International Journal of the Physical Sciences Vol. 6(20), pp. 4816-4825, 23 September, 2011
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS11.929
ISSN 1992 - 1950 ©2011 Academic Journals

Full Length Research Paper

Weighting method for modal parameter based damage detection algorithms

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Accepted 5 July, 2011

When researchers use bending modes for the purpose of damage detection, a number of available modes are adopted, each of which return a value indicating the severity of damage. Normally, there is a variance in the sensitivity of different modes and the global stiffness change is derived by taking the algebraic average, which is equivalent to the mean in statistical analysis terms. This study proposes weighting method for damage detection algorithms based on the bending mode shapes for beam structures. The results of the proposed reliability weighting method for two damage severity algorithms are presented, that is, one based on the natural frequencies and the other based on the mode shape vectors.

Key words: Modal parameters, damage algorithm, weighting.

INTRODUCTION

Inspection of structural components for damage is essential in decision making for the maintenance of structures. Dynamic testing has become an increasingly popular and important tool in structural health monitoring for identifying damage. Damage inspection of structures is important in order to come up with a planned strategy for repair and maintenance works. Vibration test has been used for damage detection since the 1970s and early 1980s in the offshore oil industry (Vandiver, 1975; Begg et al., 1976; Coppolino and Rubin, 1980). The basic idea behind this approach is that modal parameters that is, natural frequency, mode shape and modal damping, are functions of physical properties of structures namely mass, damping and stiffness. Therefore, any change in the physical properties will cause detectable changes in the modal parameters. Considerable research has been done on using the change in the natural frequencies for damage detection (Salawu, 1997). The alternative to using natural frequency as damage identification is by using mode shape, with modal assurance criteria (MAC) in determining the level of correlation between modes from the control beam and modes from the damaged beam (Doebling et al., 1998). MAC was first used by West (1984) to locate the structural damage without the use of prior finite element model. Modal parameters of lower modes were found to have satisfactory precision in detecting the crack position and depth (Ruotolo and Surace, 1997). The dynamic bending stiffness based on modal parameters was used to detect damage position and severity in RC (reinforced concrete) beams. The fundamental frequency had higher sensitivity and the third mode had less. The natural frequencies used to detect the crack location and depth was found to be able to predict the crack size with an error of 25% and detect the location with error of 12% (Lee and Chung, 2000). Natural frequencies and mode shapes were found to be useful in identifying the existence of damage (Alampalli, 2000). The trend in the natural frequencies was found to be sensitive to the corrosion deterioration state of RC beams (Abdul Razak and Choi, 2001). Frequencies were found to be affected by whether the loading configuration was symmetrical or asymmetrical, with odd modes affected more by the symmetrical configuration and the even modes affected more by the asymmetrical configuration. The MAC factor was found to be less sensitive than frequencies, but it can give an indication of the symmetrical or asymmetrical nature of damage (Ndambi et al., 2002). Modal parameters were found to underestimate the damage severity than the actual damage size (Kim and Stunns, 2002). Modal parameters

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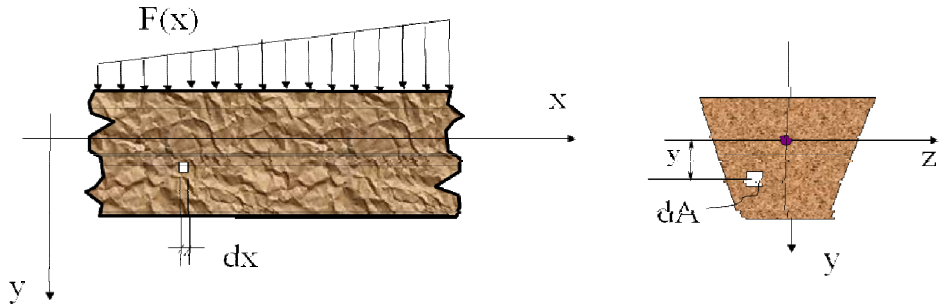

Figure 1. Beam element subjected to normal stress.

turned out as good indicators to assess damage in RC structures as compared to experimental natural frequencies, mode shapes and its derivatives (Johan, 2003). The natural frequencies were found to decrease to a larger extent as the crack size increased, with the change varying based on the mode number (Choubey et al., 2006). The change in the natural frequencies is a function of the crack length and its location and also depends on the mode shapes (Choubey et al., 2006). Jassim et al. (2010) developed an analytical model for detecting cracks in cantilever beam using the modal parameters, natural frequencies and mode shapes, and found that both modal parameters were affected by the crack regardless of position or size. Measured natural frequencies were used for damage detection and it was found that the first and third nodes are sufficient to get information regarding the damage extent and magnitude (Sathishkumar and Murthy, 2010). Lower modes were found to be more sensitive to the change in the support conditions (Fayyadh and Abdul Razak, 2010; Fayyadh et al., 2011). A new damage detection method based on the natural frequencies and the mode shape was proposed and proved to be a good indicator that needs only the natural frequencies from the datum data (Radzieński et al., 2011). The slope of the first mode shape was used for damage detection and showed good results with the only concern being that an error in identifying the mode shape can cause an error in the damage detection results (Zhua et al., 2011). A new damage detection index based on the combination between the mode shape vectors and their curvature was developed and verified to have higher sensitivity than existing algorithms (Fayyadh and Abdul Razak, 2011). Fundamental mode shape and static deflection were used for damage detection and found to have good sensitivity (Cao et al., 2011).

