International Journal of the Physical Sciences Vol. 6(11), pp. 2662-2670, 4 June, 2011
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS11.412
ISSN 1992 - 1950 ©2011 Academic Journals

Full Length Research Paper

Seismic risk analysis of Denizli (Southwest Turkey) region using different statistical models

Ibrahim COBANOGLU[1]* and Devrim ALKAYA[2]

[1]Department of Geological Engineering, Faculty of Engineering, Pamukkale University, Denizli, Turkey.
[2]Department of Civil Engineering, Faculty of Engineering, Pamukkale University, Denizli, Turkey.

Accepted 5 May, 2011

Risk analyses made in an area of seismic activity are of great importance in determining earthquake occurrence intervals and recurrence periods. Several methods, some of which include statistical methods, have been developed for this purpose. Gamma, Weibull distributions and Markov, Poisson, Gumbel are the most frequently used methods in this regard. In this study, instrumental records of 73 earthquakes of Ms ≥ 4 which occurred in Denizli (Southwest Turkey) Region were investigated. These earthquake records were obtained from the Turkish Earthquake Research Department (ERD). To explain the seismic activity of the area, the relationship between magnitude and frequency was explained by using earthquake distribution in time. The magnitude-frequency relationship of the study area was calculated by means of the "Log N = 5.91 – 0.97 M" equation. Occurrence probability and recurrence periods of the earthquakes were computed by utilizing Poisson, Gumbell and Exponential Distribution Models and the results were correlated. The recurrence period of a 5.2 magnitude earthquake was determined as 10.4 years with the exponential distribution models. Poisson and Gumbel models, on the other hand, indicated 29.0 and 9.64, respectively.

Keywords: Denizli Earthquake return period, Exponential distribution function model, Gumbel model, Poisson model, Seismic risk.

INTRODUCTION

The seismic risk analysis based on the historical earthquake data is one of the methods utilized in order to determine the seismicity of an area. Elastic rebound theory shows that earthquakes occurring on any fault or fault section are related to historical earthquakes.

The number of recurrence as well as occurence time of possible earthquakes in the future can be identified using the seismic data, which were recorded during the earthquakes. For this purpose, different distribution models such as Poisson, Gumbell and Markov are commonly utilized. From historical periods to recent, Denizli and its surrounding have been exposed to large earthquakes. Şimşek and Ceylan (2003) reported that the strongest earthquakes in the Pamukkale, ancient city, occurred in Denizli (Southwest Turkey) in 60 and 494 A.D.

Van Gelder (1997) he investigated the records of earthquakes which occurred in the last century and suggested a new statistical model in order to explain Gutenberg and Richter magnitude relations for the earthquakes resulting from the Vrancea fault in Romania.

Bağcı (2000) brought up a magnitude-frequency equation for the province of Izmir and its surroundings by investigating the 4.0 or larger magnitude earthquakes which took place in the area during 1900 to 1999. The researcher attempted to determine the recurrence probability of earthquakes in the future by utilizing Poisson and Gumbell distributions models.

In order to describe the seismicity of the Çukurova region, Çobanoğlu et al. (2006) brought up magnitude-frequency relations by means of "Log N = 6.29 to 0.96 M" formula. Their seismic risk analysis made use of the Poisson, Gumbell and Exponential distribution function models for the study area and determined the recurrence periods accordingly.

Campbell et al. (2002) formed a seismic risk model for Taiwan, depending upon tectonic and seismic data. In

*Corresponding author. E-mail: icobanoglu@pau.edu.tr Tel: +90 258 2963409. Fax: +90 258 2963382.

