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Full Length Research Paper

Stability analysis of fuzzy filtering type III

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In order to get the best corresponding answer in accordance with a reference model signal, digital filters should have the minimum error at its output using the mean square criterion. Inserting a fuzzy mechanism into its internal structure to construct an intelligent filter, the reference signal was adaptively interpreted to select and emit a decision answer in accordance with external reference signal changes, thereby updating the best correct new conditions into a process dynamically. The fuzzy filter gets the interpretation of the input signal level selecting the best weight parameter values from a set of membership functions stored into the knowledge base (KB), in order to give a signal approximation of the reference signal in natural form. The fuzzy stage improves the filter answers minimizing its convergence error using a classification of its operation into levels considering the minimum error distance. This work describes the stability properties of the fuzzy filter in accordance with the Kharitonov's polynomials theory that establishes the maximum and minimum limits intervals of the fuzzy filter and the Routh-Hurwitz criterion to get the stability analysis of the filtering process. The states of the fuzzy filtering require that all of its answers bound into the error criteria probabilistically, in accordance with the Nyquist and Shannon assumptions and finally the paper shows the simulations of the fuzzy filter into the Kalman structure using the Matlab© tools.

Key words: Digital filtering, fuzzy systems, estimation, stability.

INTRODUCTION

The development of intelligent algorithms as fuzzy filters requires a mechanism that infers or deduces its own natural external environment changes that affect the process operation. This algorithm architecture should have a classification of its own set of answers in order to discover dynamically the different operation levels of an external process. Interpreting the actual system conditions to deduce and select the best response updating its answers weights, in order to follow the best approximation condition in accordance to the external process changes by taking a decision about the actual condition to correct or update a system (Smith et al., 2003). Actually, one of the best tools used to solve this, is a digital filter with recursive structure to adjust dynamically its parameters weights in order to give bounded answers with respect to the mean square error criterion. A digital filter is an algorithm integrated into a computer software or electronic chip with different configurations depends on its uses as eliminate the noise of a system, get specific data, system identification or predict a system (Ali, 2003; García et al., 2008; Haykin, 2001).

The main problem in conventional filtering operation is that it can not characterize and infer its operation into levels; interpreting its external environment changes in order to select its answers with the smallest error and considering different operation options in dynamical

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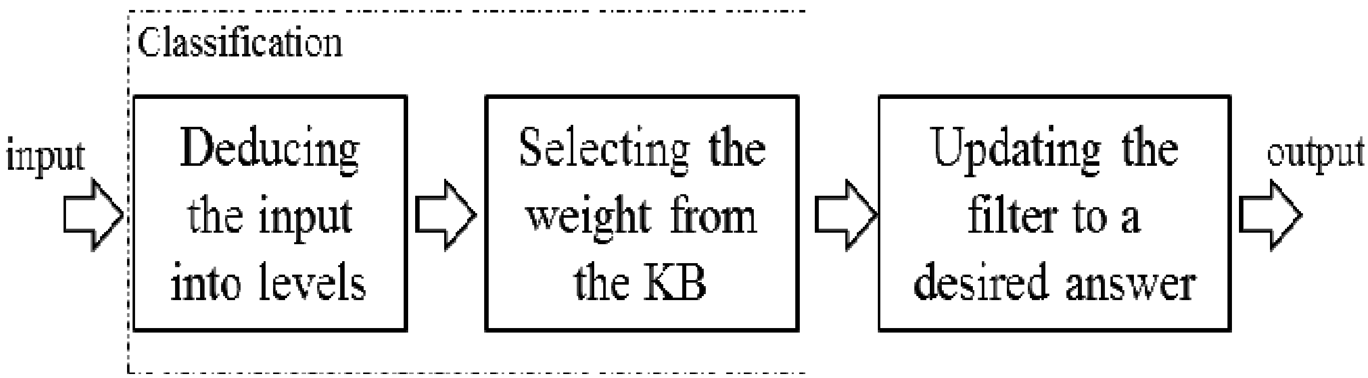

Figure 1. The fuzzy process.

sense, following the natural evolution of the reference system that interacts with the filter. In addition, a conventional filter does not have a linguistic description as levels to display its answer conditions as human interface, expressing it as levels, making it difficult to know the corresponding answer level as non-intelligent process which has some problems to develop capacities with high dynamical changing processes (Amble, 1987; Banga et al., 2011). The systems related to artificial intelligent mechanisms should use its architecture fuzzy tools (as fuzzy neural net and evolutive systems) in order to get its own perception giving the best decision answers, using this to solve complex problems, actualizing and adapting its operation levels in accordance with a reference dynamical system interactions and its possible changes, in order to get the necessary convergence conditions (Marcek, 2004; Zadeh, 1965). The goal of this kind of filter is the characterization of a system that has uncertainties in its operation, describing the natural process, with a rule set and the knowledge base. This requires a feedback law in order to follow the basic properties of a desired input signal, adjusting its parameters to give a correct solution dynamically to minimize the error criterion response (Rajen et al., 2006; Medel et al., 2008). According to this, the filter takes the best decision answer to update the system. One important problem is to probe the stability conditions of a fuzzy system (fuzzy filter in this case), but using the Kharitonov theory with the Routh-Hurwitz criterion in order to get the polynomials stability, it will be easier and it could be integrated into the fuzzy stage in order to demonstrate a system correct operation (Zadeh, 1965; Chen et al., 2005). This work describes this stability analysis in order to establish the conditions to use the Kharitonov polynomials and Ruth-Hurwitz criterion (García et al., 2010).

With this perspective, the paper integrates the FDF (Fuzzy Digital Filter) concept with its stability method, using into the simulation the Kalman filter internal structure to integrates the fuzzy stage, showing its operation by levels in natural way in order to make a specific decision and follow the natural reference model using fuzzy logic type III (Onieva et al., 2010).

THE FUZZY PROCESS

The fuzzy systems basic ideas since the first advances of fuzzy logic by Zadeh in 1965, about the fuzzy sets, have as important goals are the use of approximation reasoning, establishing a relation using the fuzzy mechanism interacting with an external process natural dynamics. According with this the fuzzy methods have been applying since approximately fifteen years ago to develop new intelligent technologies considering a stage that interpret different operational levels(García et al., 2011; Tao et al., 2005). A fuzzy system has stable operation delimiting all its variables and answers into the knowledge base (Kb) required by the process conditions, this case is limited by the filter restriction described by the mean square error criterion. A fuzzy system has a memory of all the answer levels given by the process. The answer level classification uses a set of membership functions; each one has a specific value in accordance with the operation level interpretation at the input. Using fuzzy rules with the logic connector IF, interprets the input, the fuzzy mechanism deduce the output value and with the logic connector THEN, selects the correct membership function into the Kb in order to updates a process (Mamdani, 1974; Takagi et al., 1986).