Previous studies have shown that both frequency and mode shape were used for damage detection, which means that any proposed weighting method has to be validated for both parameters. It was also observed that the change in the modal parameter affected by the presence of the damage varied based on the mode number. Moreover, the modal parameters were found to be affected by either the loading configuration was symmetrical or asymmetrical. Odd modes were more affected by the symmetrical configuration while the even modes were more affected by the asymmetrical configuration. It is therefore important to develop reliable weighting method which considers the different sensitivities of various modes and which is able to return one stiffness deterioration value.

WEIGHTING METHODS

The norm researchers have for deriving the global stiffness change is by taking the algebraic averaging, which is in the form of the mean, in terms of statistical analysis. This averaging method will always result in a constant weight for all the modes, which is equal to (1/n), where ‘n’ is the total adopted modes.

For a beam element subject to bending stresses, symmetrical in x-section about the z-axis, consider an infinitesimal volume element of length dx and area dA as shown in Figure 1.

This element is subjected to a normal stress σ_x, as follows:

$$\sigma_x = M.y/I \tag{1}$$

where M is the bending moment and I is the second moment of inertia of the cross section.

The strain energy on this element ‘U’ is as follows:

$$U = \frac{\sigma_x \, \varepsilon_x}{2} \tag{2}$$

where ε_x is the normal strain. For linear elastic material, we have:

$$U = \frac{\sigma_x^2}{2E} \tag{3}$$

where E is the elasticity modulus. Substituting, σ_x = M.y/I and multiplying by the volume of the element, we have:

$$u.dxdA = \frac{M^2 y^2}{2EI^2} .dxdA \tag{4}$$

Hence, the strain energy for a slice of the beam, of width dx, is as follows:

$$dU = \int_A \frac{\sigma^2}{2E}\, dx\, dA \quad (5)$$

$$dU = \frac{M^2}{2EI^2}\, dx \int_A y^2\, dA \quad (6)$$

and since $I_{xx} = \int_A y^2\, dA$

$$dU = \frac{M^2}{2EI}\, dx \quad (7)$$

External work '*W*' done by forces on structure equals the internal strain energy '*U*' which means:

$$W = U = \int \frac{M^2}{2EI}\, dx \quad (8)$$

Based on Equation 8, it is apparent that there is a direct relationship between the bending stiffness *EI* and the work *W*, and this implies that any change in the stiffness of the structure leads to change in the work.

For linear-elastic system deflection, δ is a linear function of the force *F*, as shown in Figure 2, and this gives the work *W* as follows:

$$W = \frac{F\delta}{2} \quad (9)$$

This means that the work equals the area under the curve of the load against deflection relationship. This concept applies to modal analysis in which the work equals the area under the curve of the mode shapes. The proposed reliability weighting method is based on the mode shapes of the bending modes for beam structures. The normal trend of the bending mode shapes will be a sine wave with number of peaks "n" corresponding to the number of the modes "n", that is, mode '1' with "one" peak and mode 'n' with "n" peaks. In order to make the beam move in accordance to the shape of any specific mode, an external work needs to be done. The work needed to move the beam in forming different modes will vary as a function of the mode shape number or the sine wave peak number, with higher number of wave peaks needing more work and vice versa.

Based on the relationship between the work and the bending stiffness as in Equation 8, it is suggested that the area under the mode shape curve of each specific mode can be used as a reliability weighting for that specific mode to the change in the bending stiffness *EI*. It is expected that the more the work needed for formation of the mode shape, the higher the reliability weighting that mode shape would have.

The area under the curves of the mode shapes will be calculated for each mode as (A_i) where *i* is the mode number and the total area (A_t) is the summation of the areas of the adopted modes. The Proposed Weighting (PW_i) of each specific mode is given in Equation 10, and the average index value of any adopted set of modes is given in Equation 11. Figure 3 shows the area under the curve for the first four bending modes for a case when four modes are adopted.

$$PW_i = \frac{A_i}{A_t} \quad (10)$$

and

$$Average_{DSA} = \sum_{i=1}^{n} DSA_i * PW_i \quad (11)$$

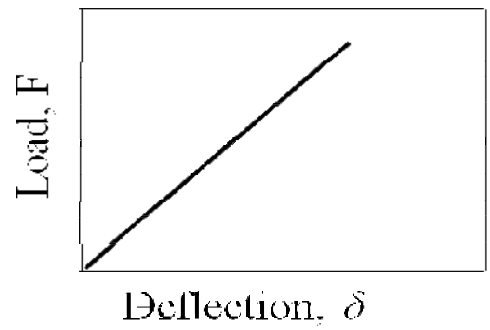

Figure 2. Work for linear elastic system.

where *i* is the mode number and DSA_i is the damage severity algorithm at mode *i*.