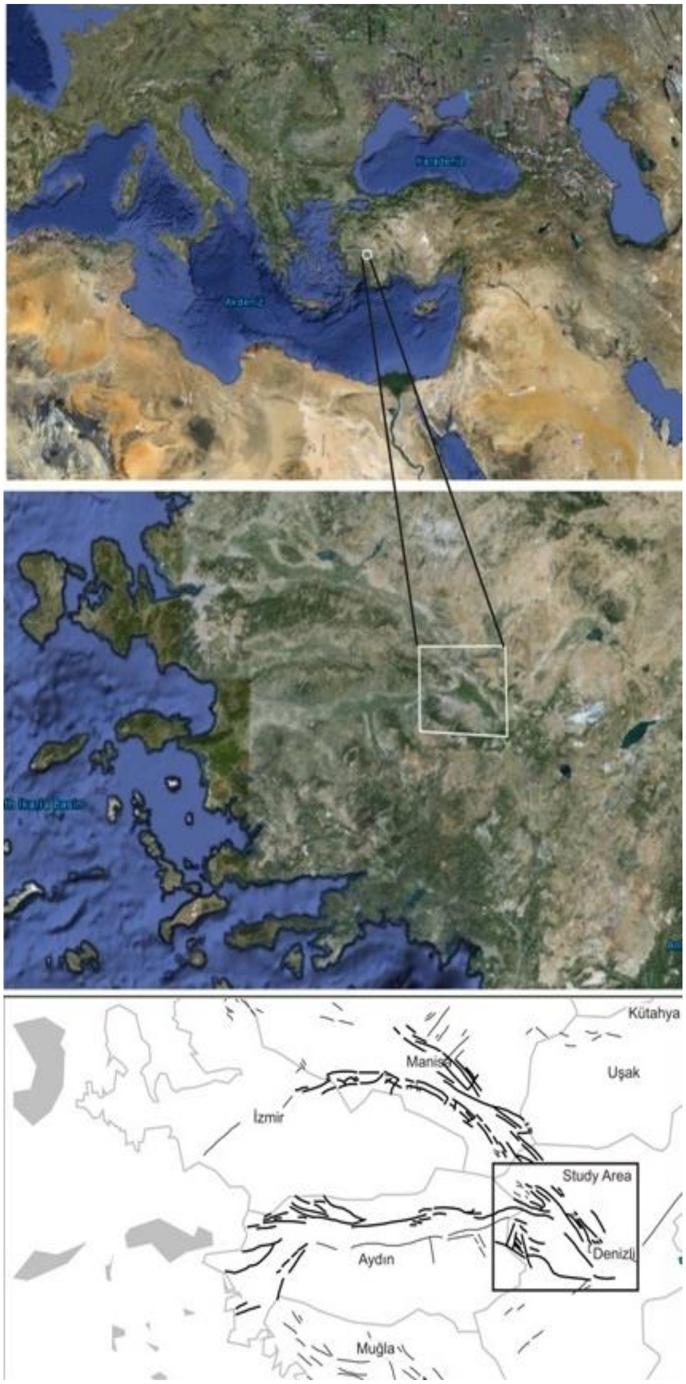

Figure 1. Location and active fault map of study area.

their study, the relationship between ML local magnitude L must be lower (interior) indice and Mw moment magnitude scale used in Taiwan were investigated and some equations were proposed. Maximum earthquake magnitude was defined by investigating the historical earthquakes in the area and recurrence periods of earthquakes with a magnitude of 4.0 to 8.0 Mw moment magnitude were calculated. According to this study, the recurrence period of earthquakes with a magnitude of 6.5 to 7.0 was determined as 100 years for that area.

STUDY AREA AND RESEARCH METHODOLOGY

This paper analyzes the earthquakes of 4 or larger magnitude values which occurred in the area situated between 35 S 646570 E / 4226319 N to 698035 E / 4226319 N to 701025 E / 4176783 N and 647354 E / 4178959 N coordinates between the years of 1900 and 2011 (Figure 1). The study area is located in the Aegean region of southern Turkey.

The seismicity of the research area was investigated by determining the earthquake occurrence number and recurrence periods. The exponential distribution function model was used together with Poisson and Gumbel models and the validity of this model for this kind of a seismicity research was investigated. No seismic risk evaluation studies using exponential distribution model has been found in literature in terms of seismicity. In this respect, this study also presents the application phases of the model in detail.