First, the fuzzy system interprets its environment changes at its input, in order to get a specific operation level of the reference system, having a characterization of the input signal $y(k)$ (desired signal) into levels. Then, the fuzzy stage choose the corresponding IF-THEN rule to get the membership function value $\hat{a}(k)$ in order to select the best parameter into the knowledge base, updating the process to a new required condition (Gustafsson, 2008; Passino, 1998). The fuzzy mechanism, with respect to the Figure 1, describes the reasoning of an external real process operation (reference model) in order to have a set of answers limited into levels. Dynamically the fuzzy stage interprets its input data describing the external process into different levels, the fuzzy stage has previously the set of all the possible values that can get from its input (classification). Then having this membership grade to deduce the answer value to be selected, automatically makes a selection of the membership value into the knowledge base (this has all possible answers values that it can give), and with this new parameter value updates the process to a new required condition (Passino, 1998; García et al., 2011). The characterization of a fuzzy system into levels could describe different kinds of process states (as low, medium or high) in accordance with the variable (temperature, velocity or pressure). The fuzzy mechanism dynamically gets a corresponding answer to update a process respect to the input changes. Using the fuzzy rules in order to get the input level and select the corresponding answer value $\hat{y}(k)$. This process consider stable conditions because the characterization of a system uses as a limit the frequency distribution probabilistically or using a set of membership grades (with triangle or Gaussian functions) to get the operation levels of a process (Ash, 1970). The membership function in a fuzzy system is a value to select with the minimum error using the actual input level from the reference process to update the process to a new condition. The operation levels are previously into the knowledge base of the fuzzy system as in artificial intelligence using supervised learning (Takagi et al., 1986).

The goal of this kind of system is to classify an external reference process into grades and get the best corresponding updating answer dynamically with the best signal process approximation avoiding and minimizing the possible errors (Zadeh, 1998). The description of the signal into a fuzzy structure is using ranks limited by the reference system and the filter criterion in order to characterize the signal levels as: First using the logic connector **IF** to get the membership grade of the input $y(k)$, second using the logic connector **THEN** to get the corresponding parameter value $\hat{a}(k)$, third the fuzzy mechanism updates the process to a new condition, and gets the output value described as $\hat{y}(k)$ with the best approximation.

FUZZY DIGITAL FILTERING (FDF)

The fuzzy filter is an intelligent algorithm that has two stages basically; the conventional adaptive filter structure considering all its conditions and the fuzzy stage integrated to the filter structure in order to characterize its operation to get a best signal approximation by levels. This stages integration works together minimizing the error criterion difference having a natural signal description in accordance with the reference model changes. On the other hand, a digital filter structure without this fuzzy stage cannot have a characterization of an external process to have a normal operation without get an answer into levels, making difficult to select the best signal approximation with the minimum error. This makes difficult to interpret the input signal of the external process and to get the best parameter value in order to update the process with the nearest value to have a better natural signal description (García et al., 2008). According with this a fuzzy digital filter is an adaptive filter adding a fuzzy stage that classifies the input signal $y(k)$, into fuzzy grades to select the best filter parameter value $\hat{a}(k)$ from the knowledge base in order to update the filter weights dynamically trough time to get the best signal approximation respect to the operation level. The goal is to give a desired operation condition each time with stable operation because this filter uses as limit answer value the mean square error criterion (Equation 1) as adaptive filtering, but the fuzzy stage minimize the error difference and gets the best signal approximation. The membership functions (parameters value set) are limited into the knowledge base (Medel et al., 2008).

$$J(k) = E\left\{(e(k))^2\right\}^{\frac{1}{2}} \tag{1}$$

The criterion $J(k)$ to reduce the filter error describes the mean square of the error between the desired signal $y(k)$ and the filter output $y(k)$, allows finding the corresponding membership function that is the best signal approximation in order to minimize the filter criterion (Haykin, 2001; García et al., 2008). The fuzzy filter with adaptive properties has an iterative searching methodology using the back propagation (BP) learning algorithm, which updates its parameters per iteration dynamically by degrees classifying its membership functions into the knowledge base in accordance to the difference between the desired response $y(k)$and the actual filter output $\hat{y}(k)$ described as the error $e(k)$. The Figure 2 shows the fuzzy filter structure:

The FDF interacts with an external reference model (real process) changes and the filter will selects the best answer to approximate the signal with minimum error (Yamakawa, 2009). Then, the fuzzy filter has next stages (Margaliot et al., 2000):

1. The fuzzy filter has the classification of the reference process model conditions as operation levels; having the knowledge of all the possible levels.
2. It interprets the desired signal $y(k)$ operation levels with the error value $e(k)$ in probabilistic form using the inference mechanism (with the logic connector ***if***).

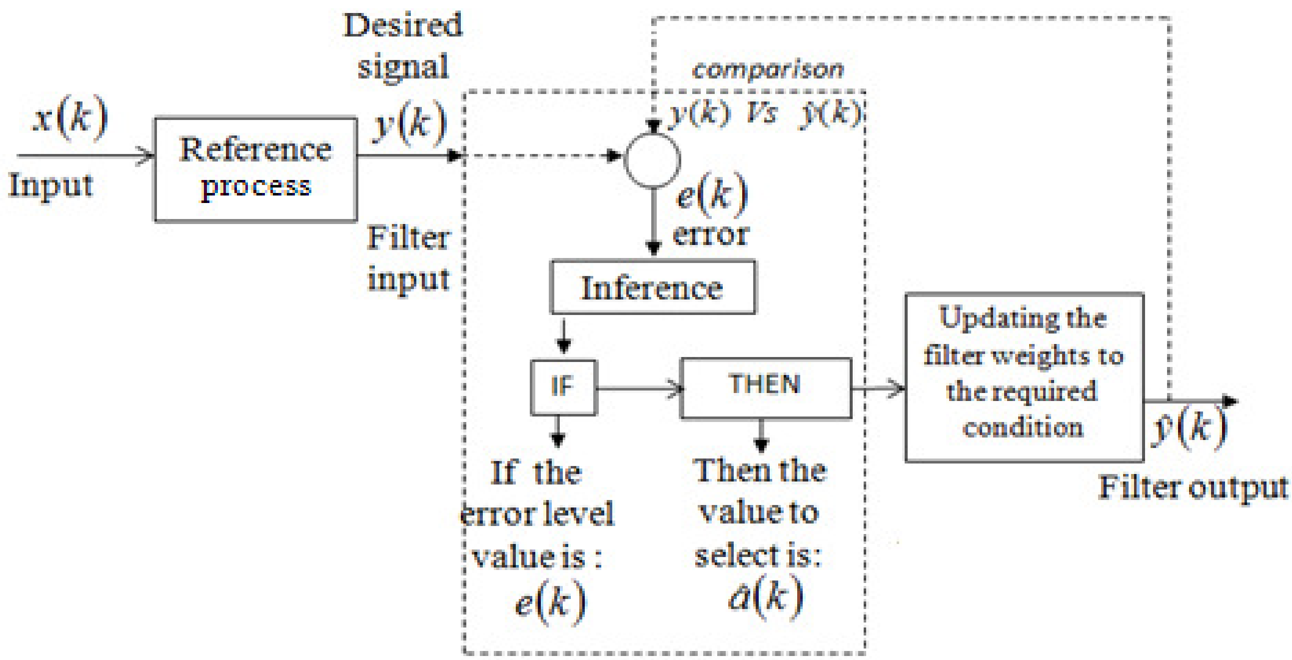

Figure 2. Fuzzy filter description.

3. It selects a corresponding membership function $\hat{a}(k)$ (parameter value) from the knowledge base (using the logic connector ***then***) according to the error level $e(k)$.
4. The parameter value $\hat{a}(k)$ selected, updates the filter operation, adapting automatically its weigh values in accordance with the square criterion, with the best approximation value of the desired signal $y(k)$.
5. The filter emits the best answer $\hat{y}(k)$, describing its responses as linguistic operational levels and continually repeats this cycle.

