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Determination of the characteristic parameters for the modified Bessel-Gauss beams

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Some characteristic parameters of the Bessel- Gaussian beam were determined theoretically by direct analytical calculations. Variation of the final beam radius (ω) with the starting beam radius (ω_o) was studied. The Rayleigh range of a Bessel-Gauss beam was calculated for each value of the minimum starting beam radius. The beam wavefront radius of curvature (*R*) was calculated as a function of starting beam radius at different distances from the source. A modified Bessel-Gauss beam was also considered. The beam intensity was calculated in the waist plane. The variation of the intensity near the center depends on whether the radius (*a)* equal, less than or greater than the starting beam radius (ω_o).

Key words: Bessel- Gauss beam, beam radius, waist plane, intensity.

INTRODUCTION

Bessel beam optics have attracted many researchers in the past few decades because of its interesting characteristics and applications. Bessel beams represent a class of diffraction free solutions to the Helmholtz equation, and have been studied extensively since the work of Durnin (1987) and Durnin et al. (1987) and recently by Lukin (2012, 2014); McGloin and Dholakia (2005); Mikutis et al. (2013); Turunen and Friberg (2010) and Trappe et al. (2005).

Durnin et al. (1987) have used an annular aperture in the focal plane of a lens to produce an approximate Bessel beam. Although successful in generating a Bessel beams, this method is highly inefficient since the aperture absorbs most of the incident radiation. This reduction in the available energy is unsuitable for applications where high intensities are needed. An axicon, or conical lens element is perhaps the most convenient and cost-effective way to generate Bessel beam (Indebetouw, 1989) (Figure 1).

When a Gaussian beam with a flat phase front is incident on the axicon, the focusing property of the axicon produces strong interference effects in the region where the deflected beam overlaps with itself (Laycock and Webster, 1990). The profile of the generated interference pattern remains invariant over the overlap region. The axial symmetry of the system results in the beam pattern having an amplitude profile that is approximated by a Bessel function of order zero (Bagini et al., 1996). Beyond the overlap region the on-axis beam amplitude falls rapidly to zero.

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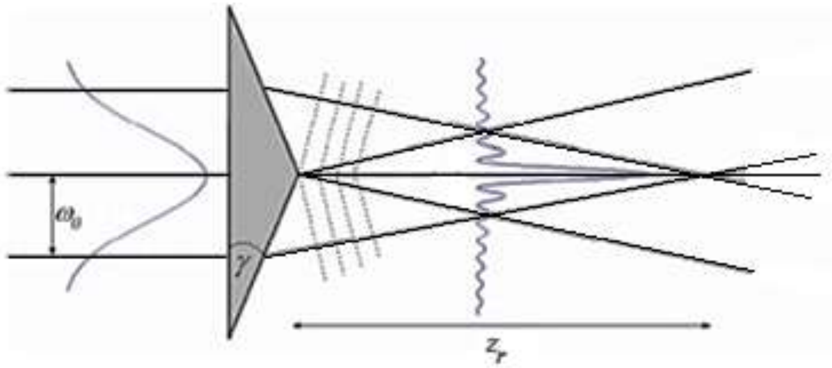

Figure 1. Propagation of a Bessel beam generated by axicon.

In this work, direct analytical calculations of some characteristic parameters of Bessel-Gauss beam are carried out. The calculations use the model proposed by Bagini et al. (1996). The intensity of the modified Bessel-Gauss beam at waist plane is also considered.

THEORETICAL BACKGROUND

The model used in this work is based on the situation that a Bessel beam is created by illuminating a Gaussian beam onto an axicon, Figure 1. In the plane z = 0 of a cylindrical reference frame (r, ϑ, z), Let us consider a field of the form (Bagini et al., 1996);

$$v_o(r,\vartheta,\alpha) = A(\alpha)\exp\left(-\frac{r^2}{\omega_o^2}\right) exp[i\beta r cos(\alpha-\vartheta)] \tag{1}$$

where ω_o is the starting beam radius and A is the amplitude. This equation represents a Gaussian beam whose mean wave vector has a projection β on the z = 0 plane, forming an angle α with respect to the x-axis, Figure 2.