Evaluation of the magnitude-frequency relations

The basic magnitude-frequency relationship suggested by Gutenberg and Richter (1954) is of great importance, since it is directly related to an earthquake occurrence. In order to uncover this relationship, 73 earthquakes which have Ms >4 magnitude were investigated in the study area. The number of earthquake occurrence was calculated by using 0.1 magnitude interval and normal frequency values are given in Table 1.

$$\text{Log } N = a - bM \tag{1}$$

In this equation;

N: Cumulative earthquake number, M: Magnitude, a and b are the coefficients.

The magnitude-frequency relation for Denizli region is identified by "Log N = 5.91 to 0.97 M" formula by using the values presented in Table 1 (Figure 2). According to this relation, a value has been calculated as 5.91 and b as 0.97. In Gutenberg-Richter function, a big value of a coefficient points to numerous small earthquakes, whereas a small value of b coefficient indicates the predominance of big earthquakes. According to this relationship, it can be concluded that small magnitude earthquakes are widespread in the Denizli region.

Identification of seismic risk using exponential distribution model

X is assumed to be a random variable having the magnitude value of M. In this study, exponential distribution model of λ and θ parameters were suggested for the *X* random variable or for magnitude 4.0 and bigger earthquake which occurred in Denizli region during 1900 to 2011. The probability density function of *X* random variable in the form of exponential function is as follows (Ramachandran, 1980):

$$f_M(x) = \lambda e^{-\lambda(x-\theta)}$$
$$\lambda > 0 \quad \theta \le x < +\infty \tag{2}$$

Table 1. Magnitude – Earthquake frequency (Log N) relationship.

M	N	Total N	Log N	M	N	Total N	Log N
4.0	9	73	1.863	5.1	0	10	1.000
4.1	6	64	1.806	5.2	2	10	1.000
4.2	5	58	1.763	5.3	2	8	0.903
4.3	6	53	1.724	5.4	1	6	0.778
4.4	4	47	1.672	5.5	1	5	0.699
4.5	11	43	1.633	5.6	1	4	0.602
4.6	2	32	1.505	5.7	3	3	0.477
4.7	7	30	1.477	5.8	0	0	---
4.8	6	23	1.362	5.9	0	0	---
4.9	3	17	1.230	6.0	0	0	---
5.0	4	14	1.146				

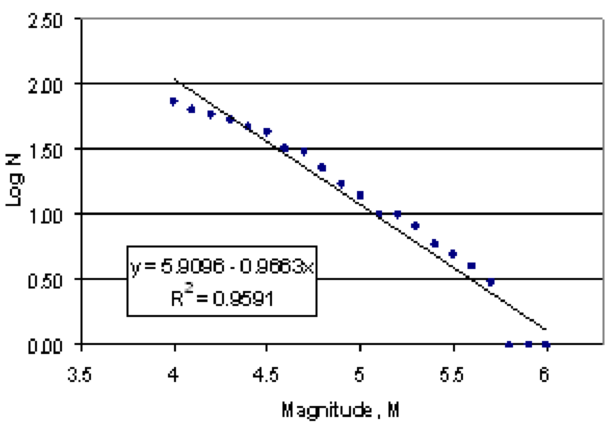

Figure 2. Magnitude – Frequency (Log N) relationship.

The value of λ parameters in exponential probability density function is calculated by the following equation:

$$\lambda = (\bar{x} - \theta)^{-1} \quad (3)$$

where, $\bar{x}$ is the mean magnitude value obtained from many earthquake data and θ stands for the smallest magnitude value. The distribution function of x random variable is found as:

$$F_M(x) = \int_{\theta}^{x} \lambda e^{-\lambda(U-\theta)} dU = 1 - e^{-\lambda(x-\theta)}$$

$$\lambda > 0 \quad \theta \le x < +\infty \quad (4)$$

by utilizing exponential probability density function (Hahn and Shapiro, 1994).

The Chi-Squared goodness of fit test

The Chi-Squared (χ^2) test, found by Karl (1989), depends on the correlation between observing value and expected value of test groups. By means of this test, the distribution of values of two or more groups can be correlated at the same time. In this study, the "Chi-Squared goodness test" was applied for 73 earthquakes observed with a view to controlling the suitability of experimental distribution to theoretical exponential distribution. In the application of the Chi-Squared goodness test, the experimental (empirical) distribution function value for each class is compared to theoretical distribution function value obtained from distribution function. If the calculated χ^2 value (χ_h^2) is smaller than χ^2 value (χ_t^2) in table, the hypothesis is considered to be true. Otherwise, the hypothesis is rejected (Hahn and Shapiro, 1994).

