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Full Length Research Paper

Mesh-free modeling of two-dimensional heat conduction between eccentric circular cylinders

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In this paper, a numerical investigation of heat conduction problem for an irregular geometry using mesh-free local radial basis function-based differential quadrature (RBF-DQ) method has been performed. This method is the combination of differential quadrature approximation of derivatives and function approximation of radial basis function. The method can be used to directly approximate the derivatives of dependent variables on a scattered set of knots. In this study, knots were distributed irregularly in the solution domain using the Halton sequences. The method is applied to a two-dimensional geometry consisting of two eccentric cylinders. The inner and outer walls are maintained at different temperatures T_h and T_c. The obtained results from numerical simulations are compared with those gained by finite volume (FV) method. Outcomes prove that current technique is in very good agreement with finite volume method and this is due to the fact that RBF-DQ method is an accurate and flexible method in solution of heat conduction problems.

Key words: Mesh-free method, conduction, radial basis function, eccentric circular cylinders.

INTRODUCTION

Traditional numerical techniques such as finite difference, finite volume and finite element methods (FDM, FVM and FEM) are routinely used to solve complex problems. It is well-known that these methods are strongly dependent on mesh properties. However, in solution of partial differential equations (PDEs) by these methods for complex geometries, mesh generation takes up very much time and sometimes is the most expensive part of the simulation. For instance, grid generation procedure will be difficult while an irregular domain is placed inside of another irregular geometry. Mesh-free methods (also called meshless methods) have generated considerable interest recently due to the need to overcome the high cost of mesh generation associated with human labor (Nayroles et al., 1992; Belytschko et al., 1994; Liu et al., 1995; Atluri and Zhu, 1998).

The smoothed particle hydrodynamics (SPH) method (Liu and Liu 2003; Lucy, 1997), diffuse approximation method (DAM) (Nayroles et al., 1991), element free galerkin (EFG) method (Belytschko et al., 1994) and finite point method (FPM) (Onate et al., 1995) are all messless techniques which are widely used in different engineering problems. In the field of heat transfer, less works were focused on heat conductions. Liu (2005) simulated the radiation transfer problem by Meshless Local Petrov-Galerkin (MLPG) Wang et al. (2006) used a developed meshless numerical model to study the transient heat conduction in non-homogeneous functionally graded materials (FGM). Sladek et al. (2005, 2007) used MLPG to analyse transient heat conduction with continuously inhomogeneous and anisotropic FGM, too. Also Wu and

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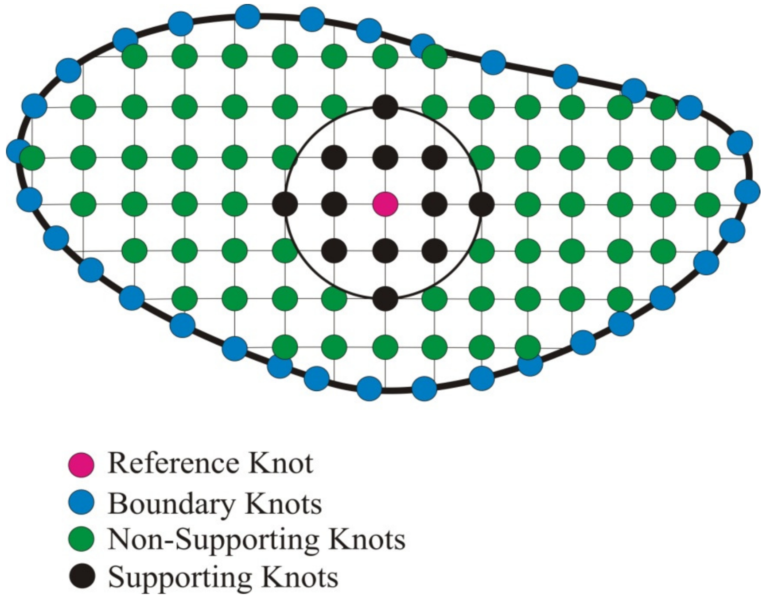

Figure 1. Supporting knots around a centered knot.