This computational model described as intelligent process may interacts with natural processes as biologic signals, using processing elements making connections to constitute an automatic updating selection to follow the best condition of a process. The control area T_N is a set of answers that the filter has with respect to all set of desired signals that the reference system emit; the control area (Equation 2) is into membership intervals inside the knowledge base; in accordance to a filtering criterion (mean square error). The set of membership function (parameters value) represents all the correct responses into the Knowledge Base (KB), according to an objective law, predefined by this natural reference process, the filter inference chooses the best correct response from the knowledge base to each reference model change.

$$T_N = \{(y(k), \hat{y}(k))\} \tag{2}$$

The desired signal $y(k)$ at the filter input is changing its operation conditions continuously, described in levels. The filter interprets the corresponding level interval and selects the best parameter value $\hat{a}(k)$ (membership function) from the knowledge base. Updating the filter operation to give a correct response $\hat{y}(k)$, the goal of the filtering process as predictor is to follow the desired signal $y(k)$ to describe the reference process into operation grades, and then the output filter $\hat{y}(k)$ will be approximately the same signal (Ash, 1970; García et al., 2011). This filtering process determines the best value to update de filter conditions and give a response with the smallest error.

FUZZY FILTERING TYPE III

The problem to solve is the integration into the identification process the fuzzy algorithm stage with time restrictions. Using the fuzzy logic type III, permitting to have a classification of the reference system answer, bounded all of them with a probability function with n∈ N. Using the system as finite differences with a set of rules, which defines a dynamic local system of n order limited into a distribution functions (Onieva et al., 2010).

$$\boldsymbol{if}\ \left(y(k-1) isin M_1^{i_1}\right)\ \boldsymbol{and} \ldots \left(y\left(k-(n-1)\right) isin M_{n-1}^{i_{n-1}}\right) \tag{3}$$

With each rule parameter described as:

$$\mu_{M(y(k))}^{t} := \omega^{i_1 \dots i_n}(y(k))$$

$$\omega^{i_1 \dots i_n}(y(k)) \in \left\{ \mu_{M_1^{t_1}(y(0))} \cdots \mu_{M_n^{t_n}(y(k-(n-1)))} \right\} \quad (4)$$

Describing the type III systems into metrics as:

$$a^{i_1 \cdots i_n}(y(k)) = 1 - \sum_{i_n = 1}^{r_n} \cdots \sum_{i_1 = 1}^{r_1} \left\{ \left[a^{i_1 \cdots i_n}\left(y(k-(n-1))\right) - a^{i_1 \cdots i_n}(y(0))\, a_0^{i_1 \cdots i_n} \right] \right\}, \forall\, i_j, j = \overline{1, n}, n \in [2,n) \quad (5)$$

And having

$$\hat{y}(k) = \left[\hat{a}_0^{i_1 \dots i_n} \quad \cdots \quad \hat{a}_n^{i_1 \dots i_n}\right]\left[y(0) \quad \cdots \quad y(k-(n-1))\right]^T \quad (6)$$

where $\{M_j^{i_1}, j = \overline{1,n}, n \in Z_+\}$ is the group of fuzzy sets for each. $y(k-(n-j))$. And having $\hat{a}(y(k))$ inside the convex set of estimated parameters as $\{\hat{a}^{1\cdots 1}, \hat{a}^{2\cdots 2}, \cdots \hat{a}^{r_1 \cdots r_n}\}$ and the pounder identification described as $(\hat{y}_p(k))$ having that:

$$\hat{y}_p(k) = \frac{\sum_{i_n=1}^{r_n} \cdots \sum_{i_1=1}^{r_1} \omega^{i_1 \cdots i_n}(y(k-(n-1)))\left[\hat{a}^{i_1 \cdots i_n} y(k-(n-1))\right]}{\sum_{i_n=1}^{r_n} \cdots \sum_{i_1=1}^{r_1} \omega^{i_1 \cdots i_n}(y(k-(n-1)))} \quad (7)$$

The classification of *y(k-(n-1))*, represents the desired signal described by the density functions with its corresponding interval for each *n*, respect to the metrics set $\left\{\mu_{M_1^{t_1}(y(0))} \cdots \mu_{M_n^{t_n}(y(k-(n-1)))}\right\}$. This permits to have an information base associated to the estimated parameters $\{\hat{a}^{i_1 \cdots i_n}[y(k-(n-1))]\}$ in order to get the identified output of the filter $\hat{y}(k)$ and its signal pondered $\hat{y}_p(k)$), using the knowledge base (KB) that has all the values set to get the natural operation conditions of the reference system (Onieva et al., 2010). Constructing the inference rules into the probability functions and its intervals in order to select from the knowledge base the best parameter of the set $\{\hat{a}^{i_1 \cdots i_n}[y(k-(n-1))]\}$; getting the best possible reference system answer approximation $\hat{y}(k)$. This process is the fuzzy approximation (Zadeh, 1998).

STABILITY DESCRIPTIONS

The fuzzy filtering interacting with a real process has all its answers bounded in order to give a correct parameter selection. But the main problem to solve is the description of the stability analysis for a filter operation with fuzzy stage properties having a description of its answers into intervals. In 1978, Kharitonov made a theory for robust systems using four polynomials (Chen et al., 2005; García et al., 2011). In accordance with the theory using four polynomials is possible to have a fuzzy stability description (into levels or ranks) using the Routh-Hurwitz criteria for lineal systems as fuzzy filters (Tao et al., 2005).

The system described in Equation 7, as Z-transform is:

$$G(z) = \frac{Y(z)}{\tilde{W}(z)} \quad (8)$$

where the proper output equation described as:

$$Y(z) := b_n z^{-n} + b_{n-1} z^{-(n-1)} + \ldots + b_1 z^{-1} + b_0$$
$$\text{with } n \in N \quad and \ \#n \in \aleph_0 \quad (9)$$

and its characteristic equation described as:

$$\tilde{W}(z) = a_n z^{-n} + a_{n-1} z^{-(n-1)} + a_{n-2} z^{-(n-2)} + \ldots + a_0 z^{-1}$$
$$\text{with } n \in N \quad and \ \#n \in \aleph_0. \quad (10)$$

Getting from each polynomial the parameter and establishing the uncertainty values as fuzzy systems bounded into intervals. The parameters are:

$$a_n \subseteq [\underline{a}_n, \overline{a}_n], \ \underline{\lim}\, a_n = \underline{a}_n, \ \overline{\lim}\, a_n = \overline{a}_n, \ n \in N. \quad (11)$$

From the polynomial described in Equation 10, in agreement to fuzzy logic parameters and the Kharitonov theory, we get two polynomials limits:

$$\tilde{W}(z) = \tilde{W}_{\text{Pair}}(z) + \tilde{W}_{\text{odd}}(z) \quad (12)$$

Separating the pair and odd parts from Equations 10 and 11, we get the next polynomials:

$$\tilde{W}(z)_{\text{pair}} = a_0 + a_2 z^{-2} + a_4 z^{-4} + \cdots$$

$$\tilde{W}(z)_{\text{odd}} = a_1 z^{-1} + a_3 z^{-3} + a_5 z^{-5} + \cdots \quad (13)$$

where $\underline{a}_n$ and $\overline{a}_n$ are the minimum and maximum limit intervals of each membership function. The nominal value of each interval $[\underline{a}_n, \overline{a}_n]$ (that is the valid value of the membership function or parameter described as a_n). Having the characteristic equation description in robust sense as:

$$\tilde{W}(z) = [\underline{a}_n, \overline{a}_n] z^{-n} + [\underline{a}_{n-1}, \overline{a}_{n-1}] z^{-(n-1)} + [\underline{a}_{n-2}, \overline{a}_{n-2}] z^{-(n-2)} + \cdots + [\underline{a}_0, \overline{a}_0] \quad (14)$$