The goodness of fit test investigates the validity of the difference between the experimental (empirical) distribution function values and the value of theoretical distribution function obtained from the distribution function. For a similar purpose, the data gathered from the examples related to earthquake numbers were checked to see if they are in accordance with the theoretical exponential distribution or not. For this aim, the following test equation was utilized;

$$\chi^2 = \sum_{i=1}^{k} \frac{(F_{M_g}(O_i) - F_{M_b}(O_i))^2}{F_{M_b}(O_i)} \quad (5)$$

where k is the class number for data, O_i, is the class mid-point value for ith class, $F_{M_g}(O_i)$ is value or cumulative percentage of empirical distribution function observed in class mid-point value for ith class, and $F_{M_b}(O_i)$ is the value of theoretical distribution function expected in class mid-point value for ith class.

A significance level is chosen for the test applied. The validity or rejection of χ^2 goodness of fit test and of hypothesis are confirmed according to this significance level. The Chi-Squared test significance level is taken as $\alpha = 0.05$ in this study.

RESULTS

Data analysis

This study analyzed the four and bigger magnitude

Table 2. Earthquake number and magnitude values (f_i) of study area earthquakes.

Magnitude *M* (*X*)	Value (f_i)
4.0 – 4.4	30
4.5 – 4.9	29
5.0 – 5.4	9
5.5 – 5.9	5
6.0 – 6.4	0

Table 3. Frequency distribution table for constructed classes.

Class number i	Class low-point boundary	Class mid-point value (O_i)	Class high-point value	Earthquake Number (f_i)	%
1	4.0	4.2	4.4	30	0.4110
2	4.5	4.7	4.9	29	0.3973
3	5.0	5.2	5.4	9	0.1233
4	5.5	5.7	5.9	5	0.0685
5	6.0	6.2	6.4	0	0.0000
		Total		73	1.0000

Table 4. Experimental and theoretical distribution functions values for the constructed model.

Class number i	Class mid-point value, (O_i)	Frequency number (f_i)	%	Empirical $F_{M_g}(x)$	Theoretical $F_{M_b}(x)$	Difference of values
1	4.2	30	0.4110	0.4110	0.3756	0.0354
2	4.7	29	0.3973	0.8082	0.8076	0.0006
3	5.2	9	0.1233	0.9315	0.9407	-0.0092
4	5.7	5	0.0685	1.0000	0.9817	0.0183
5	6.2	0	0.0000	1.0000	0.9944	0.0056

earthquakes that occurred in the area of Denizli. The total number of earthquakes and their magnitude are presented in Table 2.

$\bar{x}$ = 4.6 was found by using the following formula;

$$\bar{x} = \left(\sum_{i=1}^{k} O_i f_i\right) \Big/ \left(\sum_{i=1}^{k} f_i\right)$$

$$\bar{x} = \left(\sum_{i=1}^{5} O_i f_i\right) \Big/ \left(\sum_{i=1}^{5} f_i\right) = \frac{338}{78} = 4.63$$

and λ = 2.5 was found by using the following formula, utilizing the data in Table 2;

$$\lambda = (\bar{x} - \theta)^{-1}$$

$$\lambda = (\bar{x} - \theta)^{-1} = \frac{1}{4.6 - 4.2} = \frac{1}{0.4} = 2.5$$

The class mid-point value of first class ($\theta = O_1 = 4.2$) is taken as the smallest value for θ. In this case, the exponential probability density function turns into the following formula;

$$f_M(x) = 2.5e^{-2.5(x-4.2)} \qquad \theta \le x < +\infty$$

X random variable distribution function according to the exponential distribution function used is as follows;

$$F_M(x) = \int_{\theta}^{x} 2.5e^{-2.5(U-4.2)} dU = 1 - e^{-2.5(x-4.2)}$$

$$4.2 \le x < +\infty$$

Table 3, shows earthquake magnitude intervals, recurrence periods and percentages corresponding to those recurrence periods. Table 4 illustrates the empirical distribution function values for each class formed according to the 73 earthquake data, and the theoretical distribution function values determined by the distribution

Table 5. Return periods for different classes.