The amplitude *A* is a function of α. When α varies, the wave vector describes a cone of semiaperture φ such that $sin\varphi = \beta/k$, where k is the wave number. The superposition of all Gaussian beams of Equation 1 form the Bessel-Gauss beams of order *n.* After propagating a distance *z* from the axicon, the radius of the beam $\omega(z)$ and the wavefront radius of curvature $R(z)$ are given by Bagini et al. (1996):

$$\omega^2(z) = \omega_o^2\left[1+\left(\frac{z}{Lcos\varphi}\right)^2\right] \tag{2}$$

$$R(z) = \frac{z}{cos\varphi} + \frac{L^2 cos\varphi}{z} \tag{3}$$

Where

$$L = \frac{\pi}{\lambda}\omega_o^2 \tag{4}$$

and ω_o is the starting beam radius at plane of *z* = 0. Let us consider a superposition of Gaussian beams, having mean wave vectors parallel to the longitudinal z direction, but whose centers are placed on a circumference of radius *a* around the z axis. In this case we obtain a superposition of beams that we call modified Bessel-Gauss beams of order *n*.

The modified Bessel-Gauss beam of zero order may show a central region of uniform intensity in the waist plane. The intensity distribution of the modified Bessel-Gauss beam can be obtained by Bagini et al. (1996)

$$F(r) \approx exp\left(-2\frac{a^2}{\omega_o^2}\right)\left[1+\frac{2r^2}{\omega_o^2}\left(\frac{a^2}{\omega_o^2}-1\right)\right] \tag{5}$$

If we consider the zero order term, the intensity as a function of *r* and *z* can be formulated as;

$$F(r,z) = \left(\frac{\omega_o}{\omega(z)}\right)^2 exp\left(-2\frac{a^2}{\omega^2(z)}\right)[1+2r^2\Delta(z)] \tag{6}$$

where

$$\Delta(z) = \left[\frac{1}{\omega^2(z)}\left(\frac{a^2}{\omega^2(z)}-1\right)\right] - \left(\frac{ka}{2R(z)}\right)^2 \tag{7}$$

It is seen from these equations that the intensity may

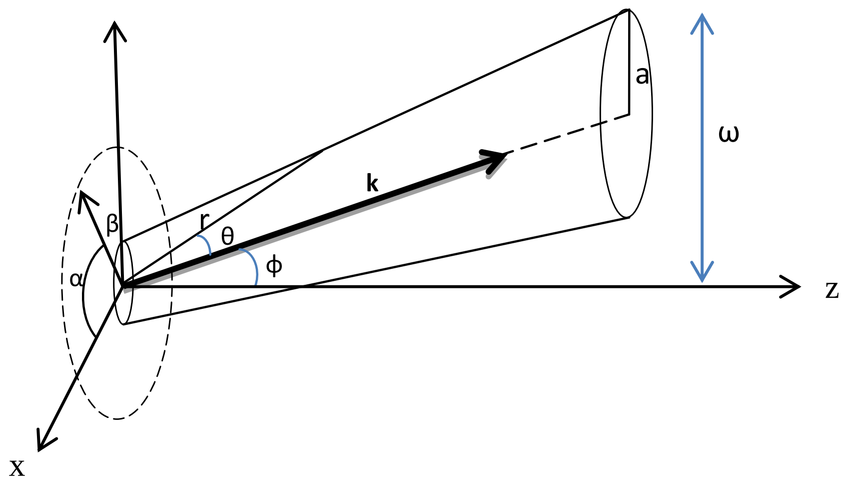

Figure 2. Illustration of the Bessel-Gauss beams.

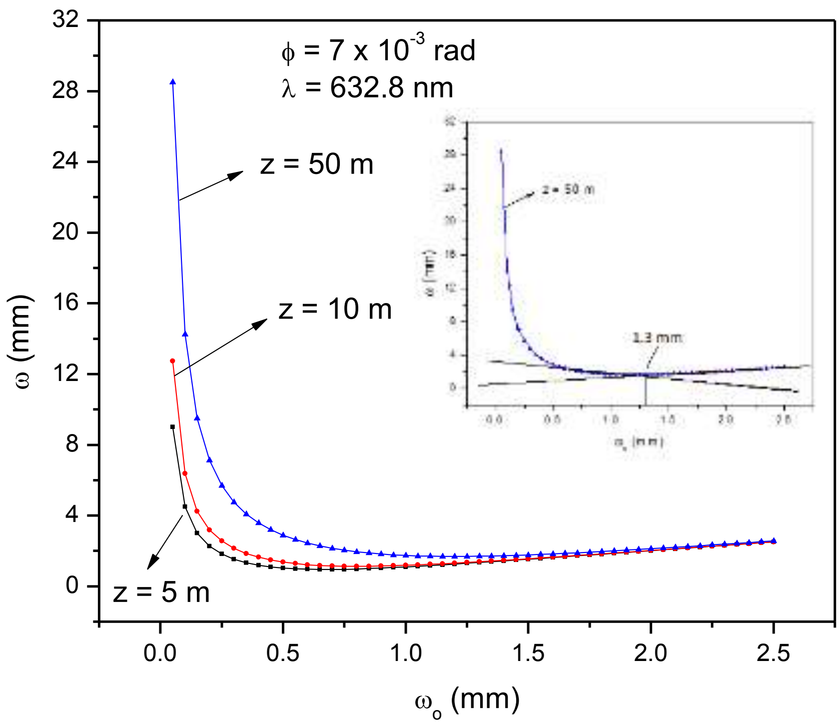

Figure 3. Beam radius as a function of starting beam radius at distances 5, 10 and 50 m from the axicon calculated from Equations 2 and 4.

show a maximum, a plateau or a minimum near $r = 0$ depending on whether Δ is negative, null or positive.