Tao (2008) applied MLPG to compute steady state heat conduction problems of irregular complex domain in 2D domain.

Recently, a new mesh-free method was proposed based on the so-called radial basis functions (RBF). Kansa (1990, 1980) introduced the direct collocation method using RBFs. It was found that RBFs were able to construct an interpolation scheme with favorable properties such as high efficiency, good quality and capability of dealing with scattered data, especially for higher dimension problems. The “truly” mesh-free nature of RBFs motivated researchers to use them to deal with partial differential equations. It should be noted that the Kansa’s RBF method is actually based on the function approximation. To approximate derivatives by using RBFs, Shu and his co-workers (Shu et al., 2003; Ding et al., 2005; Shu et al., 2005) proposed the RBF-DQ method, which combines the differential quadrature (DQ) approximation (Bellman et al., 1972; Shu and Richards, 1992, Shu and Wee, 2002; Tezer-Sezgin, 2004) of derivatives and function approximation of RBF. Previous applications showed that RBF-DQ is an efficient method to solve linear and nonlinear PDEs (Soleimani et al., 2010; Bararnia et al., 2010). Many other analytical and numerical methods can be found in open literatures (Bayat et al., 2010, 2011a, b, c, d, e, Bayat and Abdollahzadeh 2011f; Ganji, 2006; Shahidi et al., 2011).

Generally, the local RBF-DQ method is very flexible, simple in code writing and it can be easily applied to linear and nonlinear problems. In this method, the problem of ill-conditioned global matrix has been removed by replacement of global solvers by block partitioning schemes for large simulation problems as shown in Figure 1.

RADIAL BASIS FUNCTIONS

A radial basis function, denoted by $\varphi\left(\left\|x - x_j\right\|_2\right)$, is a continuous spline which depends on the separation distances of a subset of scattered points $x \in \Re^d$, while d denotes the spatial dimension. The most commonly used RBFs are

Multiquadrics (MQ):

$$\varphi(r) = \sqrt{r^2 + c^2}, c > 0, \qquad (1)$$

Thin-plate splines (TPS):

$$\varphi(r) = r^2 \log(r), \qquad (2)$$

Gaussians:

$$\varphi(r) = e^{-cr^2}, \quad c > 0, \qquad (3)$$

Inverse multiquadrics:

$$\varphi(r) = \frac{1}{\sqrt{r^2 + c^2}}, \quad c > 0, \qquad (4)$$

where $r = \left\| x - x_j \right\|_2$. Among the aforementioned four RBFs, MQ is used extensively. For scattered points, the approximation of a function $f\left(x\right)$ can be written as a linear combination of N RBFs

$$f\left(x\right) \cong \sum_{j=1}^{N} \lambda_j \varphi\left(\left\| x - x_j \right\|_2\right) + \psi\left(x\right), \tag{5}$$

where N is the number of centers or knots $x, x = (x_1, x_2, ..., x_d)$, d is the dimension of the problem, λ's are coefficients to be determined and φ is the RBF. Equation (5) can be written without the additional polynomial ψ. If Ψ_q^d denotes the space of d-variate polynomials of order not exceeding q, and letting the polynomials l $P_1, ..., P_m$ be the basis of Ψ_q^d in $\Re^d$, then the polynomial $\psi\left(x\right)$, in Equation (5), is usually written in the following form

$$\psi\left(x\right) = \sum_{1}^{m} \zeta_i P_i\left(x\right), \tag{6}$$

where $m = \left(q - 1 + d\right)! / \left(d!\left(q-1\right)!\right)$. To determine the coefficients $\left(\lambda_1, ..., \lambda_N\right)$ and $\left(\zeta_1, ..., \zeta_m\right)$, extra m equations are required in addition to the N equations resulting from the collocating Equation (5) at the N knots. This is insured by the m conditions for Equation (5), that is

$$\sum_{j=1}^{N} \lambda_j P_i\left(x_j\right) = 0 \quad i = 1, ..., m. \tag{7}$$