Class mid-point value, (O_i)	Theoretical $F_{M_b}(x)$	$f_M(x)$	F_i	Average recurrence periods (year)
4.2	0.3756	0.3756	0.2742	3.6
4.7	0.8076	0.4320	0.3154	3.2
5.2	0.9407	0.1331	0.0972	10.39
5.7	0.9817	0.0410	0.0299	33.4
6.0	0.9944	0.0126	0.0092	108.4

Table 6. Earthquake parameters for the study area.

a	b	a'	a1	a1'
5.099	0.966	5.563	3.910	3.563

function equation. The empirical distribution function values in the table have been calculated by adding the percentages as cumulative. The theoretical cumulative distribution function values presented in Table 4 were calculated by adding the theoretical distribution function values for each class cumulatively.

The appropriacy of the data obtained from 73 earthquakes analyzed in order to develop the model suitability was investigated by means of the Goodness of fit test. The Chi-Squared degree was calculated as χ^2 = 0.0196. The Chi-Squared table value was determined as χ_h^2 = 7.8147 according to unconstrained degree 3 and 0.05 significance level (Hahn and Shapiro, 1994). When the calculated and the table values are compared, it is accepted that the empirical distribution conforms to exponential distribution and thus the hypothesis is statistically validated. Table 5, which used the values of expected cumulative probability, shows the occurrence probability of earthquakes with different magnitudes ($F_M(x)$), annual expected frequency values (F_i) and average recurrence periods per year.In order to calculate the $F_M(x)$ in the table, the annual number of earthquake > 4 magnitude observed annually is required. For this purpose, the ratio sum of earthquake > 4 magnitude number to examined time periods (100 years) was utilized (0.73). Annual expected recurrence number (F_i values) in Table 5 was determined by multiplying $f_M(x)$ values by annual average observed earthquake numbers observed annually. The average recurrence periods are calculated by means of $\frac{1}{F_i}$ equation (Hahn and Shapiro, 1994). Table 5 shows that the recurrence period of a 4.2 magnitude earthquake as 3.6 years, whereas that of a 5.7 magnitude earthquake is estimated as 33.4 years.

Determination of seismic risk by poisson model

Another frequently used model in estimating earthquake occurrence is the Poisson model. According to this model, the distribution of waiting time for another earthquake is not affected by the time after the occurrence of the previous earthquake (Öztemir et al., 2000). Statistical data shows that the Poisson model is valid especially for big earthquakes. In a study carried out by Kiremidjian et al. (1992), which compared the Poisson and Markov models, it was pointed out that the Poisson model is adequate to estimate earthquake hazard in a region where frequent middle magnitude earthquakes occur.

The earthquake parameters for the investigated area for Poisson models have been computed by using the following equations and the results are listed in Table 6.

$$a' = a - \log (b * \ln 10) \quad (6)$$

$$a_1 = a - \log T \quad (7)$$

$$a_1' = a' - \log T \quad (8)$$

In these equations, T stands for the investigated time periods and was taken as 100 years. The normal frequency value used to determine seismic risk is found by this equation.

$$N(O_i) = 10^{a_1' - bO_i} \quad (9)$$

$N(O_i)$ value expresses annual average earthquake

Table 7. Obtained seismic risk and return periods according to poisson model.

x (O_i)	$N(O_i)$	Years 10	20	30	40	50	75	100	Q (O_i)
		Seismic risk (%)							
4.0	0.4982	99	100	100	100	100	100	100	2.01
4.2	0.3192	96	100	100	100	100	100	100	3.13
4.5	0.1638	81	96	99	100	100	100	100	6.11
4.7	0.1049	65	88	96	98	99	100	100	9.53
5.0	0.0538	42	66	80	88	93	98	100	18.57
5.2	0.0345	29	50	64	75	82	92	97	28.99
5.5	0.0177	16	30	41	51	59	73	83	56.50
5.7	0.0113	11	20	29	36	43	57	68	88.17
6.0	0.0058	6	11	16	21	25	35	44	171.88