(a)

$$\begin{array}{c|ccc} z^{-5} & \underline{a_5} & \overline{a_3} & \underline{a_1} \\ z^{-4} & \underline{a_4} & \overline{a_2} & \underline{a_0} \\ z^{-3} & b_4 & b_3 & 0 \\ z^{-2} & c_4 & c_3 & 0 \\ z^{-1} & d_4 & 0 & 0 \\ z^{0} & e_4 & 0 & 0 \end{array}$$

$$b_4 = -\frac{\det\begin{vmatrix} \underline{a_5} & \overline{a_3} \\ \underline{a_4} & \overline{a_2} \end{vmatrix}}{\underline{a_4}} \qquad b_3 = -\frac{\det\begin{vmatrix} \underline{a_5} & \underline{a_1} \\ \underline{a_4} & \underline{a_0} \end{vmatrix}}{\underline{a_4}}$$

$$c_4 = -\frac{\det\begin{vmatrix} \underline{a_4} & \overline{a_2} \\ b_4 & b_3 \end{vmatrix}}{b_4} \qquad c_3 = -\frac{\det\begin{vmatrix} \underline{a_4} & \underline{a_0} \\ b_4 & b_2 \end{vmatrix}}{b_4}$$

$$d_4 = -\frac{\det\begin{vmatrix} b_4 & b_3 \\ c_4 & c_3 \end{vmatrix}}{c_4} \qquad e_4 = -\frac{\det\begin{vmatrix} c_4 & c_3 \\ d_4 & d_3 \end{vmatrix}}{d_4}$$

(b)

$$\begin{array}{c|ccc} z^{0} & \overline{a_5} & \underline{a_3} & \overline{a_1} \\ z^{-1} & \underline{a_4} & \overline{a_2} & \underline{a_0} \\ z^{-2} & b_1 & b_2 & 0 \\ z^{-3} & c_1 & c_2 & 0 \\ z^{-4} & d_1 & 0 & 0 \\ z^{-5} & e_1 & 0 & 0 \end{array}$$

$$b_1 = -\frac{\det\begin{vmatrix} \overline{a_5} & \underline{a_3} \\ \underline{a_4} & \overline{a_2} \end{vmatrix}}{\underline{a_4}} \qquad b_2 = -\frac{\det\begin{vmatrix} \overline{a_5} & \overline{a_1} \\ \underline{a_4} & \underline{a_0} \end{vmatrix}}{\underline{a_4}}$$

$$c_1 = -\frac{\det\begin{vmatrix} \underline{a_4} & \overline{a_2} \\ b_1 & b_2 \end{vmatrix}}{b_1} \qquad c_2 = -\frac{\det\begin{vmatrix} \underline{a_4} & \underline{a_0} \\ b_1 & b_3 \end{vmatrix}}{b_1}$$

$$d_1 = -\frac{\det\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}}{c_1} \qquad e_1 = -\frac{\det\begin{vmatrix} c_1 & c_2 \\ d_1 & d_2 \end{vmatrix}}{d_1}$$

Figure 3. Routh-Hurwitz for the first (a) and second (b) polynomials.

The constructions of the Kharitonov's polynomials into the fuzzy systems theory have the next description:

$$\tilde{W}(z)_1 = \underline{a_0} + \overline{a_2}z^{-2} + \underline{a_4}z^{-4} + \cdots$$

$$\tilde{W}(z)_2 = \overline{a_0} + \underline{a_2}z^{-2} + \overline{a_4}z^{-4} + \cdots \qquad (15)$$

$$\tilde{W}(z)_3 = \underline{a_1}z + \overline{a_3}z^{-3} + \underline{a_5}z^{-5} + \cdots$$
$$\tilde{W}(z)_4 = \overline{a_1}z^{-1} + \underline{a_3}z^{-3} + \overline{a_5}z^{-5} + \cdots$$

Now we get the Kharitonov's polynomials describing the fuzzy filter stability as:

$$\tilde{W}_{1,3}(z) = \tilde{W}(z)_1 + \tilde{W}(z)_3 = \underline{a_0} + \underline{a_1}z^{-1} + \overline{a_2}z^{-2} + \overline{a_3}z^{-3} + \underline{a_4}z^{-4} + \underline{a_5}z^{-5} + \cdots$$

$$\tilde{W}_{1,4}(z) = \tilde{W}(z)_1 + \tilde{W}(z)_4 = \underline{a_0} + \overline{a_1}z^{-1} + \overline{a_2}s^{-2} + \underline{a_3}z^{-3} + \underline{a_4}z^{-4} + \overline{a_5}z^{-5} + \cdots$$

$$\tilde{W}_{2,3}(z) = \tilde{W}(z)_2 + \tilde{W}(z)_3 = \overline{a_0} + \underline{a_1}z^{-1} + \underline{a_2}z^{-2} + \overline{a_3}z^{-3} + \overline{a_4}z^{-4} + \underline{a_5}z^{-5} + \cdots$$

$$\tilde{W}_{2,4}(z) = \tilde{W}(z)_2 + \tilde{W}(z)_4 = \overline{a_0} + \overline{a_1}z^{-1} + \underline{a_2}z^{-2} + \underline{a_3}z^{-3} + \overline{a_4}z^{-4} + \overline{a_5}z^{-5} + \cdots \qquad (16)$$

This represents the polynomial bounds of the fuzzy filter; if all of them are Hurwitz the filtering process has stability conditions (Chen et al., 2005).

Routh-Hurwitz method

Having the Kharitonov's polynomials bounded into fifth grade, with the parameters describing the membership functions limits and using the Routh-Hurwitz criteria for each polynomial, we can observe and prove the stability conditions of the system. Having for the first (Figure 3a) and second (Figure 3b) polynomials. And now for the third (Figure 4a) and fourth (Figure 4b) polynomials. The following entails the description for the first in (Figure 3a) and the second in (Figure 3b) polynomials. The result of this method propose a modification into the parameters of the characteristic equation, because in fuzzy control we can get the mathematical structure of the system described into intervals and analyze the changes dynamically; this is the difference with respect to the classical form, this method cannot serve in classical manner, because it have to analyses all the possible parameters