Damage severity algorithms

The proposed reliability weighting method will be examined with two damage severity algorithms in order to verify its viability. One of the algorithms is based on the natural frequencies and the other on the mode shapes.

Algorithm based on natural frequency

The natural frequency, *f*, for transverse free vibration of a simply supported beam as suggested by Demeter (1973) and used by Abdul Razak and Choi (2001) is given by:

$$f = \frac{n^2\pi}{2L^2}\sqrt{\frac{EI}{m}} \quad (12)$$

where n is mode number, m is mass per unit length and L is span length, is proportional to the square root of its flexural rigidity, *EI*, given as:

$$f \propto \sqrt{EI} \quad (13)$$

Equation 13 can be rewritten as:

$$f_n^2 \propto (EI) \quad (14)$$

By introducing a constant, we have:

$$f_n^2 = A(EI) \quad (15)$$

where

$$A = 2 f_n \frac{\delta f_n}{\delta(EI)} \quad (16)$$

Equation 16 can be rearranged as:

$$\delta f_n^2 = A\,\delta(EI) \quad (17)$$

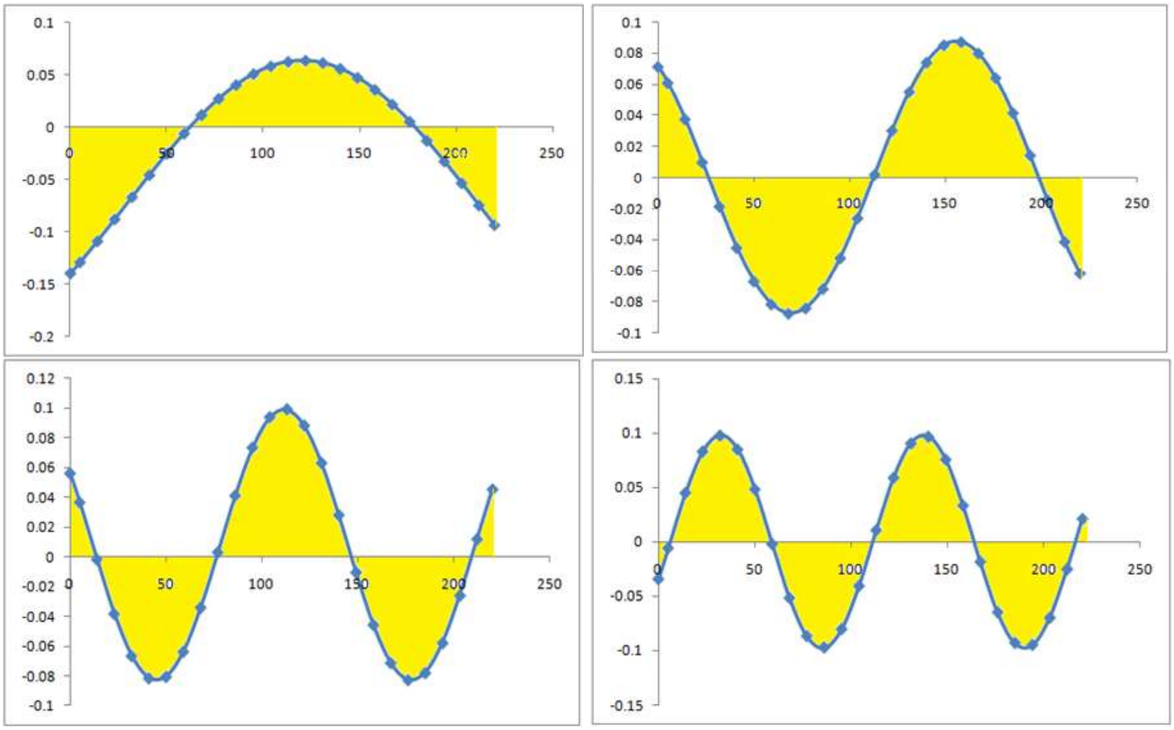

Figure 3. Area under the curve for each specific mode shape.

By substituting Equation 15 into Equation 17, the following relationship is found:

$$2\left(\frac{1}{f}\right)\partial f = \left(\frac{1}{EI}\right)\partial(EI) \qquad (18)$$

which implies that a change in flexural rigidity (EI) doubles the change in natural frequency. The stiffness change based on frequency is defined as:

$$\text{Stiffness change based on frequency} = 2.\left(1-\frac{f_{i,d}}{f_{i,c}}\right)\cdot 100\% \qquad (19)$$

where $f_{i,c}$ and $f_{i,d}$ are the natural frequency at i^{th} mode for control and damaged beam, respectively.