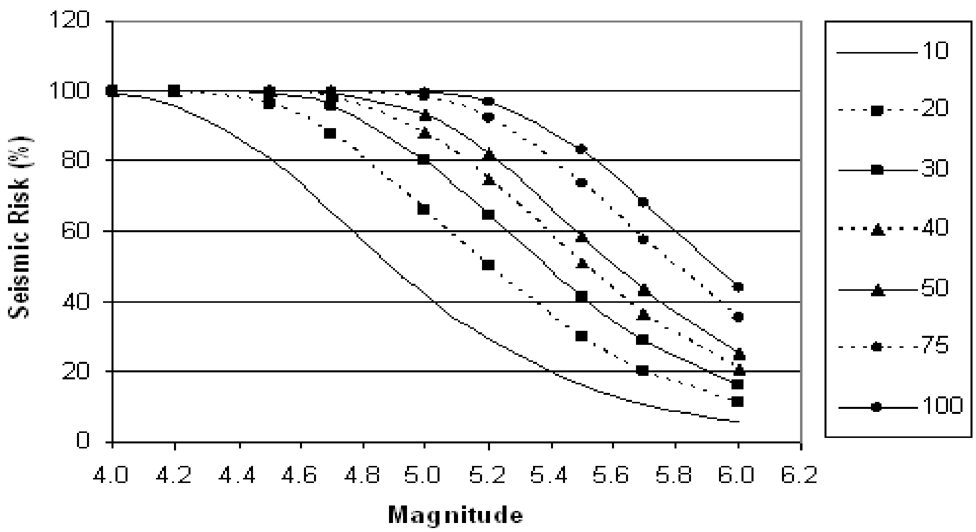

Figure 3. Seismic risk values using Poisson model.

occurrence number which is calculated according to earthquake magnitude and seismic parameters. Seismic risk values ($R(O_i)$) can be determined by the following equation:

$$R(O_i) = 1 - e^{-N(O_i)T^*} \quad (10)$$

Different from the other models, T* value in this model shows the future time portion to be used in calculating earthquake occurrence risk.

According to Poisson model, recurrence period is determined as years using the equation of

$$Q(O_i) = \frac{1}{N(O_i)} \quad (11)$$

Calculated seismic risk values and recurrence periods for the study area are shown in Table 7.

According to this model, recurrence period for 4.2 and 5.5 magnitude earthquake was found to be 3.13 and 56.50 years, respectively. Occurrence probability of 5.5 magnitude earthquake in 30 years was determined as 41%. Occurrence probability of earthquake in 100 years time period was calculated by using Poisson model and the results are shown in Figure 3.

Determination of seismic risk using Gumbel model

The Gumbel model defined by Gumbel (1958) depends on the biggest magnitude earthquake in one year. For this reason, the Gumbel model is referred to as “the biggest annuals” method. Distribution function of the method is expressed in the following equation:

$$G(M) = e^{-\alpha . e^{-\beta . M}} \quad (12)$$

where M shows earthquake magnitude, α and β shows

Table 8. Seismic risk values using Gumbel model.

M	R(M)	N(M)	Q(M)-Recurrence period
4.0	0.907	2.380	0.420
4.2	0.756	1.412	0.708
4.4	0.567	0.837	1.194
4.6	0.391	0.497	2.013
4.8	0.255	0.295	3.394
5.0	0.160	0.175	5.721
5.2	0.098	0.104	9.644
5.4	0.060	0.062	16.258
5.6	0.036	0.036	27.406
5.8	0.021	0.022	46.200
6.0	0.013	0.013	77.882

N(M), Annual average earthquake occurrence number, Q(M), recurrence periods, R(M), seismic risk value.

