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Full Length Research Paper

Accelerated flows of a magnetohydrodynamic (MHD) second grade fluid over an oscillating plate in a porous medium and rotating frame

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The aim of this paper is to determine the exact solutions for the velocity field related to the magnetohydrodynamic (MHD) and rotating flow of a second grade fluid in a porous medium induced by accelerated flows over an oscillating plate. This is accomplished by using the Fourier sine and Laplace transforms. Two explicit flow situations of the fluid are considered. In each case, both sine and cosine oscillations of the plate are incorporated. Finally, some graphical results of the fluid's velocity profiles are presented correspondingly for different values of the emerging parameters. The physical interpretations for these parameters are discussed with the help of these graphical illustrations.

Key words: Fourier and Laplace transforms, accelerated flow, magnetohydrodynamic (MHD) second grade fluid, porous medium, rotating frame.

INTRODUCTION

Non-Newtonian fluids have received considerable attention because of its numerous applications in geophysics, engineering and industry. Such applications include the extension of polymer fluids, solidification of liquid crystals, personal care products, exotic lubricants and colloidal and suspension solutions (Zhaosheng and JianZhong, 1998). The non-Newtonian fluids have been mainly classified under the differential, rate and integrals types. The second grade fluids are the subclass of non-Newtonian fluids and are the simplest subclass of differential type fluids which can show the normal stress effects. It was employed to study various problems due to their relatively simple structure. Moreover, one can reasonably hope to obtain exact solutions for the velocity field from this type of second grade fluid. For this reason, we have chosen this type of fluid in this study. The exact solution is important, because it provides reference for checking the accuracies of many approximate solutions which can be numerical or empirical. It can also be used as test for validating numerical schemes that are developed for investigating more complex flow problems.

Exact solution of the problem is given by invoking the Fourier sine and Laplace transforms method. This method has already been successfully applied by various workers (Fetecau et al., 2011a, b ; Faisal et al., 2011a, b). As expected, the traditional Fourier sine and Laplace transforms method has the following important feature, that is, it is still a powerful technique for solving analytically these types of problems, which literally transforms the original linear differential equation into an elementary algebraic expression.

The analysis of the effects of rotation and magnetohydrodynamic (MHD) flows through a porous medium and rotating frame have gained an increasing interest due to the wide range of applications in engineering, such as the optimization of the solidification process of metals and metal alloys, the control underground spreading of chemical wastes and pollutants. MHD is the study of the interaction of conducting fluids with electromagnetic phenomena. The flow of an

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electrically conducting fluid in the presence of magnetic field is of importance in various areas of technology and engineering, such as MHD power generation and MHD pumps. Therefore, many have discussed the flows of second grade fluid in different configurations and there are attempts in the literature to include the effects of rotation and MHD (Shen et al., 2006; Tan and Mazuoka, 2005; Erdogan and Imrak, 2005; Hussain et al., 2010; Fetecau and Corina, 2005; Jamil et al., 2011; Corina et al., 2009; Islam et al., 2011).

The objective of this work is to establish exact solutions for the velocity field induced by accelerated flows over an oscillating plate for second grade fluid. The fluid occupies the porous space and is electrically conducting. In addition, the whole system is also rotating. Two flow problems of the fluid are considered and their exact solutions for the velocity field are established. In the first problem, the fluid occupying the half space is bounded by an oscillating and accelerated rigid plate. In the second problem, we modified the Stokes' second problem which deals with the flow between the two plates. The upper plate is taken stationary, while the lower one is accelerated and oscillating. In these problems, both sine and cosine oscillations are considered. Graphs of the solutions are also plotted and discussed.

FORMULATION OF THE FLOW PROBLEM

Let us consider a Cartesian coordinate system (x, y, z). We consider a fluid saturated porous half space bounded by an infinite plate at $z = 0$ ($z-$axis is taken normal to the plate). The whole system is rotating uniformly with a constant angular velocity Ω about the $z-$axis. The porous space is described by the modified Darcy's law. A constant magnetic field $B_\circ$ acts in the $z-$ direction, that is, the fluid is electrically conducting in the presence of an applied magnetic field $\boldsymbol{B} = (0, 0, B_\circ)$ the magnetic Reynolds number is assumed small and hence the induced magnetic field is neglected.