Differential quadrature method (DQ)

The DQ method is a numerical discretization technique for approximation of derivatives which was initiated from the idea of conventional integral quadrature. The essence of the DQ method is that the partial derivative of an unknown function with respect to an independent variable can be approximated by a linear weighted sum of functional values at all mesh points in that direction. Suppose that a function $f(x)$ is sufficiently smooth. Then its mth order derivative with respect to x at a point x_i can be approximated by DQ as

$$f_x^{(m)}\left(x_i\right) = \sum_{j=1}^{N} w_{ij}^{m} f\left(x_j\right), \quad i = 1, 2, ..., N, \tag{8}$$

where x_j are the discrete points in the domain, $f\left(x_j\right)$ and $w_{ij}^{(m)}$ are the function values at these points and the related weighting coefficients, respectively.

Local MQ-DQ (LMQDQ method) method formulation

Following the work of Shu et al. (2003) in the LMQDQ method, The MQ RBFs used as basis functions to determine the weighting coefficients in the DQ approximation of derivatives for a two-dimensional problem. However, the method can be easily extended to three-dimensional problems. Suppose that the solution of a partial differential equation is continuous, which can be approximated by MQ RBFs, and only a constant is included in the polynomial term $\psi\left(x\right)$. Then, the function in the domain can be approximated by MQ RBFs as

$$f\left(x, y\right) = \sum_{j=1}^{N} \lambda_j \sqrt{\left(x - x_j\right)^2 + \left(y - y_j\right)^2 + c_j^2} + \lambda_{N+1}. \tag{9}$$

To make the problem be well-posed, one more equation is required. From Equation (7), we have

$$\sum_{j=1}^{N} \lambda_j = 0 \Rightarrow \lambda_i = -\sum_{j=1, j \neq i}^{N} \lambda_j. \tag{10}$$

Substituting Equation (10) into Equation (9) gives

$$f\left(x, y\right) = \sum_{j=1, j \neq i}^{N} \lambda_j g_j\left(x, y\right) + \lambda_{N+1}. \tag{11}$$

where

$$g_j\left(x, y\right) = \sqrt{\left(x - x_j\right)^2 + \left(y - y_j\right)^2 + c_j^2} - \sqrt{\left(x - x_i\right)^2 + \left(y - y_i\right)^2 + c_i^2}. \tag{12}$$

λ_{N+1} can be replaced by λ_i and Equation (11) can be written as

$$f\left(x, y\right) = \sum_{j=1, j \neq i}^{N} \lambda_j g_j\left(x, y\right) + \lambda_i. \tag{13}$$

$f\left(x, y\right)$ in Equation (13) constitutes N-dimensional linear vector space V^N with respect to the operation of addition and multiplication. From the concept of linear independence, the bases of a vector space can be considered as linearly independent subset that extends across the entire space. In the space V^N one set of base vectors is $g_i\left(x, y\right) = 1,$ and $g_j\left(x, y\right), j = 1, ..., N$ but $j \neq i$ given by Equation (12). From the property of a linear vector space, if all the base functions satisfy the linear Equation (8), so does any function in the space V^N represented by Equation (13). From Equation (13), while all the base functions are given, the function $f(x, y)$ is still unknown since the coefficients λ_i are unknown. However, when all the base functions satisfy Equation (8), we can guarantee that $f(x, y)$ also satisfies Equation (8). In other words, we can guarantee that the solution of a partial differential equation approximated by the radial basis function satisfies Equation (8). Thus, when the weighting coefficients of DQ approximation are determined by all the base functions, they can be used to discretize the derivatives in a partial differential equation. That is the essence of the RBF-DQ method. Substituting all the base functions into Equation (8), we have

$$0 = \sum_{k=1}^{N} w_{ik}^{(m)}, \tag{14}$$

$$\frac{\partial^m g_j(x_i, y_i)}{\partial x^m} = \sum_{k=1}^{N} w_{ik}^{(m)} g_j(x_k, y_k), \; j = 1, 2, ..., N, \text{ but } j \neq i, \tag{15}$$