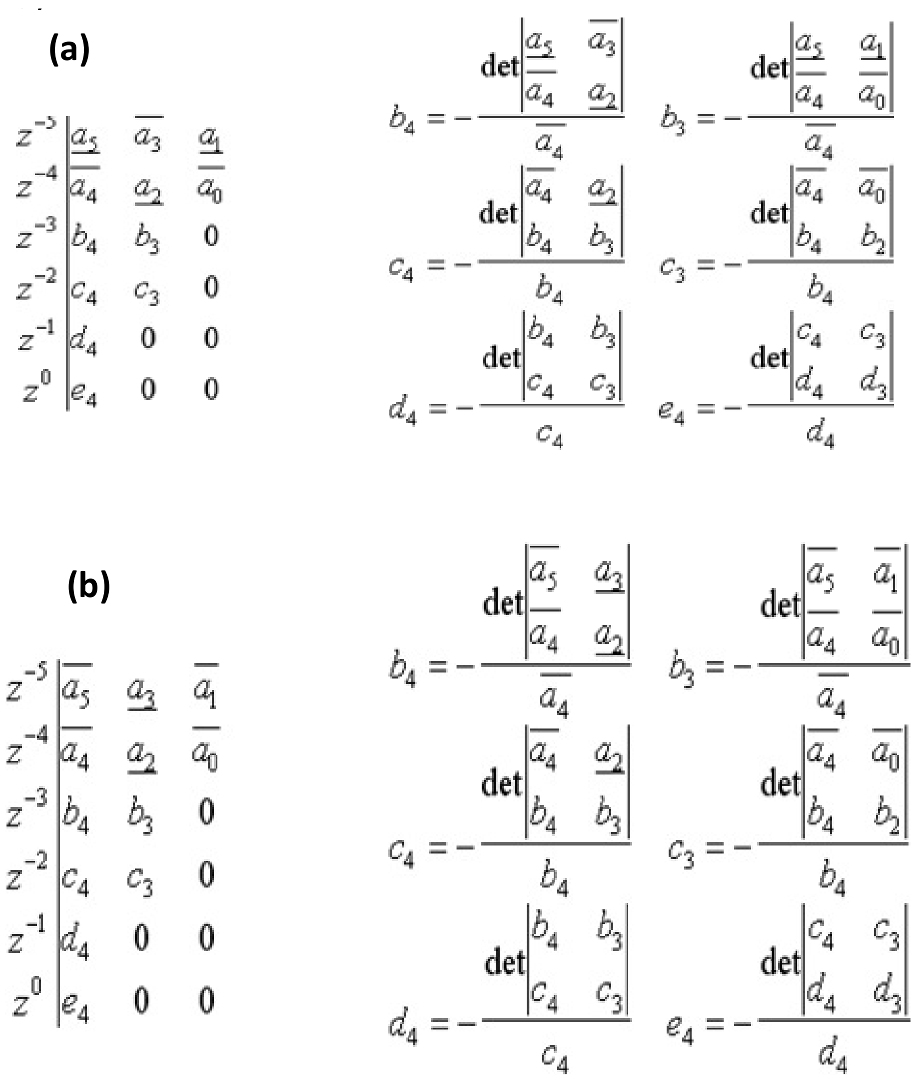

Figure 4. Routh-Hurwitz for the first (c) and second (d) polynomials.

combinations into the intervals (Takagi et al., 1986; García et al., 2011). The set of parameters bounded into the maximum and minimum limits use the four polynomials of Kharitonov that describes a fuzzy process and for each polynomial we get the Routh-Hurwitz criteria, in order to get the stability in robust sense. The four polynomials region is a description of the Dasgupta rectangle to describe the system stability as: (1) A region into the left semi plane S for the Routh-Hurwitz, for continuous systems. 2) A region into the unitary circle into the Z plane, for discrete systems (Tao et al., 2005).

Simulation results

For the simulation of the fuzzy filter, we integrated the Kalman filter structure with the fuzzy stage in order to get a characterization of the filter operation, showing graphically the fuzzy filter response. The Kalman filter in this case uses the identification configuration, having a dynamical parameter selection in order to get the best desired signal (from the reference model) approximation. The filter has a set of membership functions describing all the filter changes limited by the error criterion (Equation 1). The reference model into the simulation is an ARMA model (Autoregressive Mobile Average), which interacts with the fuzzy filter in order to get the best answers. This has a description in discrete states space, expressed by the fifth order difference as:

$$x(k+1) = \sum_{i=k}^{1} a(k-i)x(k-i) + w(k) \qquad (17)$$

The fifth order corresponds to $k = 5$ and its output has the next description:

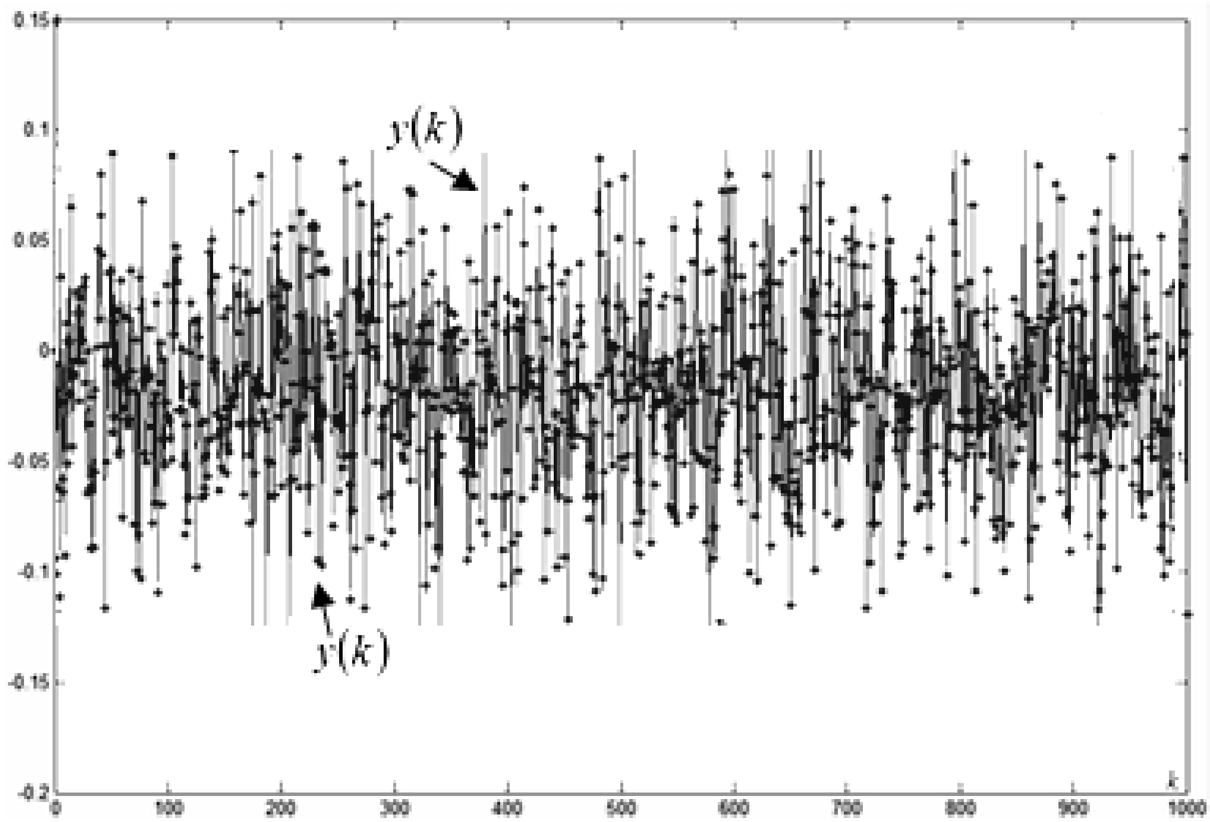

Figure 5. Desired signal approximation.

$$y(k) = x(k) + v(k) \ : \quad x(k),\ w(k),\ v(k) \in R \qquad (18)$$

where x(k) is the reference model internal states sequence, a(k) is the parameters sequence, w(k) is the reference model noise, y(k) is the desired signal from the reference model to the filter input and v(k) is the output vector noises.