The equations governing the present flow are as given in the work Hayat et al. (2008).

$$\rho\left(\frac{\partial u}{\partial t} - 2\Omega\, \upsilon\right) = \mu\frac{\partial^2 u}{\partial z^2} + \alpha_1\frac{\partial^3 u}{\partial z^2\partial t} - \sigma B_\circ^2 u - \frac{\mu\phi}{k}\left(1 + \frac{\alpha_1}{\mu}\frac{\partial}{\partial t}\right)u \tag{1}$$

$$\rho\left(\frac{\partial \upsilon}{\partial t} + 2\Omega\, u\right) = \mu\frac{\partial^2 \upsilon}{\partial z^2} + \alpha_1\frac{\partial^3 \upsilon}{\partial z^2\partial t} - \sigma B_\circ^2 \upsilon - \frac{\mu\phi}{k}\left(1 + \frac{\alpha_1}{\mu}\frac{\partial}{\partial t}\right)\upsilon\,. \tag{2}$$

In the aforementioned equations $u, \upsilon, \rho, t, \mu, \sigma, \alpha_1, \phi$ and k, respectively indicate the velocity field components in x and y directions, fluid density, time, dynamic viscosity, electrical conductivity, material parameter of second grade fluid, porosity and the permeability of porous medium.

The initial and boundary conditions are:

$$u = \upsilon = 0 \text{ when } t = 0,\ z > 0, \tag{3}$$

$$u(0,t) = At + UH(t)\cos\omega t \text{ or}$$
$$u(0,t) = At + UH(t)\sin\omega t \text{ for } t > 0, \tag{4}$$

$$u, \frac{\partial u}{\partial z}, \upsilon, \frac{\partial \upsilon}{\partial z} \to 0\text{, as } z\to\infty,\quad t>0 \tag{5}$$

where A is the constant acceleration, U the amplitude , $H(t)$ is the Heaviside unit step function and ω the frequency of oscillation of the plate.

SOLUTION OF THE FIRST PROBLEM

Defining a complex function $F = u + i\upsilon$, Equations 1 and 2 can be combined as:

$$\frac{\partial F}{\partial t} + \left(2i\Omega + \frac{\sigma B_\circ^2}{\rho}\right)F = \nu\frac{\partial^2 F}{\partial z^2} + \frac{\alpha_1}{\rho}\frac{\partial^3 F}{\partial z^2\partial t} - \frac{\nu\phi}{k}\left(1 + \frac{\alpha_1}{\mu}\frac{\partial}{\partial t}\right)F, \tag{6}$$

where ν is the kinematic viscosity. The appropriate boundary and initial conditions are:

$$F(0,t) = At + UH(t)\cos\omega t \text{ or}$$
$$F(0,t) = At + UH(t)\sin\omega t,\quad t > 0, \tag{7}$$

$$F(z,t), \frac{\partial F(z,t)}{\partial z} \to 0 \text{ as } as z\to\infty, t > 0.$$

$$F(z,0) = 0, z > 0. \tag{8}$$

In order to solve the linear partial differential Equation 6 with initial and boundary conditions 7 and 8, we shall apply the Fourier sine and Laplace transforms. For a greater generality we consider the boundary condition $F(0,t) = U_\circ(t)$ with $U_\circ(0) = 0$ and apply the Fourier sine transform with respect to z . We then obtain:

$$\frac{\partial F_s(\eta,t)}{\partial t} + \frac{\left[\nu\eta^2 + \nu\frac{\phi}{k} + c\right]}{\left[1 + \alpha\eta^2 + \alpha\frac{\phi}{k}\right]}F_s(\eta,t) = \frac{\eta\sqrt{\frac{2}{\pi}}\left[\nu U_\circ(t) + \alpha U_\circ'(t)\right]}{\left(1 + \alpha\eta^2 + \alpha\frac{\phi}{k}\right)}; t > 0, \tag{9}$$

where

$$\alpha = \frac{\alpha_1}{\rho}\,,\quad c = 2i\Omega + \frac{\sigma B_\circ^2}{\rho}$$

and the Fourier sine transform $F_s(\eta,t)$ of $F(z,t)$ has to satisfy the conditions:

$$F_s(\eta,0) = 0; \eta > 0. \tag{10}$$

Employing the Laplace transform to Equation 9, using the initial condition of Equation 10, and then we found that:

$$\bar{F}_s(\eta,q)=\frac{\eta\sqrt{\frac{2}{\pi}}\left[\nu+\alpha q\right]\bar{U}_\circ(q)}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)+q\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)}, \quad (11)$$

where q is the transform parameter, while $\bar{F}_s(\eta,q)$ and $\bar{U}_\circ(q)$ are the Laplace transform of $F_s(\eta,t)$ and $U_\circ(t)$, respectively.