For the given i , equation system (14 to 15) has N unknowns with N equations. So, solving this equation system can obtain the weighting coefficients $w_{ik}^{(m)}$. From Equation (12), one can easily obtain the first order derivative of $g_j(x, y)$ as

$$\frac{\partial g_j(x_i, y_i)}{\partial x^m} = \frac{x - x_j}{\sqrt{(x-x_j)^2 + (y-y_j)^2 + c_j^2}} - \frac{x - x_i}{\sqrt{(x-x_i)^2 + (y-y_i)^2 + c_i^2}}, \tag{16}$$

In the matrix form, the weighting coefficient matrix of the x-derivative can then be determined by

$$[G][W^n]^T = \{G_x\}, \tag{17}$$

where $[W^n]^T$ is the transpose of the weighting coefficient matrix $[W^n]$, and

$$[W^n] = \begin{bmatrix} w_{1,1}^{(n)} & w_{1,2}^{(n)} & \dots & w_{1,N}^{(n)} \\ w_{2,1}^{(n)} & w_{2,2}^{(n)} & \dots & w_{2,N}^{(n)} \\ \vdots & \vdots & \ddots & \vdots \\ w_{N,1}^{(n)} & w_{N,2}^{(n)} & \dots & w_{N,N}^{(n)} \end{bmatrix}$$

$$[G] = \begin{bmatrix} 1 & 1 & \dots & 1 \\ g_1(x_1, y_1) & g_1(x_2, y_2) & \dots & g_1(x_N, y_N) \\ \vdots & \vdots & \ddots & \\ g_N(x_1, y_1) & g_N(x_2, y_2) & \dots & g_N(x_N, y_N) \end{bmatrix}, \tag{18}$$

$$[G_x] = \begin{bmatrix} 0 & 0 & \dots & 0 \\ g_x^n(1,1) & g_x^n(1,2) & \dots & g_x^n(1,N) \\ \vdots & \vdots & \ddots & \\ g_x^n(N,1) & g_x^n(N,2) & \dots & g_x^n(N,N) \end{bmatrix}.$$

With the known matrices $[G]$ and $[G_x]$, the weighting coefficient matrix of x-derivative $[W^n]$ can be obtained by using a direct or iterative method such as LU decomposition or SOR. The weighting coefficient matrix of the y-derivative can be obtained in a similar manner. Using these weighting coefficients, we can discretize the spatial derivatives and transform the governing equations into a system of algebraic equations, which can be solved by iterative or direct method.

SHAPE PARAMETER (c) IN LOCAL MQ-DQ METHOD

As mentioned before, the MQ approximation of the function contains a shape parameter c that could be knot-dependent and must be determined by the user. Some related point of the shape parameter is described subsequently.

Normalization of supporting region

In the local MQ-DQ method, the shape parameter c has a strong influence on the accuracy of numerical results. The optimal value of c is affected mainly by the number of supporting knots and the size of the supporting region. In this method, the number of supporting knots is usually fixed for an application. When the knots are randomly generated, the scale of supporting region for each reference knot could be different, and the optimal shape parameter c for accurate numerical results may also be different. The difficulty of assigning different values of c at different knots can be removed by normalization of scale in the supporting region. The essence of this idea is to transform the local support to a unit square for the two dimensional case or a unit box for the three dimensional case. The coordinate transformation has the form