RESULTS AND DISCUSSION

Direct calculations were carried out to determine some characteristic parameters of the Bessel-Gaussian beam. The final beam radius (ω) was calculated as a function of the starting beam radius (ω_o) at a fixed distance (z) using Equations 2 and 4. Figure 3 shows the variation of the final beam radius with the starting beam radius at distances 5, 10 and 50 m from the axicon. From the figure one can notice that the beam radius reaches a

Table 1. The optimum values of the starting beam radius and the Rayleigh range at different distances.

z (m)	5	10	50
$(\omega_o)_{opt}$ (mm)	0.56	0.7	1.3
z_r (mm)	10.98	13.73	25.49

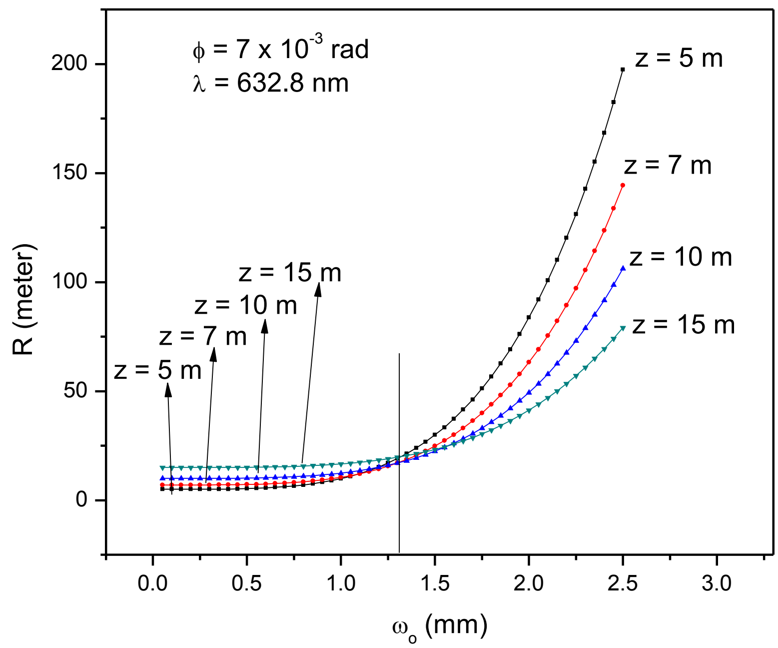

Figure 4. Wavefront radius of curvature versus starting beam radius calculated using equations 3 and 4 at distances 5, 7, 10 and 15 m.

minimum value at a certain starting beam radius. The initial beam radius corresponding to minimum beam radius over the distance (z) is known as optimum starting beam radius $(\omega_o)_{opt}$. For example at Z = 50 m, the optimum starting beam radius $(\omega_o)_{opt} \approx 1.3$ mm and this may represent the best combination of minimum beam diameter and minimum beam spread over a distance 50 m, see inset of Figure 3.

The Rayleigh range of a Bessel-Gauss beam can then be calculated for each value of the optimum ω_o from the relation (Duocastella, and Arnold, 2012):

$$z_r = \frac{(\omega_o)_{opt}}{(n-1)\gamma} \quad (8)$$

where n is the refractive index of the axicon and γ is the opening angle of the axicon. The Rayleigh range represents the largest distance after the axicon in which Bessel-Gauss beam will propagate where the central maximum will not exhibit diffractive spreading. The optimum ω_o values as well as the Rayleigh range at different distances z are listed in Table 1 (taking γ = 0.1 rad and n = 1.51). Figure 4 represents a plot of the wavefront radius of curvature (R) versus starting beam radius at different distances using equations 3 and 4. It is noticed that R increases with the increase of ω_o.

The change of the wavefront radius of curvature with the distance z is somehow irregular. Before approximately $\omega_o = 1.3\ mm$, R increases as the distance is increased while after $\omega_o = 1.3\ mm$ the radius of curvature decreases with the increase of z. Therefore $\omega_o = 1.3\ mm$ may represent a suitable value of the starting beam radius at which the beam radius of curvature is being a constant regardless of the distance apart the beam source.