Algorithm based on mode shape

The method used to ascertain configuration errors between experimental mode shapes and eigenvectors predicted from the finite element model is called modal assurance criterion (MAC) (Ewins, 2000). It is a correlation between experimental mode shapes and curve-fitted mode shapes with the correlation for the *i*th element given by the following formula:

$$[MAC_{XA}]_{ii} = \frac{\left(\{\tilde{\varphi}_i\}^T\{\varphi_i^X\}\right)^2}{\left(\{\varphi_i^X\}^T\{\varphi_i^X\}\right)\left(\{\tilde{\varphi}_i\}^T\{\varphi_i^X\}\right)} \qquad (20)$$

where $[\tilde{\varphi}] = [\varphi^A][C] = [\varphi^A][\varphi^A]^+[\varphi^X]$. Matrix $[\varphi^A]$ contains the analytical model mode shapes, $[\varphi^A]^+$ is the pseudo-inverse of the matrix and $[C]$ is the curve-fitting matrix. The values of the diagonal elements of the MAC matrix give the curve-fitting results.

Utilizing the concept in the previous paragraph, the stiffness deterioration indicator can be considered as the reduction in MAC values for the damage cases based on the datum cases as in Equation 21. The stiffness change based on MAC is defined as:

$$\text{Stiffness change based on MAC} = 1 - \frac{\left|\sum_{i=1}^{n}\varphi_{i,c}\cdot\varphi_{i,d}\right|^2}{\left(\sum_{i=1}^{n}\varphi_{i,c}\cdot\varphi_{i,c}\right)\left(\sum_{i=1}^{n}\varphi_{i,d}\cdot\varphi_{i,d}\right)} \qquad (21)$$

and $\varphi_{i,c}$ and $\varphi_{i,d}$ are the mode shapes at *i* th mode for control and damaged beam, respectively.

CASE STUDY

To demonstrate the significance and capability of the proposed weighting method, a finite-element beamlike structure model was generated to represent control and damaged cases. The span length of the beam was 3250 mm with a cross-sectional area of 150 mm by 250 mm. The reliability level of the new method attempted to detect different damage levels and locations. Since the dynamic parameters are related to the stiffness of the structural element, the damage is presented by reducing the modules of elasticity *E* values. The stiffness reduction ratio (SRR) was adopted as notation for the damage level and can be calculated as:

SRR = $(1 - E_d/E_c).100\%$ (22)

where *Ed* is the modulus of elasticity value for the damage cases and *Ec* is the modulus of elasticity for the control case.

Four levels of stiffness were adopted that is, the control where *E* is 200 GPa, and the SRR was 0%. The first damage level is the smallest level where the *SRR* was 1% (*E* is 198 GPa), the second damage level is the medium level where the *SRR* was 5% (*E* is 190 GPa) and the third damage level is the highest level where the SRR was 12.5% (*E* is 175 GPa). The same damage levels were examined for different damage locations; the first damage was located at the mid-span and the second damage was located at the quarter-span. A total of six cases for different SRR levels and different damage locations were adopted. Table 1 shows the SRR adopted in the present study. Figure 4 shows the beamlike structure model for the control beam, mid-span damage and quarter-span damage.

Utilizing a general-purpose finite-element package that is based on the displacement method, a two-dimensional finite-element model was constructed to represent the beamlike structure model. The beam model was built by using a 4-node plane stress element. Figure 5 shows a typical model for the beam constructed using FE software. The physical and material properties of the beam were Poisson's ratio of 0.2, mass density of 7850 kg/m^3 and Young's modulus of 200 GPa for the control case. These vary from one damage case to another. The self-weight was computed by taking gravitational acceleration as 9.81 m/s^2 in the $-y$ direction.

Initially, eigen analyses were performed so that modal parameters for the control beam model could be approximated. Next, the damage was created on the beam model by changing the values of the modules of elasticity first at the mid-span (Figure 4b) and secondly at the quarter-span (Figure 4c). At each damaged case, eigenvalue analysis was again performed to obtain the modal parameters relevant to the damage case induced. Finally, the PW_i for each mode was calculated based on the area under the curve of the mode shapes, and then the average value was calculated for each algorithm. The PW_i for each mode are as shown in Table 2.

RESULTS AND DISCUSSION

This part present the results from the analytical modelling of the beamlike structure used in the present study. The stiffness change indices based on frequency and MAC were calculated for the first four modes of the control-beam model and three different damaged levels as 1, 5 and 12.5% reduction in *E* value at the mid-span and quarter-span of the beam model (Figure 4b and c). Figures 6 and 7 show the comparison of the stiffness change based on frequency values at different damage levels for damage located at mid-span and quarter-span, respectively.

The results show that the first and the third modes are the most sensitive to detect damage severity when the damage is located at mid-span, while modes two and three are the most sensitive when the damage is located at quarter-span. The higher values for all the modes were at the highest damage level at SRR 12.5% for both damage locations. The first mode shows the highest value for damage located at mid-span, while mode two has the highest value when the damage is located at the quarter-span. Figures 8 and 9 show graphically the comparison of stiffness change based on MAC at different damage levels for damage located at mid-span and quarter-span, respectively.