$$\bar{x} = \frac{x}{D_i}, \qquad \bar{y} = \frac{y}{D_i}. \tag{19}$$

where (x, y) represents the coordinates of supporting region in the physical space, $(\bar{x}, \bar{y})$ denotes the coordinates in the unit square, D_i is the diameter of the minimal circle enclosing all knots in the supporting region for the knot i . The corresponding MQ test functions in the local support now become

$$\varphi = \sqrt{\left(\bar{x} - \frac{x_i}{D_i}\right)^2 + \left(\bar{y} - \frac{y_i}{D_i}\right)^2 + \bar{c}^2}, \qquad i = 1, ..., N, \tag{20}$$

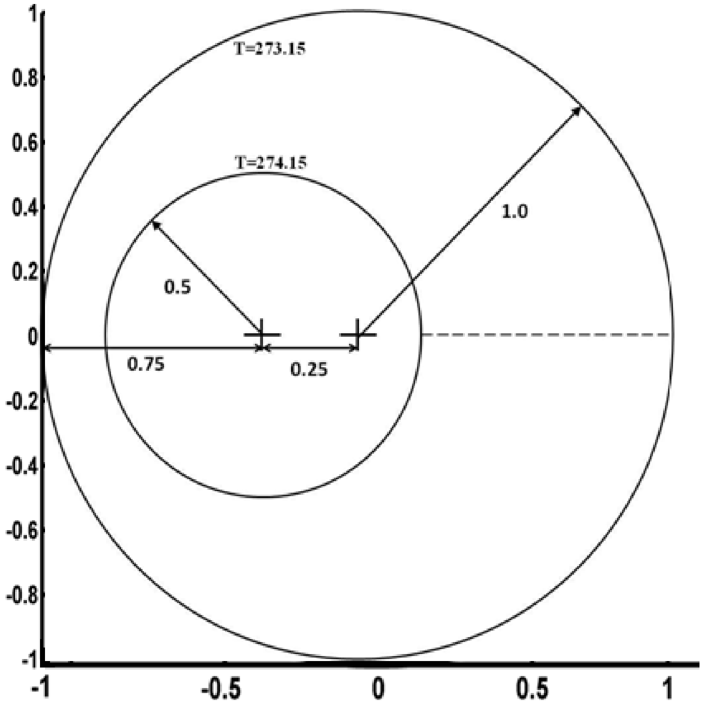

Figure 2. Geometries and boundary conditions.

where N is the total number of the knots in the support. It can found that the shape parameter c is equivalent to $\bar{c}D_i$. The coordinate transformation also changes the formulation of the weighting coefficients in the local MQ-DQ approximation. For instance, using the differential chain rule, the first order partial derivative with respect to x can be written as

$$\frac{\mathrm{d}f}{\mathrm{d}x} = \frac{\mathrm{d}f}{\mathrm{d}\bar{x}}\frac{\mathrm{d}\bar{x}}{\mathrm{d}x} = \frac{1}{D_i}\frac{\mathrm{d}f}{\mathrm{d}\bar{x}} = \frac{1}{D_i}\sum_{j=1}^{N} w_j^{(1x)} f_j = \sum_{j=1}^{N}\frac{w_j^{(1x)}}{D_i} f_j, \quad (21)$$

where $w_j^{(1x)}$ are the weighting coefficients computed in the unit square, $w_j^{(1x)}/D_i$ are the actual weighting coefficients in the physical domain. $\bar{c}$ is chosen as a constant. Its optimal value depends on the number of supporting knots. In our application, $\bar{c}$ is chosen as a constant, and is taken as 0.12, and the number of supporting knots is taken as 16 based on the previous work of Ding et al. (2005).

MATHEMATICAL MODELING AND NUMERICAL PROCEDURE

Considered geometry of the problem with related boundary condition is depicted in Figure 2.

