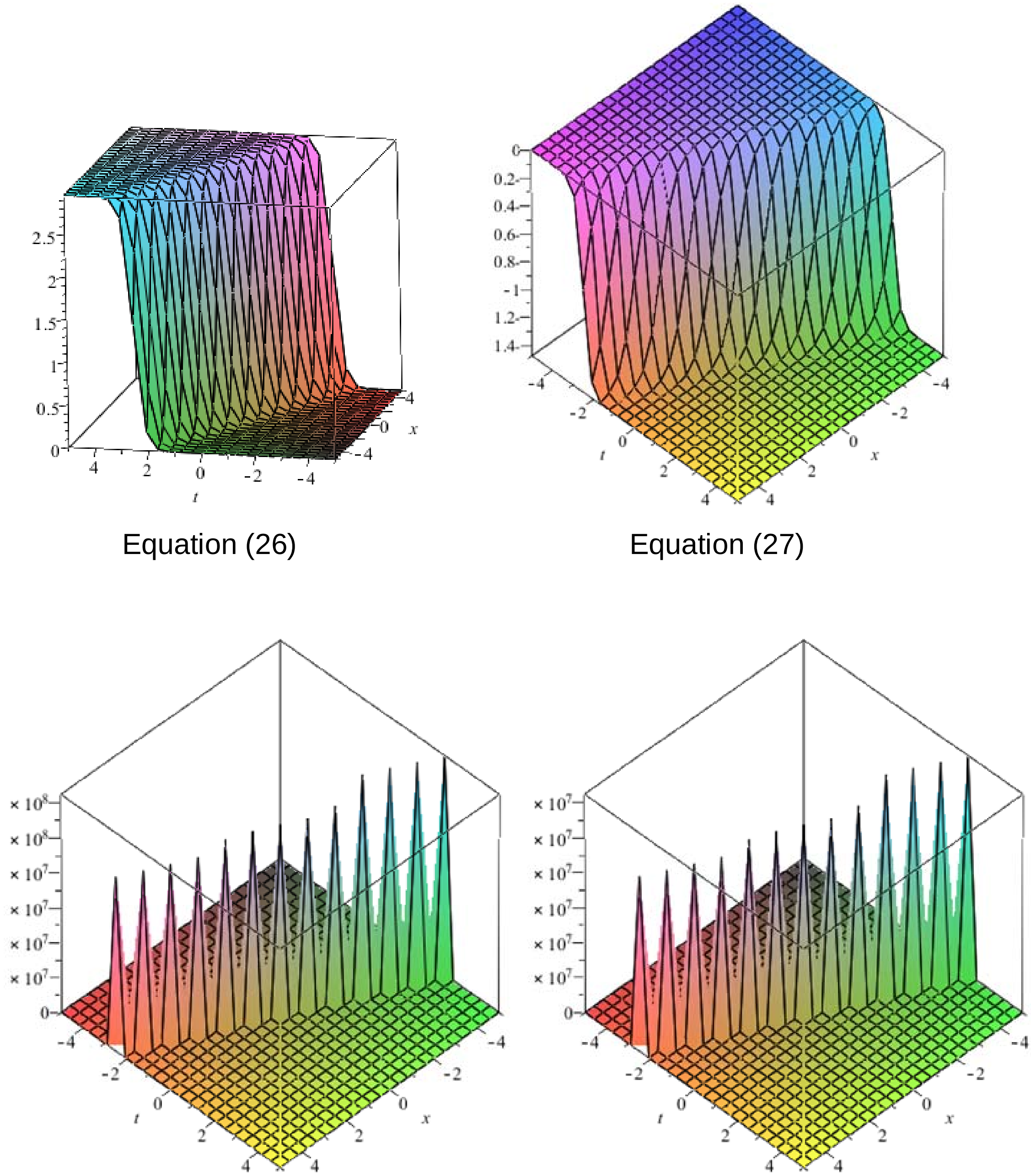
$$u(\xi) = \frac{-k \pm\sqrt{k^2 - 4\beta}}{-2} \pm \frac{E_1}{\sqrt{2}}\left(1 + \tanh\left[\frac{E_1}{2}\xi\right]\right), \quad (37)$$

and

$$v(\xi) = \frac{1}{\sqrt{\delta}}\left(\frac{-k \pm\sqrt{k^2 - 4\beta}}{-2} \pm \frac{E_1}{\sqrt{2}}\left(1 + \tanh\left[\frac{E_1}{2}\xi\right]\right)\right), \quad (38)$$

While, if $c_4 = -1$, we get the solitary wave solutions

$$u(\xi) = \frac{-k \pm\sqrt{k^2 - 4\beta}}{-2} \pm \frac{E_1}{\sqrt{2}}\left(1 + \coth\left[\frac{E_1}{2}\xi\right]\right), \quad (39)$$