Stiffness change based on MAC values had lower sensitivity than stiffness change based on frequency, with the higher value being 0.05% for the third mode when the *SRR* was 12.5% for damage located at quarter-span. However, Modes 3 and 4 had higher sensitivity than the first two modes.

The results prove that for the stiffness change based on frequency, the first mode was the most sensitive when the damage was located at mid-span and the second mode was the most sensitive when it was located at quarter-span. For the stiffness change based on MAC, the third mode was the most sensitive regardless of the damage location. It is difficult to judge the stiffness deterioration quantitatively based on both modal parameters algorithms, since there is a variance between different modes. In order to come out with one stiffness deterioration amount at each damage level and location for both algorithms, the proposed weighting method (PWM) is used and compared to the normal averaging method (NAM) and the results are shown in Figures 10 and 11 for stiffness change based on frequency and damage located at mid-span and quarter-span, respectively.

The proposed weighting method (PWM) as compared to the normal averaging method (NAM) for the stiffness change based on MAC and the results are shown in Figures 12 and 13 for damage located at mid-span and quarter-span, respectively.

The results showed that the proposed weighting method is more reliable than the normal averaging method for both algorithms and at both damage locations. The PWM return higher deterioration value than the NAM, where for frequency algorithm the PWM return values higher than the NAM by 4.9% for both damage locations. For algorithm based on MAC the PWM return values higher than the NAM by 4 and 6.6% for damage located at mid-span and quarter-span, respectively. This proves the assumption that PWM is more reliable and closer to the actual deterioration amount than the NAM.

The results show that the PWM returns one value at each damage level for both algorithms and at both damage locations, which makes it easier to decide the stiffness deterioration amount. The proposed method does not affect the sensitivity of the algorithms, but only helps to average its values for the set of the adopted modes in a more reliable and easily calculable method. The results show that when the damage is located at quarter-span, more stiffness deterioration occurs based on both algorithms. The sensitivity of frequency is more than MAC and increases corresponding to the increase in the induced damage ratio. For 12.5% reduction in the E value, there is a change of only about 0.5% in the natural frequencies and 0.017% in the mode shapes, which

Table 1. Damage cases adopted in present study.

Stiffness damage cases	*SRR* (%)	Damage location
Control C	0	N/A
SD1	1	Mid-Span
SD2	5	Mid-Span
SD3	12.5	Mid-Span
SD4	1	Quarter-Span
SD5	5	Quarter-Span
SD6	12.5	Quarter-Span

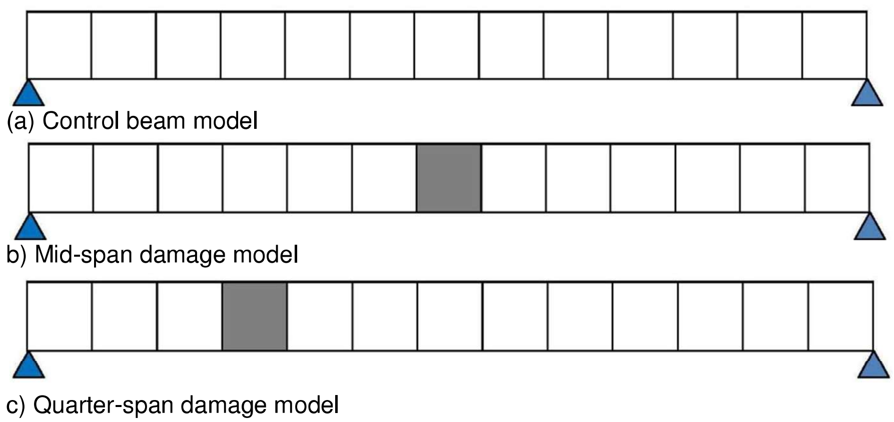

Figure 4. Beam-like structure model used in present study.