Case 1

$$F(0,t)=At+UH(t)\cos\omega t$$

In this case $\bar{U}_\circ(q)=\frac{A}{q^2}+\frac{Uq}{q^2+\omega^2}$ and then Equation 11 takes the form:

$$\bar{F}_s(\eta,q)=\frac{\eta\sqrt{\frac{2}{\pi}}\left[\nu+\alpha q\right]\left[\frac{A}{q^2}+\frac{Uq}{q^2+\omega^2}\right]}{q\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)+\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)}. \quad (12)$$

Applying the inverse Laplace transform to Equation 12, we then arrive at:

$$F_s(\eta,t)=\frac{\eta\sqrt{\frac{2}{\pi}}A}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2}\left[(\nu-\alpha c)\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)\right]e^{\frac{-\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)t}{\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)}}$$

$$+\eta\sqrt{\frac{2}{\pi}}U\left[\frac{(\nu-\alpha c)\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)^2}\right]e^{\frac{-\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)t}{\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)}}$$

$$+\eta\sqrt{\frac{2}{\pi}}U\cos\omega t\left[\frac{\nu\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)+\alpha\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)^2}\right]$$

$$+\eta\sqrt{\frac{2}{\pi}}U\omega\sin\omega t\left[\frac{\nu-\alpha c}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)^2}\right]$$

$$+\frac{\eta\sqrt{\frac{2}{\pi}}A\left[\nu(t+1)\right]}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)} \quad (13)$$

Inversion of Fourier sine transform in Equation 13 gives:

$$F(z,t)=A(t+1)e^{-\left(\sqrt{\frac{\phi}{k}+\frac{c}{\nu}}\right)z}+\frac{2A}{\pi}\int_0^\infty\left[\frac{\eta\left[(\nu-\alpha c)\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)\right]}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2}e^{\frac{-\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)t}{\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)}}\right]\sin(z\eta)d\eta$$

$$-\frac{2UH(t)}{\pi}\int_0^\infty\left[\frac{(\nu-\alpha c)\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)^2}e^{\frac{-\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)t}{\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)}}\right]\eta\sin(z\eta)d\eta$$

$$+\frac{2}{\pi}UH(t)\cos\omega t\int_0^\infty\left[\frac{\left(\nu^2\eta^2+\nu^2\frac{\phi}{k}+\nu c\right)+\alpha\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)^2}\right]\eta\sin(z\eta)d\eta$$

$$+\frac{2}{\pi}UH(t)\omega\sin\omega t\int_0^\infty\left[\frac{\eta(\nu-\alpha c)}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)^2}\right]\sin(z\eta)d\eta \quad (14)$$

Alternative form of the solution (Equation 14) is:

$$F(z,t)=A(t+1)e^{-\left(\sqrt{\frac{\phi}{k}+\frac{c}{\nu}}\right)z}$$

$$+UH(t)\cos\omega t\left(1-\frac{2}{\pi}\right)\times\int_0^\infty\left[\frac{\left(\nu\frac{\phi}{k}+c\right)\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)+\omega^2\left(1+\alpha\frac{\phi}{k}\right)\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)}{\eta\left[\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)^2\right]}\right]\sin(z\eta)d\eta$$

$$+\frac{2}{\pi}UH(t)\omega\sin\omega t\int_0^\infty\left[\frac{\eta(\nu-\alpha c)}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)^2}\right]\sin(z\eta)d\eta$$

$$-\frac{2UH(t)}{\pi}\int_0^\infty\left[\frac{(\nu-\alpha c)\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)^2}e^{\frac{-\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)t}{\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)}}\right]\eta\sin(z\eta)d\eta$$

$$+\frac{2A}{\pi}\int_0^\infty\left[\frac{\eta\left[(\nu-\alpha c)\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)\right]}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2}e^{\frac{-\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)t}{\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)}}\right]\sin(z\eta)d\eta. \quad (15)$$

Expression (Equation 15) is the exact solution for the velocity field induced by accelerated flows over an oscillating plate for MHD second grade fluid in a porous medium and rotating frame.

The resulting steady state solution $F_{st}(z,t)$ (valid for large times) can be expressed as:

$$F_{st}(z,t)=A(t+1)e^{-\left(\sqrt{\frac{\phi}{k}+\frac{c}{\nu}}\right)z}$$

$$+UH(t)\cos\omega t\left(1-\frac{2}{\pi}\right)\times\int_0^\infty\left[\frac{\left(\nu\frac{\phi}{k}+c\right)\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)+\omega^2\left(1+\alpha\frac{\phi}{k}\right)\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)}{\eta\left[\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)^2\right]}\right]\sin(z\eta)d\eta$$

$$+\frac{2}{\pi}UH(t)\omega\sin\omega t\int_0^\infty\left[\frac{\eta(\nu-\alpha c)}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)^2}\right]\sin(z\eta)d\eta. \quad (16)$$

The solution (Equation 16) can also be written as (Appendix) (Gradhsteyn and Rhyzik, 1994):

$$F_{st}(z,t)=A(t+1)e^{-\left(\sqrt{\frac{\phi}{k}+\frac{c}{\nu}}\right)z}+UH(t)e^{-Bz}\cos(\omega t-Cz) \quad (17),$$

where

$$2B^2=\sqrt{b^4+S^2}+b^2,$$

$$2C^2=\sqrt{b^4+S^2}-b^2,$$

$$b^2=\left[\frac{\phi}{k}+\frac{(\nu c+\alpha\omega^2)}{(\nu^2+\alpha^2\omega^2)}\right]$$ and

$$S^2=\left[\frac{\omega(\nu-\alpha c)}{(\nu^2+\alpha^2\omega^2)}\right]^2 \quad (18)$$