Considering the output recursive form with respect to Equations 17 and 18,

$$y(k) = \bar{a}\bar{r}(k) + \tilde{w}(k) \qquad (19)$$

Where

$\tilde{w}(k) = f(\{v(k-i)\}, w(k-i), v(k)) \subseteq N(\mu_{\tilde{w}}, \sigma_{\tilde{w}}^2 < \infty)$,

$y(k),\ \tilde{w}(k) \in R,\ \ \bar{a} := [a(k-1) \vdots \cdots \vdots a(0)] \subseteq R^{[k \times 1]}$,

$\bar{r}(k) := [y(k-1) \vdots \cdots \vdots y(0)]^T \subseteq R^{[1 \times k]}$ and Equation 19 with upper (*U*) and lower (*L*) boundaries

$$\bar{y}(k) = \bar{a}_U \bar{r}(k) + \tilde{w}(k)$$

$$\underline{y}(k) = \bar{a}_L \bar{r}(k) + \tilde{w}(k), \qquad (20)$$

Respectively, where $\bar{a}_U$ and $\bar{a}_L$ are the boundaries corresponding to each membership function. The nominal value set of these intervals $[\bar{a}_U, \bar{a}_L]$ is the parameters vector $\bar{a}$. Now considering (Equation 19) the Z-transform, the characteristic equation is:

$$\tilde{W}(z) = (1 - \bar{a}[z^0 \vdots \cdots \vdots z^{-(k-1)}]^T) \qquad (21)$$

In robust sense,

$$\tilde{W}(z) = (1 - [\bar{a}_U, \bar{a}_L][z^0 \vdots \cdots \vdots z^{-(k-1)}]^T) \qquad (22)$$

It is described in explicit form as

$$\tilde{W}(z) = 1 - ([\underline{a}_0, \bar{a}_0] + [\underline{a}_1, \bar{a}_1]z^{-1} + [\underline{a}_2, \bar{a}_2]z^{-2} + [\underline{a}_3, \bar{a}_3]z^{-3} + \cdots + [\underline{a}_{(k-1)}, \overline{a_{(k-1)}}]z^{-(k-1)}) \qquad (23)$$

With the Kalman fuzzy filter, the different operation levels of the filter proposed must match inside the filter error criterion, in accordance with the desired signal $y(k)$, the membership parameter selection of $\hat{a}(k) \subseteq R^{([1 \times 6], k)}$ into the filter structure, and the filter output $\hat{y}(k) \in R$, in accordance with a mathematical selection process into the fuzzy filter. The Figure 5 shows the desired signal

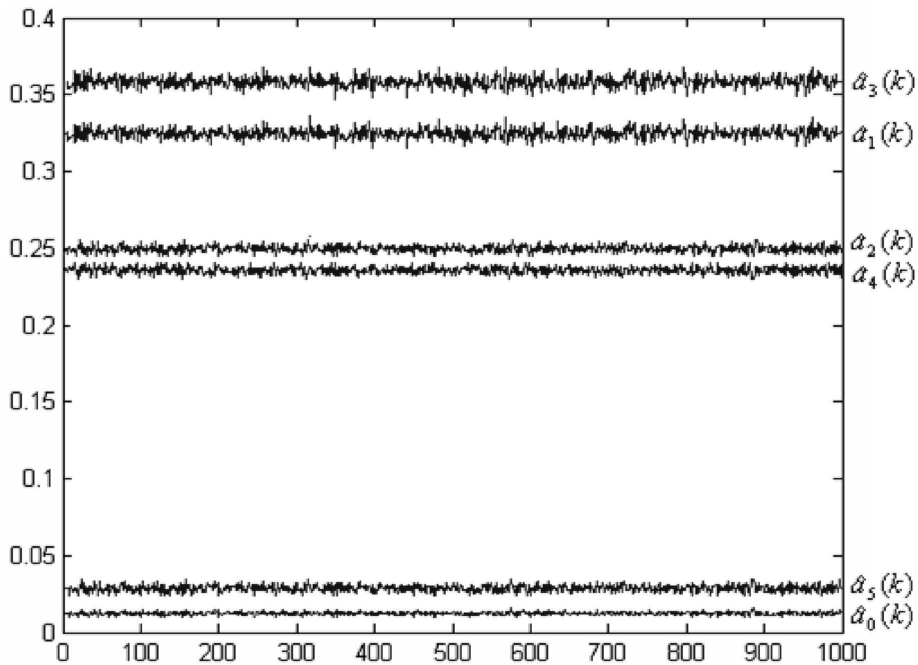

Figure 6. Membership value selection.

$y(k) \in R$ approximation with the fuzzy filter output described as $\hat{y}(k) \in R$. In accordance with the desired signal levels, the fuzzy stage makes a selection process by the fuzzy rules to get dynamically the parameter value (membership value) changing its values through time. Figure 6, shows this process schematically. Figure 8 shows the fuzzy stage making a classification process and describing the parameter values into levels dynamically. Next has a selection of the best correspondence with the reference process changes, in order to get the best signal approximation considering the lowest error value. In accordance to Equation 23 and having five delays:

$$[\underline{a_0}, \overline{a_0}] = [0.012, 0.0125], [\underline{a_1}, \overline{a_1}] = [0.32, 0.321],$$
$$[\underline{a_2}, \overline{a_2}] = [0.249, 0.25], [\underline{a_3}, \overline{a_3}] = [0.357, 0.36],$$
$$[\underline{a_4}, \overline{a_4}] = [0.24, 0.241], [\underline{a_5}, \overline{a_5}] = [0.028, 0.029],$$

With the form:

$$\tilde{W}(z) = 1 - \left([0.012, 0.0125] + [0.32, 0.321]z^{-1} + [0.249, 0.25]z^{-2} + [0.357, 0.36]z^{-3} + [0.24, 0.241]z^{-4} + [0.028, 0.029]z^{-5}\right) \quad (24)$$

The Kharitonov's polynomials considered in Equation 24 with the fuzzy estimation regions described as follows:

$$\tilde{W}(z)_1 = 0.18 + 0.09z^{-2} + 0.1z^{-4},$$

$$\tilde{W}(z)_2 = 0.22 + 0.03z^{-2} + 0.16z^{-4}, \quad (25)$$

$$\tilde{W}(z)_3 = 0.32z + 0.3z^{-3} + 0.028z^{-5},$$

$$\tilde{W}(z)_4 = 0.37z^{-1} + 0.23z^{-3} + 0.032z^{-5}.$$

and observing from Equation 25 that the polynomials coefficients does not have sign changes using Routh-Hurwitz. Figure 7 has the state identification process $x(k)$ of the reference model, using the fuzzy filter with a description of the internal process by $\hat{x}(k)$. This internal state describes the reference model information, with respect to the dynamical changes. With the fuzzy inference using the error levels and the set of fuzzy rules, the Figure 9 shows the membership error classification in order to select the best answer. Finally, in accordance with the fuzzy filter selection process the Figure 10, shows the different operational levels of the filter output described by $\hat{y}(k)$. The FDF had a description of the real signal into operational levels minimizing the error difference with the filter mechanism, as we can see the graphics above, the FDF did the classification of a signal,

$$\begin{array}{c|ccc} z^{-5} & \underline{a_5} & \overline{a_3} & \underline{a_1} \\ z^{-4} & \underline{a_4} & \overline{a_2} & \underline{a_0} \\ z^{-3} & b_4 & b_3 & 0 \\ z^{-2} & c_4 & c_3 & 0 \\ z^{-1} & d_4 & 0 & 0 \\ z^{0} & e_4 & 0 & 0 \end{array}$$

$$b_4 = -\frac{\det\begin{vmatrix} 0.028 & 0.36 \\ 0.24 & 0.25 \end{vmatrix}}{0.24} = 0.3308 \qquad b_3 = -\frac{\det\begin{vmatrix} 0.028 & 0.32 \\ 0.24 & 0.012 \end{vmatrix}}{0.24} = 0.3186$$