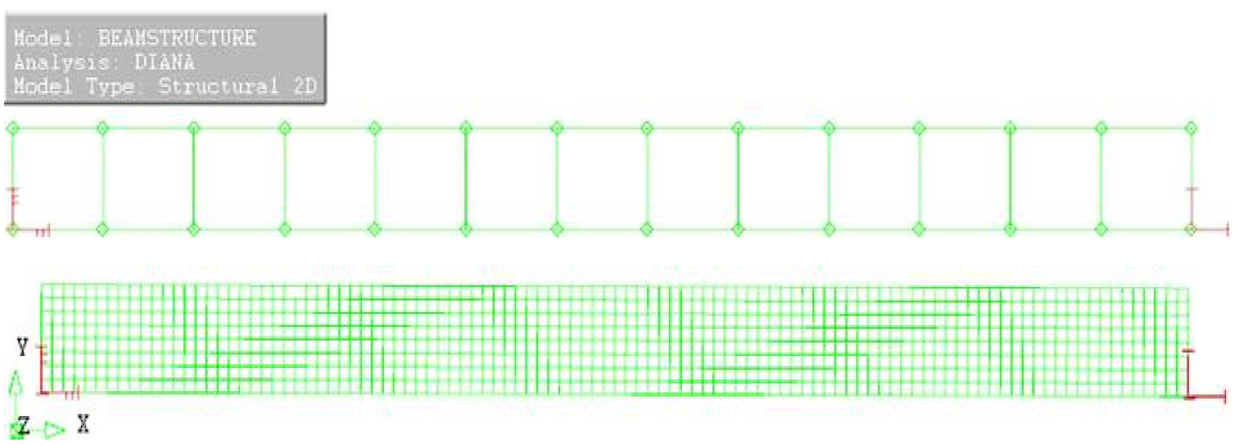

Figure 5. Finite element modelling for the beam-like structure model.

Table 2. Area under the curve and reliability weighting for each mode.

Mode no.	Area under the curve of the mode shape	PW_i
Mode 1	9.69	0.217
Mode 2	10.56	0.237
Mode 3	11.23	0.252
Mode 4	13.10	0.294
Total	44.58 square unit	1.000

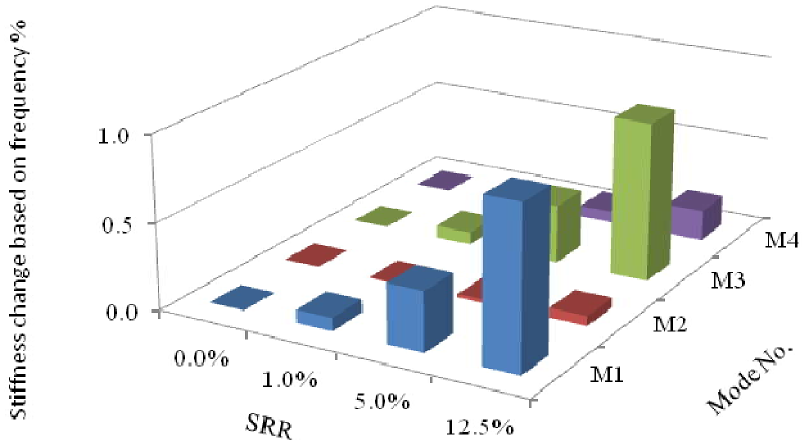

Figure 6. Stiffness change based on frequency for damage located at mid-span.

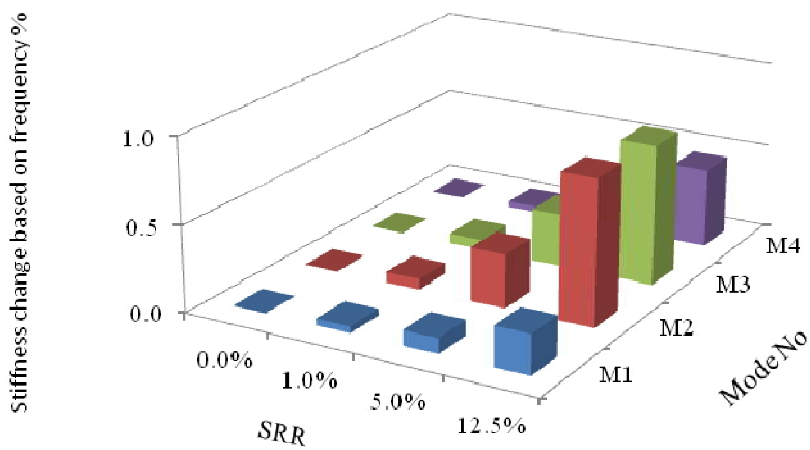

Figure 7. Stiffness change based on frequency for damage located at quarter-span.

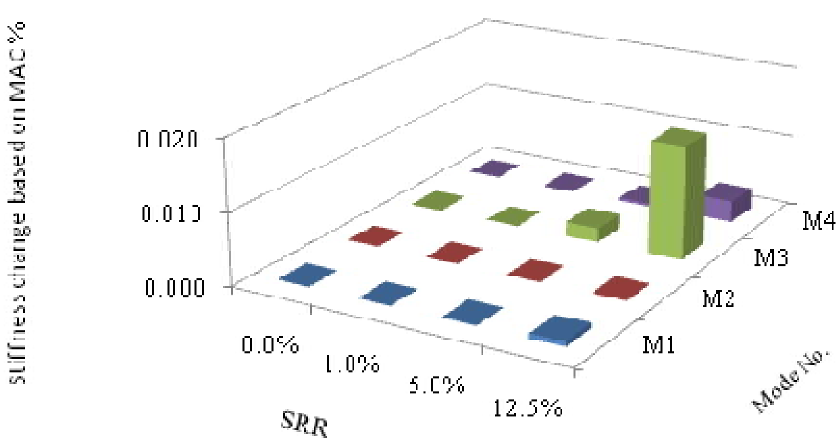

Figure 8. Stiffness change based on MAC for damage located at mid-span.