For the case $F(0,t)=At+UH(t)\sin\omega t$

In this case, we apply similar procedure as previously applied, and then the starting solution $F(z,t)$ and steady state solution $F_{st}(z,t)$ are respectively presented as follows:

$$F(z,t)=A(t+1)e^{-\left(\sqrt{\frac{\phi}{k}+\frac{c}{\nu}}\right)z}+\frac{2A}{\pi}\int_0^\infty\left[\frac{\eta\left[(\nu-\alpha c)\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)\right]}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2}e^{\frac{-\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)t}{\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)}}\right]\sin(z\eta)d\eta$$

$$+\frac{2UH(t)\omega}{\pi}\int_0^\infty\left[\frac{(\nu-\alpha c)\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)^2}e^{\frac{-\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)t}{\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)}}\right]\eta\sin(z\eta)d\eta$$

$$+\frac{2}{\pi}UH(t)\sin\omega t\int_0^\infty\left[\frac{\nu\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)+\alpha\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)^2}\right]\eta\sin(z\eta)d\eta$$

$$-\frac{2}{\pi}UH(t)\omega\cos\omega t\int_0^\infty\left[\frac{\nu\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)-\alpha\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)^2}\right]\eta\sin(z\eta)d\eta \quad (19)$$

$$F_{st}(z,t)=A(t+1)e^{-\left(\sqrt{\frac{\phi}{k}+\frac{c}{\nu}}\right)z}$$

$$+UH(t)\sin\omega t\left(1-\frac{2}{\pi}\right)\times\int_0^\infty\left[\frac{\left(\nu\frac{\phi}{k}+c\right)\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)+\omega^2\left(1+\alpha\frac{\phi}{k}\right)\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)}{\eta\left[\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)^2\right]}\right]\sin(z\eta)d\eta$$

$$-\frac{2}{\pi}U\omega\cos\omega t\int_0^\infty\left[\frac{\eta(\nu-\alpha c)}{\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)^2}\right]\sin(z\eta)d\eta \quad (20)$$

The solution (Equation 20) can be reduced to a closed form:

$$F_{st}(z,t)=A(t+1)e^{-\left(\sqrt{\frac{\phi}{k}+\frac{c}{\nu}}\right)z}+UH(t)e^{-Bz}\sin(\omega t-Cz) \quad (21)$$

MODIFIED STOKES' SECOND PROBLEM

Here, we consider the electrically conducting fluid between the two plates which are of distance d apart. The upper plate is fixed, whiles the lower one is oscillating and accelerating. This stated problem is governed by Equations 6 and 7, and the following conditions:

$$F(d,t)=0 \quad t>0, \quad (22)$$

$$F(z,0)=0\ ;\ 0<z<d \quad (23)$$

For the case $F(0,t)=At+UH(t)\cos\omega t$

The starting solution here is:

$$F(z,t)=UH(t)\left(1-\frac{z}{d}\right)\cos\omega t-\frac{2}{d}UH(t)\cos\omega t\sum_{n=1}^\infty\left[\frac{\left(\nu\frac{\phi}{k}+c\right)\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)+\omega^2\left(1+\alpha\frac{\phi}{k}\right)\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)}{\lambda_n\left[\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)^2\right]}\right]\sin(\lambda_n z)$$

$$+\frac{2}{d}UH(t)\omega\sin\omega t\sum_{n=1}^\infty\left[\frac{\lambda_n(\nu-\alpha c)}{\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)^2}\right]\sin(\lambda_n z)$$

$$+\frac{2A}{d}\sum_{n=1}^{\infty}\left[\frac{\lambda_n \nu\left[t+1\right]}{\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)}\right]\sin(\lambda_n z)$$

$$-\frac{2UH(t)}{d}\sum_{n=1}^{\infty}\left[\frac{(\nu-\alpha c)\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)}{\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)^2}e^{\frac{-\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)t}{\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)}}\right]\lambda_n\sin(\lambda_n z)$$

$$+\frac{2A}{d}\sum_{n=1}^{\infty}\left[\frac{\lambda_n\left[(\nu-\alpha c)\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)\right]}{\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)^2}e^{\frac{-\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)t}{\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)}}\right]\sin(\lambda_n z) \quad .(24)$$

and the steady state solution is:

$$F(z,t)=UH(t)\left(1-\frac{z}{d}\right)\cos\omega-\frac{2}{d}UH(t)\cos\omega\sum_{n=1}^{\infty}\left[\frac{\left(\nu\frac{\phi}{k}+c\right)\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)+\omega^2\left(1+\alpha\frac{\phi}{k}\right)\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)}{\lambda_n\left[\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)^2\right]}\right]\sin(\lambda_n z)$$