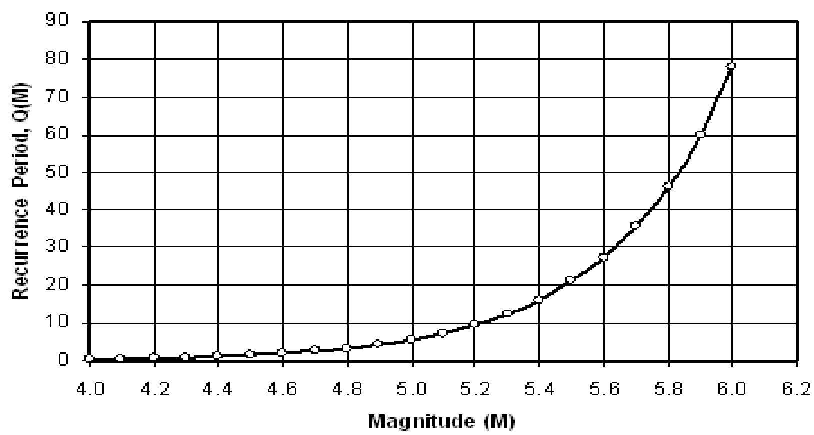

Figure 4. Return period values obtained from using Gumbel model.

regression coefficient depending on seismicity. α and β are calculated by using the following equations:

$$\alpha = 10^{a} \quad (13)$$

$$\beta = b * \ln(10) \quad (14)$$

a and b values were calculated as 4.912 and 1.134 respectively for the study area. Table 8 and Figure 4 illustrate the results regarding earthquake recurrence at 0.2 unit (earthquake magnitude) interval.

DISCUSSION

It is seen that the exponential distribution function model and Gumbel models give consistent values for this region. As for the annual occurrence number, the exponential distribution function model gives similar values to those of the other models especially for 5.2 and bigger magnitude earthquakes (Figure 5).When recurrence periods are investigated according to earthquake magnitude, exponential distribution function and Gumbel models give close values. These models produce smaller values of recurrence periods compared to the Poisson model (Figure 6).

CONCLUSIONS

It is known that statistical methods give almost correct results for the evaluation of seismic risk analysis when suitable data is used. For this reason, the Poisson and Gumbel models are the methods frequently used in the world.

In this study, magnitude-frequency relation was determined with Log N = 5.91 to 0.97 M formula using earthquake data from the investigated area. In addition, the validity of exponential distribution function model was examined and the consistency of results with other methods was investigated.

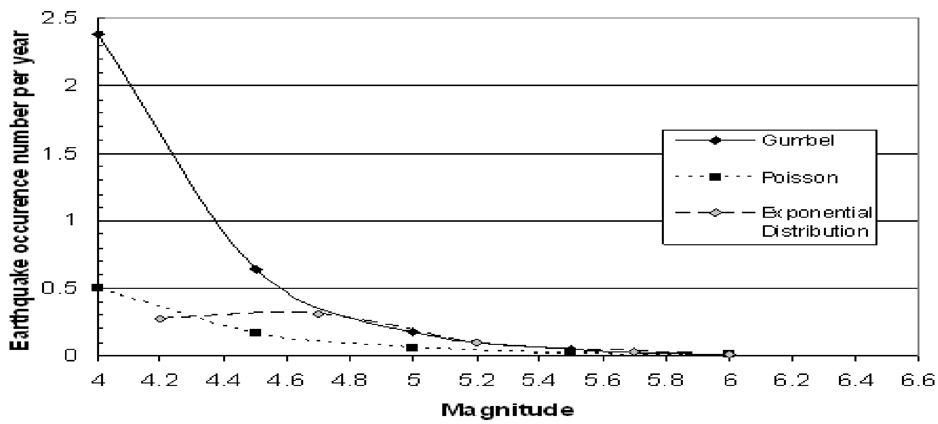

Figure 5. Earthquake occurrence numbers per year according to the models for the different magnitude values.