Equation (26) Equation (27)

Equation (28) Equation (29)

Figure 1. The plots of the solutions (26) to (29) when $A_0 = 0,\ A_1 = \sqrt{2},\ \beta = \dfrac{1}{12}, k = \dfrac{7}{12},\ c = -2,\ \delta = 4.$

and

$$v(\xi) = \frac{1}{\sqrt{\delta}}\left(\frac{-k \pm \sqrt{k^2 - 4\beta}}{-2} \pm \frac{E_1}{\sqrt{2}}\left(1 + \coth\left[\frac{E_1}{2}\xi\right]\right)\right), \quad (40)$$

Note that the graphical representations of the solutions (26) to (29) are drawn in Figure 1 as follows:

Example 2: The nonlinear Bogoyavlenskii equations

Here, we determine the exact solutions and the solitary wave solutions of Equation (3). To this end, we use the wave transformation (6) to reduce Equation (3) to the following nonlinear system of ordinary differential equations:

$$\begin{cases} -4cu' + u''' - 4u^2u' - 4u'v = 0, \\ \dfrac{u^2}{2} = v. \end{cases} \quad (41)$$

Substituting the second Equation of (41) into the first one, and integrating the resultant equation, we have

$$u'' - 2u^3 - 4cu = 0, \quad (42)$$

With zero constant of integration. Balancing u'' with u^3 in Equation (42), we get $N = 1$. Consequently, we get the same formal solution (12). Substituting Equation (12) to (14) into Equation (42) and equating the coefficients of $\psi^{-3}, \psi^{-2}, \psi^{-1}, \psi^0$ to zero, we obtain

$$\psi^{-3} : 2A_1\psi'^3\left(1 - A_1^2\right) = 0, \quad (43)$$

$$\psi^{-2} : -3A_1\psi'\left(\psi'' + 2A_0A_1\psi'\right) = 0, \quad (44)$$

$$\psi^{-1} : A_1\left(\psi''' - \left(6A_0^2 + 4c\right)\psi'\right) = 0, \quad (45)$$

$$\psi^0 : -2A_0\left(A_0^2 + 2c\right) = 0. \quad (46)$$

From Equations (43) and (43) we deduce that

$A_1 = \pm 1, \quad A_0 = 0 \; or \; A_0 = \pm\sqrt{-2c}$, where $c < 0$.

Case 1. If $A_0 = 0, \; A_1 = \pm 1$

In this case Equations (44) and (45) yield $\psi' = 0$, so this case is rejected.

Case 2. If $A_0 = \pm\sqrt{-2c}, A_1 = \pm 1$

In this case, we deduce from Equations (44) and (45) that

$$\psi' = \frac{-1}{2A_0A_1}\psi'', \quad (47)$$

and

$$\psi''' - \left(6A_0^2 + 4c\right)\psi' = 0. \quad (48)$$

Equations (47) and (48) yield

$$\frac{\psi'''}{\psi''} = E_0, \quad (49)$$

Where $E_0 = \dfrac{-6A_0^2 - 4c}{2A_0A_1}$. Integrating Equation (49) and using Equation (47) we have

$$\psi' = \frac{-c_1}{2A_0A_1}\exp[E_0\xi], \quad (50)$$

$$\psi = \frac{-c_1}{2E_0A_0A_1}\exp[E_0\xi] + c_2, \quad (51)$$

Where c_1 and c_2 are arbitrary constants of integration. Substituting Equations (50) and (51) into Equation (12) we have the exact solutions:

$$u(\xi) = \pm\sqrt{-2c} \pm E_0\left(\frac{\exp[E_0\xi]}{\exp[E_0\xi] + c_2}\right), \quad (52)$$

and

$$v(\xi) = \frac{1}{2}\left(\pm\sqrt{-2c} \pm E_0\left(\frac{\exp[E_0\xi]}{\exp[E_0\xi] + c_2}\right)\right)^2, \quad (53)$$

Where $c_1 = -2E_0A_0A_1$.

(i) If $c_2 = 1$, we have the solitary wave solutions

$$u(\xi) = \pm\sqrt{-2c} \pm \frac{E_0}{2}\left(1 + \tanh\left[\frac{E_0}{2}\xi\right]\right), \quad (54)$$

and

$$v(\xi) = \frac{1}{2}\left(\pm\sqrt{-2c} \pm \frac{E_0}{2}\left(1 + \tanh\left[\frac{E_0}{2}\xi\right]\right)\right)^2. \quad (55)$$

(ii) If $c_2 = -1$, we get the solitary wave solutions:

$$u(\xi) = \pm\sqrt{-2c} \pm \frac{E_0}{2}\left(1 + \coth\left[\frac{E_0}{2}\xi\right]\right), \quad (56)$$

and

$$v(\xi) = \frac{1}{2}\left(\pm\sqrt{-2c} \pm \frac{E_0}{2}\left(1 + \coth\left[\frac{E_0}{2}\xi\right]\right)\right)^2. \quad (57)$$