Dotted lines in Figure 2 were used to calculate the temperature distribution for comparison with FV solution. The dimensionless form of the governing equation can be written as

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial X^2} + \frac{\partial^2 T}{\partial Y^2}\right), \quad (22)$$

The steady-state results are obtained from unsteady conduction equation, Equation (22). Where T denotes the dimensionless temperature, X and Y are the dimensionless lengths and $\alpha = \rho\, C_p / k$ is the thermal diffusivity of the material.

Discritizing of space derivation for Equation (22) using RBF-DQ is gained as follow:

$$\frac{T_i^{n+1} - T_i^n}{\Delta t} = \alpha\left(\sum_{k=1}^{n_i} w_{i,k}^{2x} T_i^k - \sum_{k=1}^{n_i} w_{i,k}^{2y} T_i^k\right) \quad (23)$$

where T_i represents the function value at knot *i*, T_i^k represents the function value at the *k*th supporting knot for knot *i*. $w_{i,k}^{2x}$ and $w_{i,k}^{2y}$ represent the computed weighting coefficients in the DQ approximation for the second order derivatives in the *x* and *y* direction, respectively.

RESULTS AND DISCUSSION

Unlike mesh generation in conventional FD, FE or FV methods, there is no pre-requirement for the distribution

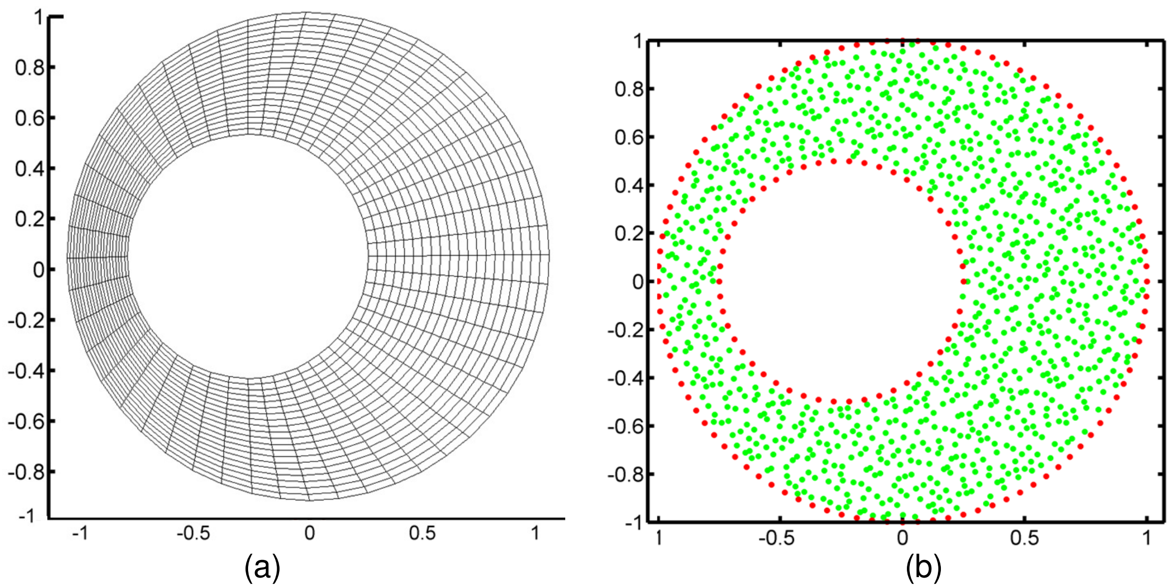

Figure 3. (a) Finite volume grids and (b) Irregular nodes for RBF-DQ method solution.