Figure 5 demonstrates the variation of the beam radius

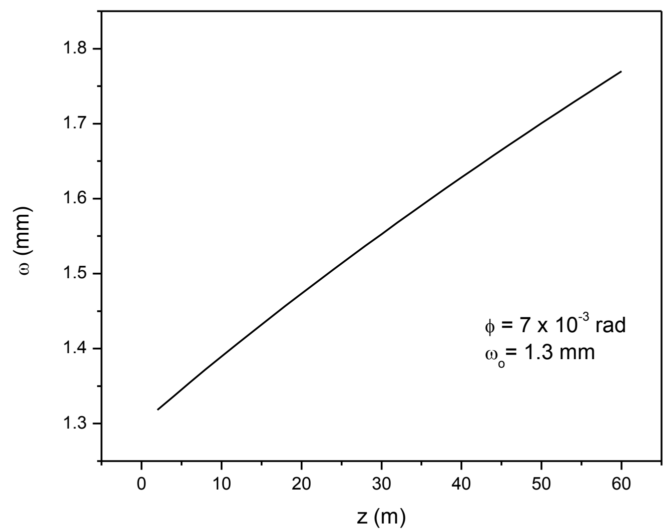

Figure 5. Variation of the beam radius with the distance at fixed value of the starting beam radius.

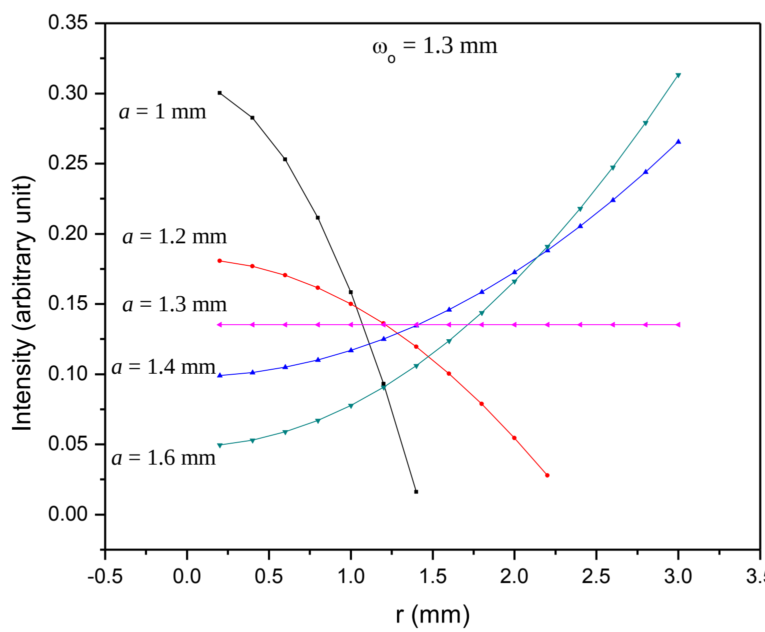

Figure 6. Intensity distribution of the modified beam of zero order on the waist plane as a function of r, for ω_o = 1.3 mm and different values of a. The calculations were carried out using Equations 6 and 7.

with the distance until 60 m from the axicon at fixed value of the starting beam radius. The relationship between the two parameters is linear indicating that as the beam travels apart from the source, it will spread out.

The intensity of the modified Bessel-Gauss beam of zero order is considered. Calculation of the beam intensity in the waist plane using Equations 6 and 7 was carried out. Figure 6 represents the beam intensity as a function of (r) at different values of (a). The starting beam radius (ω_o) was kept at 1.3 mm. From the figure one can deduce that if the radius a equals ω_o, the intensity near the axis is constant. If ω_o is greater than a, the intensity will decrease by raising of r while the intensity will increase when ω_o is less than *a*. This can be explained by considering that when the radius *a* is increased, the maxima of the constituent Gaussian beams recede from one another so that a central dip appears.

Conclusion

The parameters characterizing the Bessel-Gaussian beam were determined. It was found that the beam radius reaches a minimum value at a certain starting beam radius at any distance. The change of the wavefront radius of curvature with the distance *z* is somehow irregular. It was concluded that starting beam radius equals 1.3 mm represents the suitable value of the starting beam radius at which the beam radius of curvature is being a constant regardless of the distance from the beam source. Calculation of the beam intensity of the modified Bessel-Gauss beam in the waist plane was proceeded. The results are dependent on weither *a* is smaller, equal or greater than ω_o.

Conflict of interests

The authors have not declared any conflict of interests.

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