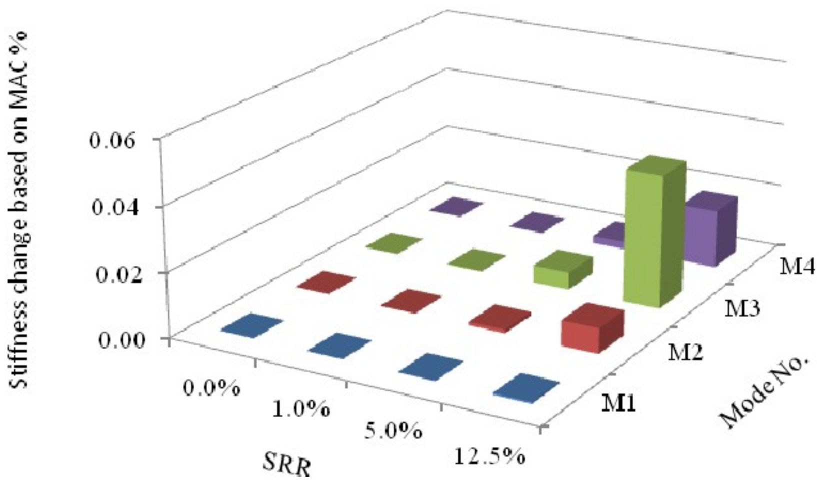

Figure 9. Stiffness change based on MAC for damage located at quarter-span.

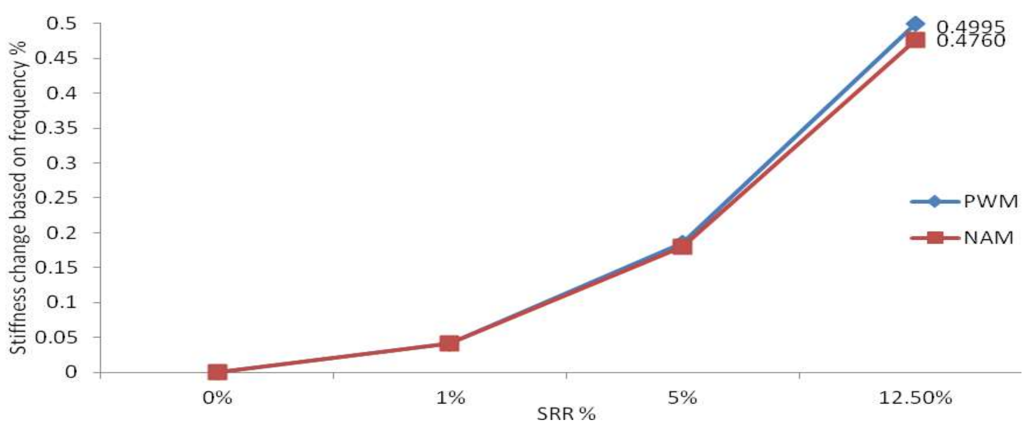

Figure 10. Compare PWM and NAM for stiffness change based on frequency and damage located at mid-span.

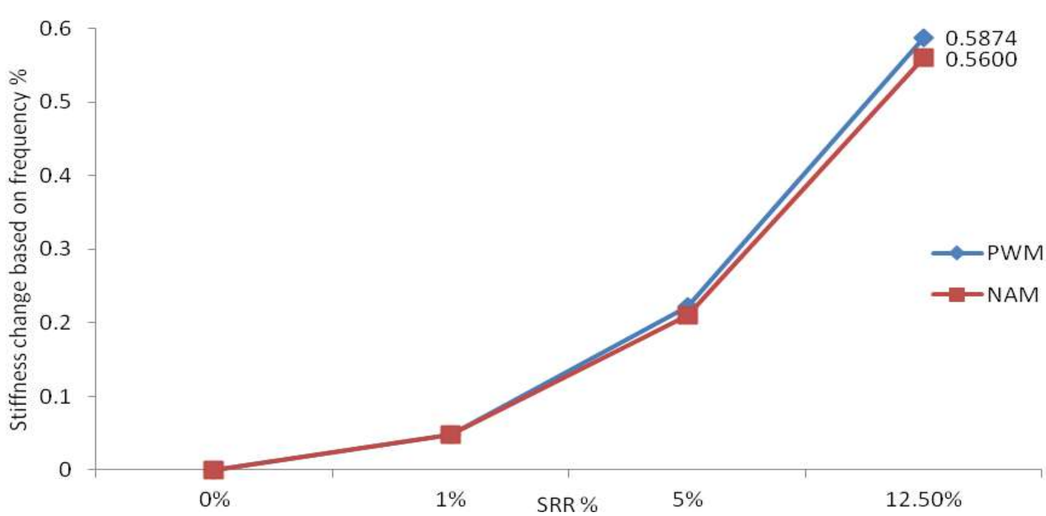

Figure 11. Compare PWM and NAM for stiffness change based on frequency and damage located at quarter-span.