$$c_4 = -\frac{\det\begin{vmatrix} 0.24 & 0.25 \\ 0.3308 & 0.3186 \end{vmatrix}}{0.3308} = 0.01885 \qquad c_3 = -\frac{\det\begin{vmatrix} 0.24 & 0.012 \\ 0.3308 & 0 \end{vmatrix}}{0.3308} = 0.012$$

$$d_4 = -\frac{\det\begin{vmatrix} 0.3308 & 0.3186 \\ 0.01885 & 0.012 \end{vmatrix}}{0.01885} = 0.10801 \qquad e_4 = -\frac{\det\begin{vmatrix} 0.01885 & 0.012 \\ 0.10801 & 0 \end{vmatrix}}{0.10801} = 0.012$$

$$\begin{array}{c|ccc} z^{-5} & \overline{a_5} & \underline{a_3} & \overline{a_1} \\ z^{-4} & \underline{a_4} & \overline{a_2} & \underline{a_0} \\ z^{-3} & b_4 & b_3 & 0 \\ z^{-2} & c_4 & c_3 & 0 \\ z^{-1} & d_4 & 0 & 0 \\ z^{0} & e_4 & 0 & 0 \end{array}$$

$$b_4 = -\frac{\det\begin{vmatrix} 0.029 & 0.357 \\ 0.24 & 0.25 \end{vmatrix}}{0.24} = 0.32679 \qquad b_3 = -\frac{\det\begin{vmatrix} 0.029 & 0.321 \\ 0.24 & 0.012 \end{vmatrix}}{0.24} = 0.31955$$

$$c_4 = -\frac{\det\begin{vmatrix} 0.24 & 0.25 \\ 0.32679 & 0.31955 \end{vmatrix}}{0.32679} = 0.01531 \qquad c_3 = -\frac{\det\begin{vmatrix} 0.24 & 0.012 \\ 0.3654 & 0 \end{vmatrix}}{0.3654} = 0.012$$

$$d_4 = -\frac{\det\begin{vmatrix} 0.32679 & 0.31955 \\ 0.01531 & 0.012 \end{vmatrix}}{0.01531} = 0.06339 \qquad e_4 = -\frac{\det\begin{vmatrix} 0.01531 & 0.012 \\ 0.06339 & 0 \end{vmatrix}}{0.06339} = 0.012$$

$$\begin{array}{c|ccc} z^{-5} & \underline{a_5} & \overline{a_3} & \underline{a_1} \\ z^{-4} & \overline{a_4} & \underline{a_2} & \overline{a_0} \\ z^{-3} & b_4 & b_3 & 0 \\ z^{-2} & c_4 & c_3 & 0 \\ z^{-1} & d_4 & 0 & 0 \\ z^{0} & e_4 & 0 & 0 \end{array}$$

$$b_4 = -\frac{\det\begin{vmatrix} 0.028 & 0.36 \\ 0.241 & 0.249 \end{vmatrix}}{0.241} = 0.3310 \qquad b_3 = -\frac{\det\begin{vmatrix} 0.028 & 0.32 \\ 0.241 & 0.0125 \end{vmatrix}}{0.241} = 0.3185$$

$$c_4 = -\frac{\det\begin{vmatrix} 0.241 & 0.249 \\ 0.3310 & 0.3185 \end{vmatrix}}{0.3310} = 0.017043 \qquad c_3 = -\frac{\det\begin{vmatrix} 0.241 & 0.0125 \\ 0.3310 & 0 \end{vmatrix}}{0.3310} = 0.0125$$

$$d_4 = -\frac{\det\begin{vmatrix} 0.3310 & 0.3185 \\ 0.017043 & 0.0125 \end{vmatrix}}{0.017043} = 0.07573 \qquad e_4 = -\frac{\det\begin{vmatrix} 0.017043 & 0.0125 \\ 0.07573 & 0 \end{vmatrix}}{0.07573} = 0.0125$$

$$\begin{array}{c|ccc} z^{0} & \overline{a_5} & \underline{a_3} & \overline{a_1} \\ z^{-1} & \overline{a_4} & \underline{a_2} & \overline{a_0} \\ z^{-2} & b_4 & b_3 & 0 \\ z^{-3} & c_4 & c_3 & 0 \\ z^{-4} & d_4 & 0 & 0 \\ z^{-5} & e_4 & 0 & 0 \end{array}$$

$$b_4 = -\frac{\det\begin{vmatrix} 0.029 & 0.357 \\ 0.241 & 0.249 \end{vmatrix}}{0.241} = 0.327 \qquad b_3 = -\frac{\det\begin{vmatrix} 0.029 & 0.321 \\ 0.241 & 0.0125 \end{vmatrix}}{0.241} = 0.3194$$

$$c_4 = -\frac{\det\begin{vmatrix} 0.241 & 0.249 \\ 0.327 & 0.3194 \end{vmatrix}}{0.327} = 0.0136 \qquad c_3 = -\frac{\det\begin{vmatrix} 0.241 & 0.0125 \\ 0.327 & 0 \end{vmatrix}}{0.327} = 0.0125$$

$$d_4 = -\frac{\det\begin{vmatrix} 0.327 & 0.3194 \\ 0.0136 & 0.0125 \end{vmatrix}}{0.0136} = 0.01884 \qquad e_4 = -\frac{\det\begin{vmatrix} 0.0136 & 0.0125 \\ 0.01884 & 0 \end{vmatrix}}{0.01884} = 0.0125$$

Figure 7. Routh-Hurwitz stability for the four polynomials.

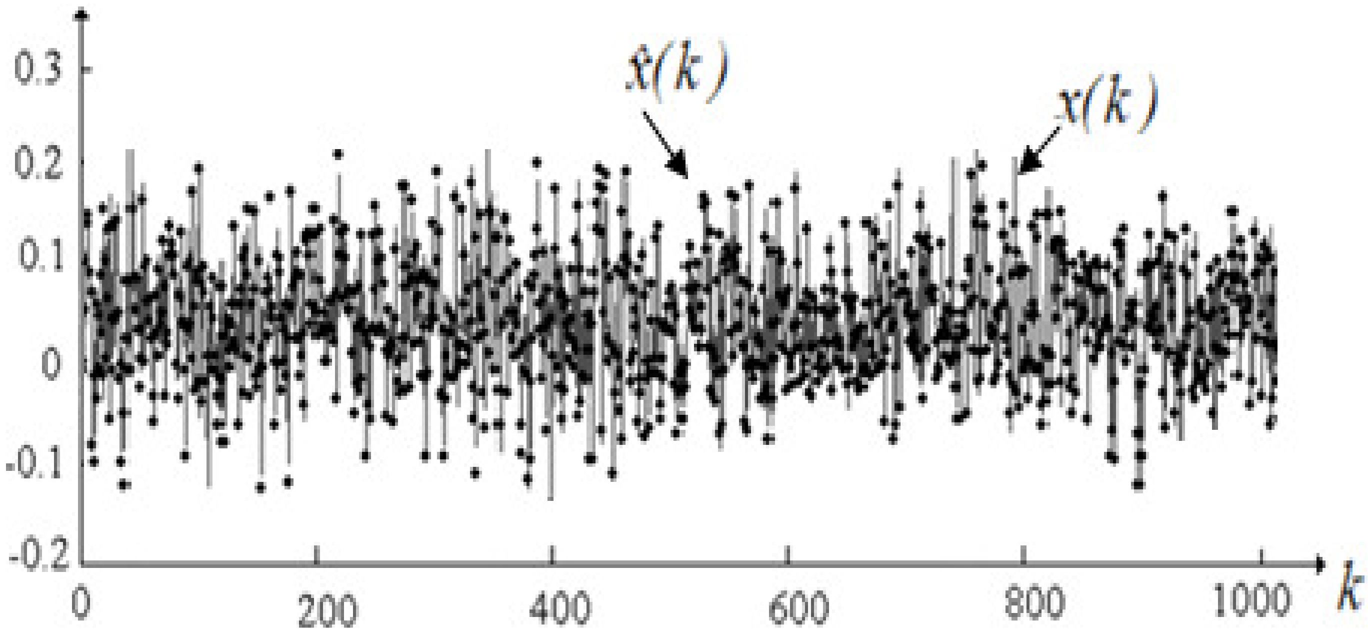

Figure 8. Internal states identification.