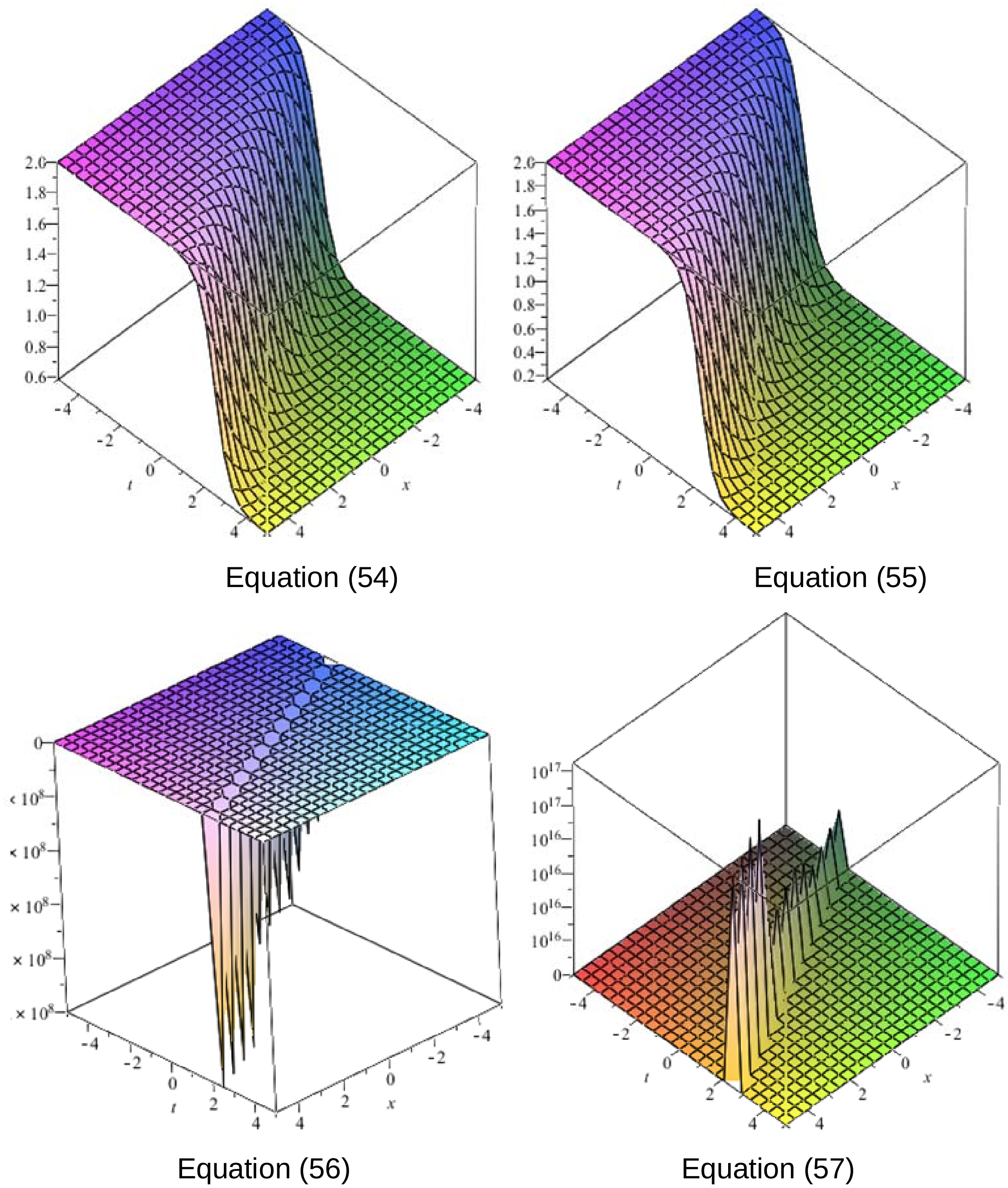


Figure 2. The plots of the solutions (54) to (57) when $A_0 = \sqrt{-2c}$, $A_1 = 1$, $c = -2$, $y = 0$.

Note that the graphical representations of the solutions Equations (54) to (57) are drawn in Figure 2 as follows:

Conclusions

The modified SEM had been used successfully to find the exact traveling wave solutions of nonlinear evolution equations. As an application, the traveling wave solutions for the nonlinear diffusive predator-prey system of equations and the nonlinear Bogoyavlenskii equations had been constructed using the modified SEM. It could be concluded that this method was reliable and proposed a variety of exact solutions of nonlinear differential equations.

Conflict of Interest

The authors have not declared any conflict of interest.

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