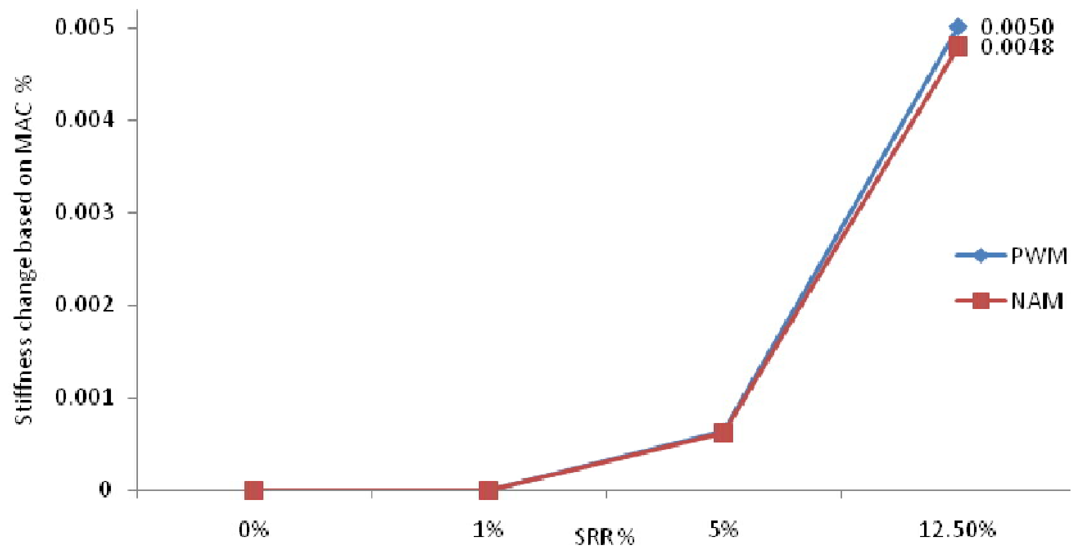

Figure 12. Compare PWM and NAM for stiffness change based on MAC and damage located at mid-span.

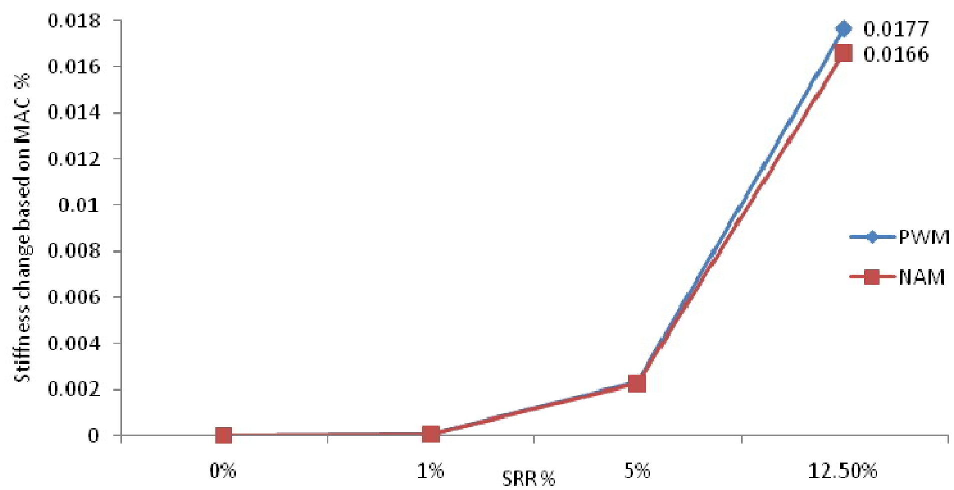

Figure 13. Compare PWM and NAM for stiffness change based on MAC and damage located at quarter-span.

highlights that the existing algorithms are underestimating.

Conclusions

Based on the results obtained using the finite element modelling of a beam like structure suffering deterioration in stiffness by means of reducing the elasticity modulus value, the following main conclusions are drawn:

1. Different modes have different sensitivity to the deterioration level based on the damage location and the modal parameter used.
2. Proposed weighting method proved to be more reliable and closer to actual deterioration severity than the normal averaging method.
3. Using the simple calculation of the proposed method helped to return one value regarding the stiffness deterioration amount based on the adopted set of modes.
4. The proposed method does not affect the sensitivity of the damage algorithms based on the natural frequencies or mode shapes, and only helps to apply simple reliability calculations for each specific mode.

5. Damage at quarter-span was found to have higher influence on the overall stiffness than damage at mid-span.
6. The algorithm based on natural frequencies was found to have higher sensitivity to the damage, regardless of its amount and location, than the algorithm based on the mode shapes.
7. Existing algorithms based on the modal parameters are underestimating.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the financial assistance provided by University of Malaya through a research grant entitled “Development of Algorithms for Structural Health Monitoring using Modal Parameters” (RG090/10AET). The authors would also like to thank all the people who have contributed either directly and indirectly, in making this research possible.

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