of points in the present method. Point generation in the present method is consequently easier in complex domains as compared with mesh-based methods. There are many ways to generate the points for practical problems. For example, for applications in the regular domain, where the solution is sufficiently smooth, the points in the domain can be generated in the same manner as the regular grid in conventional finite-difference schemes. The boundary points can simply be the intersections of the grid with boundary, irrespective of its shape. For problems with irregular domain, we can get adequate nodes in the domain either by random point generation or by using algebraic function. The main difficulty for point generation algorithms lies in deciding whether a generated point is within the domain or outside it. Furthermore, for many boundary-value applications, the solution may need different resolutions for different regions; high resolution being typically required for regions near boundaries. Thus, when we use either mesh-based or point-based methods, the density of mesh or point distribution should reflect that need. In such circumstances, the distribution of the nodes or points in computational domain must be generated either adaptively or by using known information about the specific physical problem. Both of them can be implemented easily in the present method as we can freely add or delete nodes instead of re-meshing. Another advantage of this method is that user can explicitly determine the grid spacing in the normal direction of the boundary. In general, there are three grid generation algorithms that can be selected according to the practical applications (Ding et al., 2004) which are:

1. The grid generation algorithm of conventional finite-difference schemes if the geometry of the domain is very simple, such as rectangles or circles.
2. Using the random point generation algorithm, if the geometry of the domain is complex and only concerned with Dirichlet boundary condition.
3. Using a fast hyperbolic grid or algebraic formulation to produce several layers of locally orthogonal grids near the boundary, and using the random point generation algorithm to generate other nodes in the rest of the computational domain if the geometry of the domain is complex and Neumann boundary conditions are involved.

In the present study, the second algorithm is applied for the problem. All numerical results are obtained for $\alpha = 8.4e-5(m^2/s)$ and compared with the FV solution. Finite volume grids and RBF-DQ nodes used for current problems are illustrated in Figure 3.

Structured grids and irregular nodes were used for FV and RBF-DQ, respectively. In the current investigation, knots were distributed in the domain using the Halton sequences (Fasshauer, 2007). In the present code, we have used the all-pair search approach (Liu and Lui, 2003) for identifying the nearest supporting knots.

The increase of knots' number has been stopped since an appropriate accuracy obtained. We selected 841 knots totally, with 51 and 107 knots on the internal and external boundaries, respectively. The temperature distribution by RBF-DQ and FV is obtained and illustrated for steady