$$+\frac{2}{d}UH(t)\omega\sin\omega t\sum_{n=1}^{\infty}\left[\frac{\lambda_n(\nu-\alpha c)}{\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)^2}\right]\sin(\lambda_n z)$$

$$+\frac{2A}{d}\sum_{n=1}^{\infty}\left[\frac{\lambda_n \nu\left[t+1\right]}{\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)}\right]\sin(\lambda_n z) \quad (25)$$

with

$$\lambda_n=\frac{n\pi}{d}.$$

For the case $F\left(0,t\right)=At+UH(t)\sin\omega t$

The starting solution here is:

$$F(z,t)=UH(t)\left(1-\frac{z}{d}\right)\sin\omega-\frac{2}{d}UH(t)\sin\omega\sum_{n=1}^{\infty}\left[\frac{\left(\nu\frac{\phi}{k}+c\right)\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)+\omega^2\left(1+\alpha\frac{\phi}{k}\right)\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)}{\lambda_n\left[\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)^2\right]}\right]\sin(\lambda_n z)$$

$$-\frac{2}{d}UH(t(\omega\cos\omega t\sum_{n=1}^{\infty}\left[\frac{\lambda_n\nu\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)-\alpha\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)}{\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)^2}\right]\sin(\lambda_n z)$$

$$+\frac{2A}{d}\sum_{n=1}^{\infty}\left[\frac{\lambda_n \nu\left[t+1\right]}{\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)}\right]\sin(\lambda_n z)$$

$$+\frac{2UH(t)\omega}{d}\sum_{n=1}^{\infty}\left[\frac{(\nu-\alpha c)\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)}{\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)^2}e^{\frac{-\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)t}{\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)}}\right]\lambda_n\sin(\lambda_n z)$$

$$+\frac{2A}{d}\sum_{n=1}^{\infty}\left[\frac{\lambda_n\left[(\nu-c)\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)\right]}{\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)^2}e^{\frac{-\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)t}{\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)}}\right]\sin(\lambda_n z) \quad (26)'$$

And the steady state solution is:

$$F(z,t)=UH(t)\left(1-\frac{z}{d}\right)\sin\omega-\frac{2}{d}UH(t)\sin\omega\sum_{n=1}^{\infty}\left[\frac{\left(\nu\frac{\phi}{k}+c\right)\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)+\omega^2\left(1+\alpha\frac{\phi}{k}\right)\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)}{\lambda_n\left[\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)^2\right]}\right]\sin(\lambda_n z)$$

$$-\frac{2}{d}UH(t(\omega\cos\omega t\sum_{n=1}^{\infty}\left[\frac{\lambda_n\nu\left(1+\alpha\eta^2+\alpha\frac{\phi}{k}\right)-\alpha\left(\nu\eta^2+\nu\frac{\phi}{k}+c\right)}{\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)^2+\omega^2\left(1+\alpha\lambda_n^2+\alpha\frac{\phi}{k}\right)^2}\right]\sin(\lambda_n z)$$

$$+\frac{2A}{d}\sum_{n=1}^{\infty}\left[\frac{\lambda_n \nu\left[t+1\right]}{\left(\nu\lambda_n^2+\nu\frac{\phi}{k}+c\right)}\right]\sin(\lambda_n z) \quad (27)$$

RESULTS AND DISCUSSION

Here, we present the graphical illustrations of the steady state of the velocity profiles which have been determined for the MHD second grade fluid in a porous medium and rotating frame induced by accelerated flows over an oscillating plate. The discussion on the physical interpretation of the emerging parameters then follow suit.

These profiles show the behaviours of the velocity field with regards to various values of the emerging parameters. These parameters are defined here as:

1. Ω is the rotating parameter,
2. $M=\frac{\sigma B_{\circ}^2}{\rho}$ is the magnetic field parameter,
3. α is the second grade parameter,
4. $P=\frac{\phi}{k}$ is the porous medium.

In order to illustrate the role of these parameters on the real and imaginary parts of the velocity field $F\left(z\right)$,