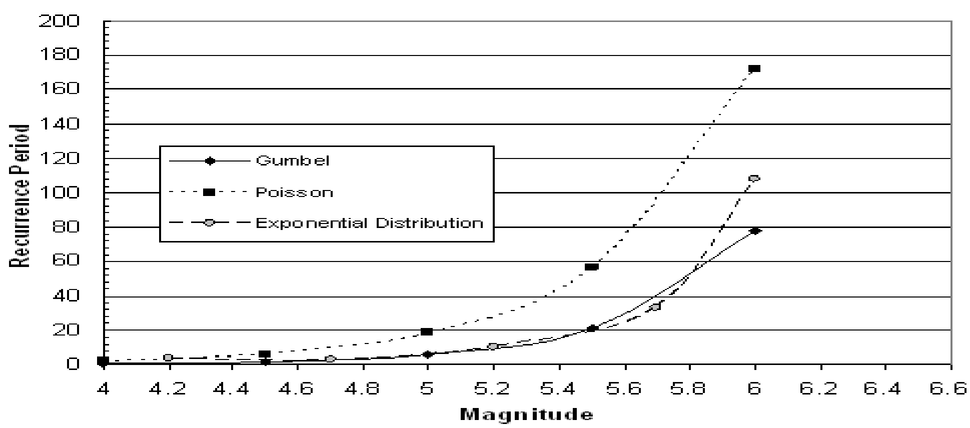

Figure 6. Recurrence periods graphic for different earthquake magnitudes for investigated models.

The recurrence period of 5.2 magnitude earthquake has been calculated as 10.39 years with the exponential distribution function model, and 28.99 years according to the Poisson models. On the contrary, the Gumbel model gave 9.64 years for the same magnitude. The exponential distribution model led to values similar to those computed by using Gumbel model. It was found out that the Poisson model resulted in the biggest values than others related to recurrence periods.

It should not be overlooked that the results obtained from this kind of study are directly connected to the distribution amount and total number of the data used. For this reason, active tectonic data of the region should be evaluated very carefully to identify regional limits.

The exponential distribution function model, as well as Poisson and Gumbel models, can be utilized for seismic risk analyses. However, instead of using only one method, the use of different distribution models in combination will be very important to evaluate and interpret the results accurately.

REFERENCES

Bağcı G (2000). İzmir ve Çevresinin Deprem Riski, Batı Anadolu'nun Depremselliği Sempozyumu Bildiriler Kitabı. (in Turkish). İzmir, pp. 239-248.

Campbell KW, Thenhaus, PC, Barnhard TP, Hampson DB (2002). Seismic Hazard Model for Loss Estimation and Risk Management in Taiwan. Soil Dyn. Earthquake Eng., 22: 743-754.

Çobanoğlu İ, Bozdağ Ş, Dinçer İ, Erol H (2006). A Statistical Approaches to Estimating the Recurrence of Earthquakes in the Eastern Mediterranean Region. Yerbilimleri Dergisi. İstanbul Ünv. Müh. Fak. Yayını. C., 19: 91-100.

Gumbel EJ (1958). Statics of Extremes. Columbia University Press. New York.

Gutenberg B, Richter CF (1954). Earthquake Magnitüde, Intensity, Energy and Acceleration, Bull. Seism. Soc. Am., 32(3): 162-191.

Hahn GJ, Shapiro SS (1994). Statistical Models in Engineering. Third Edition. John Wiley & Sons. New York.

Kiremidjian AS, Mortgat C, Shah H (1992). Stanford Seismic Hazard Analysis-STASHA. The John Blume Earthq. Engrg. Ctr., Stanford Univ. Stanford CA.

Öztemir F, Necioğlu A, Bağcı G (2000). Antakya ve Çevresinin Depremselliği ve Odak Mekanizması Çözümleri. Jeofizik. JFMO Yayını. Ankara (in Turkish), 14(1-2).

Ramachandran G (1980). Extreme values theory and earthquake

insurance. Trans. 21st Int. Congr. Actuaries, Switzerland, 1: 337-353.
Şimsek C, Ceylan A (2003). Leodikeia'da tespit edilen bir deprem ve Diocletianus'a ithaf edilen bir yazıt (Lykos Leodikeia'sı). Anadolu Arşivleri Dergisi, AÜDTC (in Turkish), 6.
Van Gelder PHAJM, Lungu D (1997). New statistical model for Vrancea earthquakes using prior information from earthquakes before 1900. ESREL'97. Lisbon, 2: 1535-1542.