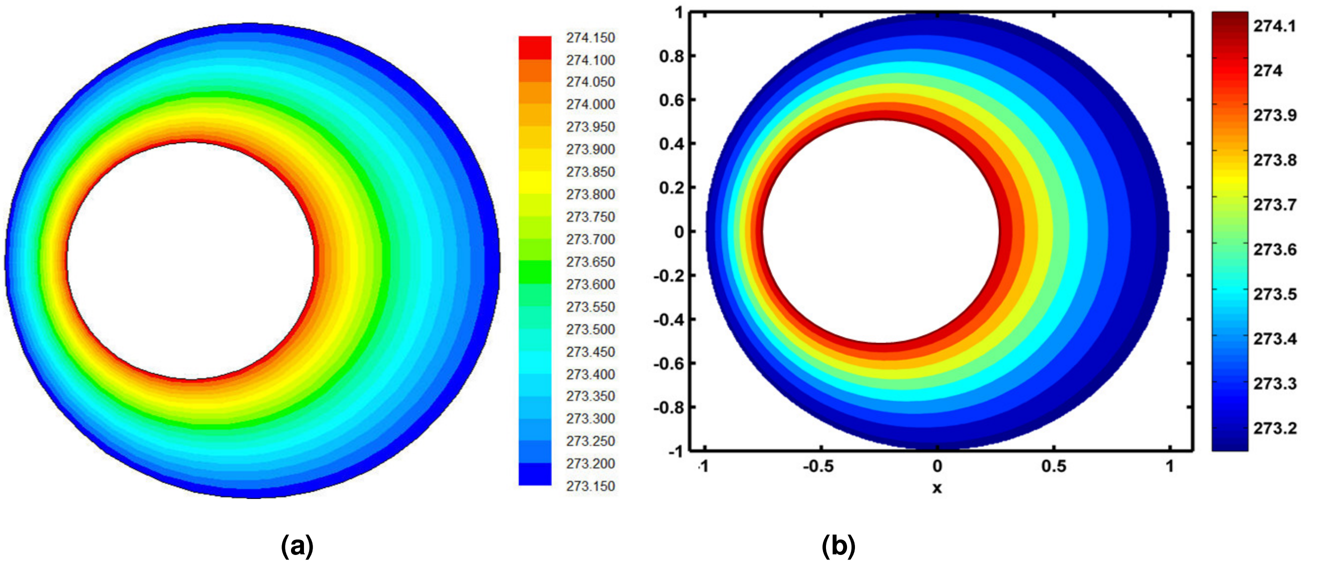

Figure 4. Temperature distribution for Case (a): FV solution (b): RBF-DQ method solution.

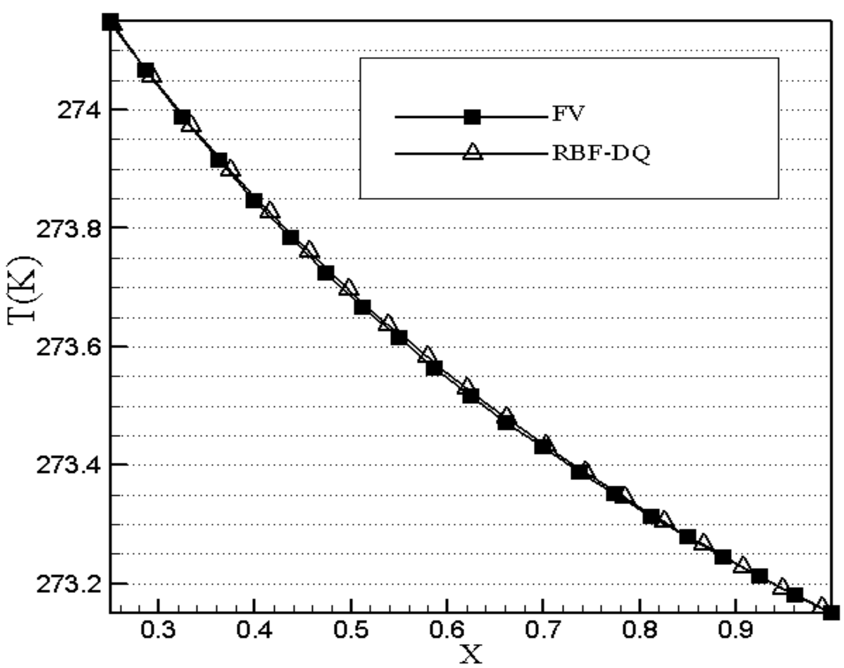

Figure 5. Temperature variations for Case (a) along the dashed line.

state in Figure 4.

In order to compare the results in details, temperature variation was calculated along the dashed line and is shown in Figure 5. Outomes gained by RBF-DQ is in excellent agreement with FVsolution.

Conclusions

Two dimensional heat conduction equations for regular and irregular geometries applicable in engineering is analyzed numerically by mesh-free Local RBF-DQ method.

Results are compared with solutions achieved by a FE code. Comparison of the present outcomes with FEM demonstrates that the Local RBF-DQ method is an attractive approach in terms of accuracy, capability and flexibility in programming for heat transfer problems, even for complex irregular boundaries.

Nomenclature

Shape parameter	c
Specific Heat	C_p
Thermal conductivity	k
Dimensionless time	t
Dimensionless temperature, (K)	T
Weighting Coefficient Matrix	W^n
Normalized coordiantes	$\overline{x}$, $\overline{y}$
Dimensionless Cartesian coordinates	X,Y

Greek symbols

Thermal diffusivity, ($m^2.s^{-1}$)	α
Density of material	ρ
Time step	Δt

Subscripts

Cold	c
Hot	h

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