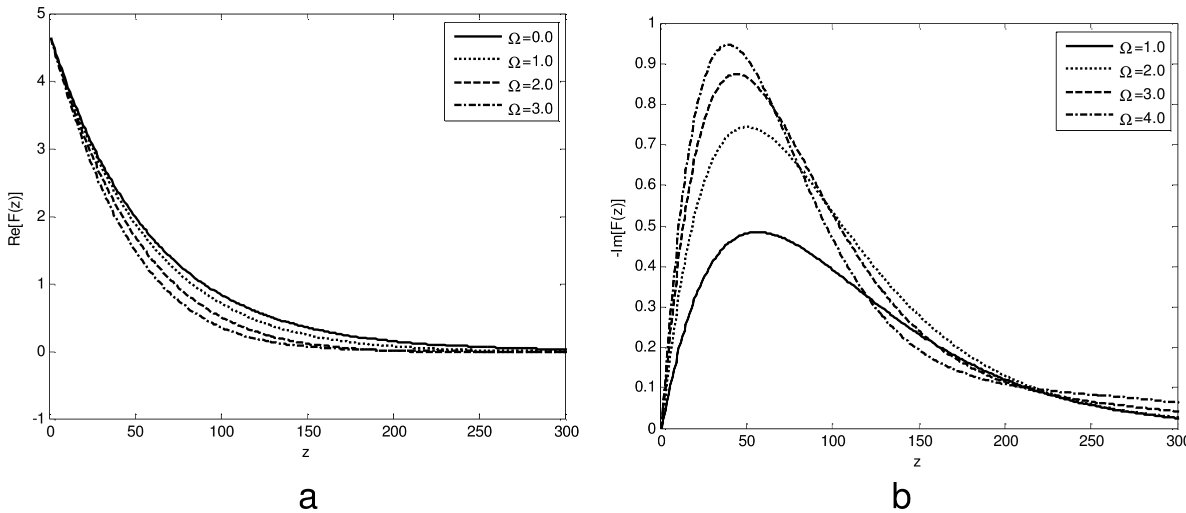

Figure 1. Profiles of the normalized steady state velocity $F(z)$ for various values of Ω when $A=.5, \alpha=1, P=1, M=2, t=2\pi$).

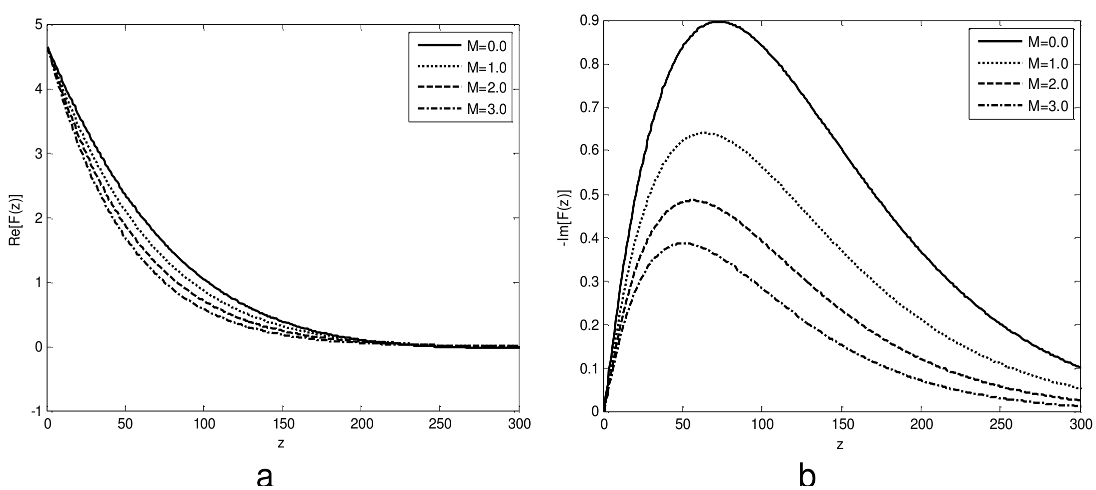

Figure 2. Profiles of the normalized steady state velocity $F(z)$ for various values of M when ($A=.5, \alpha=1, P=1, \Omega=1, t=2\pi$).

Figures 1 to 4 have been displayed. In these Figures, panels (a) depict the variations of Re [$F(z)$] and panels (b) indicate the variations of – Im [$F(z)$]. Figure 1a shows that the real part of the velocity profile decreases for various values of rotation Ω, with respect to the increase in z. As the number of rotation Ω increases, it is found that the velocity profile decreases. Figure 1b indicates that the magnitude of imaginary part of the velocity profile increases initially and later decreases for various values of rotation Ω, with respect to the increase in z. As Ω increases, the velocity profile also increases correspondingly.

Figure 2a is prepared to show the effects of the applied magnetic field M on the real part of the velocity profile. Keeping P, α, Ω, t fixed and varying M, it is noted that the real part of the velocity profile decreases by increasing the magnetic field parameter M. Figure 2b is portrayed to see the effects of the applied magnetic field on the imaginary part of the velocity profile. Keeping α, P, Ω, t fixed and varying M, it is observed that the imaginary part increases initially and later decreases. Clearly, we observe that with increasing values of M, the velocity profile of $F(z)$ decreases in both real and imaginary parts of the velocity profiles in Figure 2a and b. In fact this is because of the effects of transverse magnetic field on an electrically conducting fluid which gives rise to a resistive type force called the Lorentz force which tends to slow down the motion of the fluid.