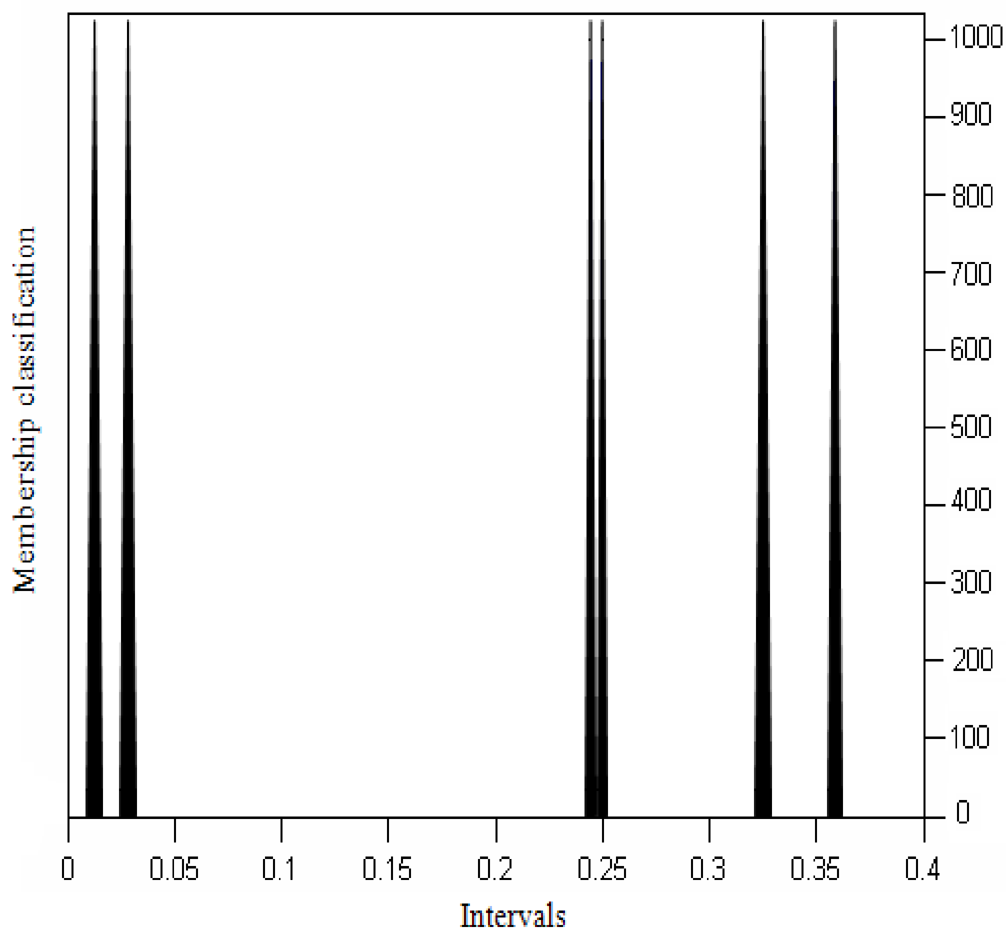

Figure 9. The error levels classification.

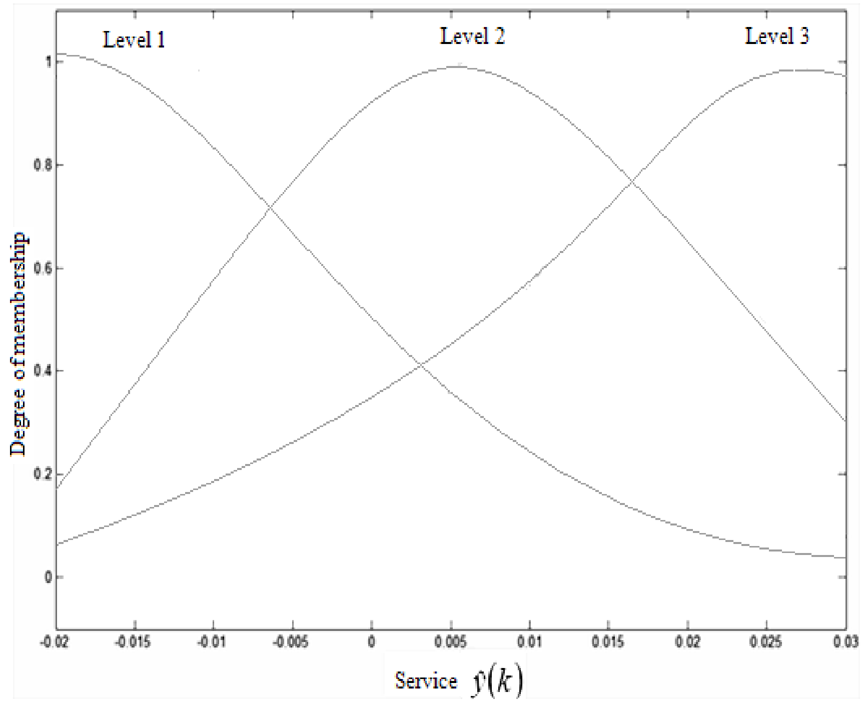

Figure 10. The filter service level.

having an inference that interpret the input (desired signal) and get the best parameter value in order to approximate its output in accordance to the changes.

Conclusion

The fuzzy filtering characterizes a signal in order to get the best answers in accordance with the changes on its environment. Which as artificial intelligent systems has a signal reasoning in order to describe the operation into levels, using logic rules into the fuzzy stage that interpret the input from an external process, and get the best membership value selected dynamically into the knowledge base, this process updates the filter weights inorder to get the best signal approximation (Shannon, 1948). This work establishes how to construct and characterize the membership values of the knowledge base in a probabilistic manner by its distribution function. Having a decision rules set and making a description of the real process signal using the fuzzy filter working together in order to act adaptively with the changes. The Kharitonov's theory help to establish the analysis of the filter process considering the maximum and minimum limits of the system in order to describe the stability of all possible combinations of the polynomials. Then with the Routh-Hurwitz criteria we could get the robust stability of each polynomial.

Finally, this work showed a simulation of the FDF operation using the Matlab tool and the Kalman filter structure to integrate the fuzzy mechanism, considering three answer levels, having an accurate filtering time response with respect to the reference model and the minimum error difference as natural process. The next trend in applications for this kind of intelligent filters would be the integration with technologies that needs reasoning into its architecture as artificial intelligence systems. For example future advanced devices that communicate with our brains describing our external and internal processes in order to deduce the best action to do dynamically, as biological human processes and its external environment changes.

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