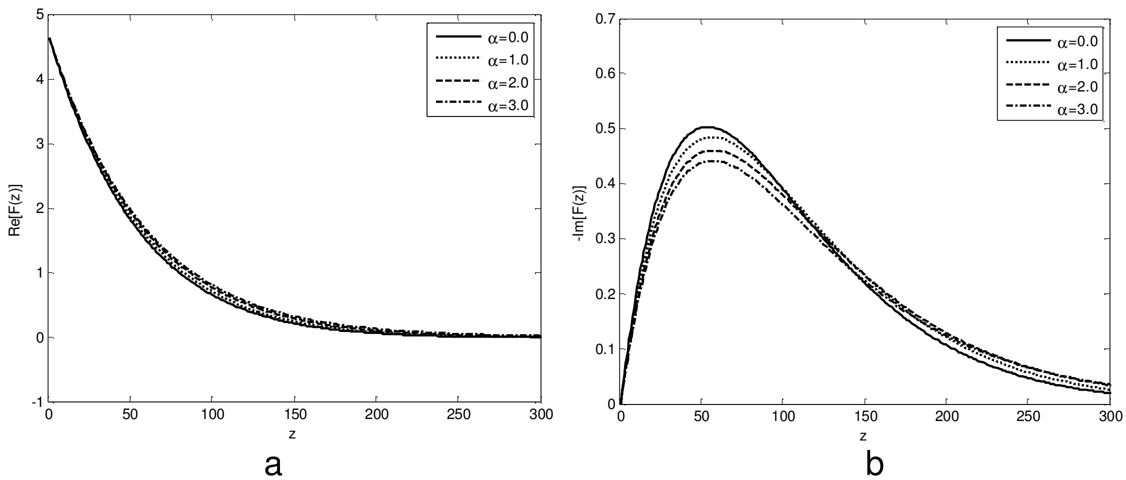

Figure 3. Profiles of the normalized steady state velocity $F(z)$ for various values of α when $(A=.5, M=2, P=1, \Omega=1, t=2\pi)$.

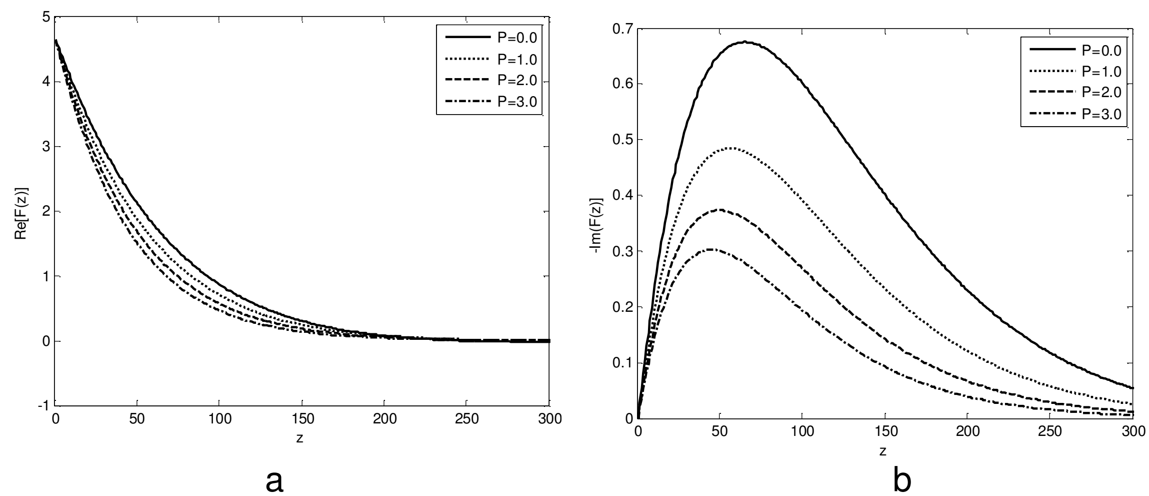

Figure 4. Profiles of the normalized steady state velocity $F(z)$ for various values of P when $(A=.5, \alpha=1, M=2, \Omega=1, t=2\pi)$.

Figure 3a shows the effects of second grade parameter α of second grade fluid on real part of velocity profile when P, M, Ω, t are fixed. It is interesting to notice that increasing the second grade parameter α, would lead to increase in the real part of the velocity profile. Figure 3b is obtained when P, M, Ω, t are fixed and the second grade parameter α is increased, and this would lead to imaginary part of the velocity profile increasing initially and later decreases. In both parts of Figure 3a and b, the velocity profile increases with increase in the second grade parameter α. This is because, the fact that increasing values of α would reduce the friction forces, and thus assists the flow of the fluid considerably; and hence, the f luid moves with greater velocity. Figure 4a indicates the variations of the porous parameter P. Keeping M, α, Ω, t fixed, it is noted that increasing the porous parameter P, would lead to decrease in the real part of the velocity profile. Keeping M, α, Ω, t fixed and varying P, it is noted that the imaginary part increases initially and later decreases (Figure 4b). Here, both the real and imaginary parts of velocity profiles in Figure 4a and b decrease with an increasing values of the porous parameter P. This is due to the fact that increasing values of P would lead to increase in the friction forces which thus slow down the motion of the fluid.

The graphical illustrations showed the behaviours of the steady state velocity profiles which have been determined for the MHD second grade fluid in a porous medium and rotating frame, which are being induced by accelerated flows over an oscillating plate. These profiles depict the performances of the velocity field with regards to various values of the emerging parameters Ω, M, α, P.

Conclusion

The exact solutions is established for the velocity field corresponding to the motion of MHD second grade fluid in a porous medium and rotating frame induced by accelerated flows over an oscillating plate. This is achieved by means of the Fourier sine and Laplace transforms technique. It is found that the solutions satisfy all the governing equations and all the imposed boundary conditions. Clearly the graphical results on the solutions portray the behaviour of the corresponding velocity fields with respect to the various emerging parameters.

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REFERENCES

Zhaosheng Y, Jianzhong L (1998). Numerical research on the coherent structure in the viscoelastic second-order mixing layers. Appl. Math. Mech., 19: 717-723.

Fetecau C, Hayat T, Khan M, Corina F (2011a). Erratum to: Unsteady flow of an Oldroyd-B fluid induced by the impulsive motion of a plate between two side walls perpendicular to the plate. Acta Mech., 216: 359–361.

Fetecau C, Hayat T, Zierep J, Sajid M (2011b). Energetic balance for the Rayleigh- Stokes problem of an Oldroyd-B fluid, Non-linear Anal. Real World Appl., 12: 1–3.

Faisal S, Zainal AA, Dennis LCC (2011a). New exact solution for Rayleigh – Stokes problem of Maxwell fluid in a porous medium and rotating frame. Results Phys., 1: 9–12.

Faisal S, Zainal AA, Dennis LCC (2011b). New exact solutions for MHD transient rotating flow of a second grade fluid in a porous medium, J. Appl. Math., doi: 10.1155/2011/823034.

Shen F, Tan WC, Zhao Y, Masuoka T (2006). The Rayleigh-Stokes problem for a heated generalized second grade fluid with fractional derivative model, Nonlinear Anal. Real World Appl., 7: 1072-1080.

Tan WC, Masuoka T (2005). Stokes' first problem for a second grade fluid in a porous half space with heated boundary. Int. J. Non-linear. Mech., 40: 515–522.

Erdogan ME, Imrak CE (2005). On unsteady unidirectional flows of a second grade fluid. Int. J. Non-linear. Mech., 40: 1238–1251.

Hussain M, Hayat T, Asghar S, Fetecau C (2010). Oscillatory flows of second grade fluid in a porous space, Nonlinear Anal. Real World Appl., 11: 2403-2414.

Fetecau C, Corina F (2005). Starting solutions for unsteady unidirectional flows of a second grade fluid. Int. J. Eng. Sci., 43: 781–789.

Hayat T, Fetecau C, Sajid M (2008). Analytic solution for MHD transient rotating flow of a second grade fluid in a porous space. Nonlinear Anal. Real World Appl., 9: 1619–1627.

Jamil M, Rauf A, Fetecau C, Khan NA (2011). Helical flows of second grade fluid due to constantly accelerated shear stresses. Comm. Nonlinear Sci. Numer. Simulat, 16: 1959 -1969.

Corina F, Jamil M, Fetecau C, Siddique I (2009). A note on the second problem of Stokes for Maxwell fluids. Int. J. Non-Linear Mech., 44: 1085-1090.

Islam S, Haroon T, Manzoor Elahi, Murad Ullah, Siddiqui AM (2011). Steady and unsteady exact inverse solutions for the flow of a viscous fluid. Int. J. Phys. Sci. 6: 6234-6240.

Grandshteyn IS, Ryzhik IM (1994). Tables of Integrals, Series and Products 5th ed. Academic Press, New York,

APPENDIX

In order to obtain Equation 17 from Equation 16, we use the result in this work (Grandshteyn and Ryzhik, 1994):

$$\int_0^\infty \frac{x\sin(ax)}{(x^2+\in b^2)+f}dx = \frac{\pi}{2f}\exp(-aB)\sin(aC) \quad ,$$

$$\int_0^\infty \frac{x(x^2+\in b^2)\sin(ax)}{(x^2+\in b^2)+f}dx = \frac{\pi}{2}\exp(-aB)\cos(aC) \, ,$$

where

$$2B^2 = \sqrt{b^4+f^2}+\in b^2 \ , \ 2C^2 = \sqrt{b^4+f^2}-\in b^2$$

and

$\in = \pm 1$.