

*Full Length Research Paper*

# A stochastic facility layout model in cellular manufacturing systems

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**Facility layout which is the arrangement of facilities in the shop-floor has a great impact on the performance of manufacturing systems, while in volatile manufacturing systems; any kinds of change can degrade the performance of the system and leads to inefficiency of the layout. In this paper, a new mathematical model for layout problems in cellular manufacturing systems has been proposed that considers the stochastic nature of demand. The model minimizes the total material handling cost (both inter-cell and intra-cell material handling cost), and it is solved by two solution approaches, Lingo optimization software, and an enumeration method. The obtained results show the validity of the mathematical model.**

**Key words:** Cellular manufacturing, facility layout, stochastic model, enumeration, Lingo.

## INTRODUCTION

Facility layout problem (FLP) is the placing and distributing of the facilities in the plant area in order to maximize efficiency of service and production (Ye and Zhou, 2007). It is how to locate facilities in the shop-floor to have the most efficient layout (Krishnan et al., 2009a). An effective placement of facilities can decrease 10 to 30% of the material handling cost (Tompkins et al., 2003). Furthermore, it decreases manufacturing lead time and work-in-process and increases through put rate (Raman et al., 2009). Facility layout problem is a strategic decision and any modifications or re-arrangements in existing layout results in a large amount of expenses (Singh and Singh, 2010). Hence, in today's volatile environment of manufacturing system that high competition in the global market drastically changes the demand of products; the layout of facilities must be suitable for different demand scenarios (Aiello and Enea, 2001). Stochastic FLP is an approach that can aim to arrange the facilities in the shop-floor in such a way that

the efficiency of the layout does not intensely degrade over the time by changes in the production environment (Kulturel-Konak, 2007).

Cellular manufacturing system (CMS) as a successful implementation of group technology (Luo and Tang, 2009), incorporates the efficiency of flow lines and the flexibility of job shops (Wang et al., 2009). In CMS, parts based on their similarities in their design and manufacturing requirements are grouped into part families and dedicated to a group of machines named "machine cell" to be produced (Bhatnagar and Saddikuti, 2010). Implementation of CMS has some major advantages such as, reduction in throughput time and setup time, lowering the level of work-in-process (WIP) and lot sizes, improvement in the quality of products and enhancement in the flexibility of the system (Deljoo et al., 2010; Tavakkoli-Moghaddam et al., 2010).

In order to implement a CMS, several decisions should be made; the cell formation problem, the layout of machine cells in the shop floor and the layout of facilities in each of the machine cells (Sangwan and Kodali, 2009). The process of determining part families and machine cells in CMS is referred as cell formation (Liu et al., 2010). Cell formation (CF) is well a studied area and

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comprehensive summaries and taxonomies devoted to CF problem is extensive and abundant in this field of study (Sarker and Xu, 1998; Yin and Yasuda, 2005; Yin and Yasuda, 2006; Papaioannou and Wilson, 2010), but the layout problem has not absorbed the attention of researchers as much as cell formation (Ahi et al., 2009; Ariafar et al., 2011; Ariafar and Ismail, 2009).

The aim of this paper is to propose a non-linear mathematical model for layout problems in a cellular manufacturing system under stochastic demand, when the distribution of demand is a uniform distribution. The proposed model minimizes the total material handling cost, (both inter-cell and intra-cell material handling cost). The mathematical model is solved by two solution approach; Lingo 12.0 optimization software, and an enumeration method.

## BACKGROUND AND LITERATURE REVIEW

Uncertainty in production mix and demand declines the performance of CMS (Saad, 2003). Stochastic facility layout problem is an approach that reduces the level of deterioration of performance in volatile manufacturing systems (Kulturel-Konak, 2007). Most of the existing stochastic facility layout models have been formulated as quadratic assignment problems (QAP). Li and Smith (1995) addressed the problem of determining the layout and location of facilities under stochastic congestion in a traffic circulation system. They proposed a heuristic algorithm to solve their problem. Although their algorithm does not guarantee to find the optimal solution for the problem, it provides acceptable sub-optimal solutions in a reasonable time. Aiello and Enea (2001) proposed a methodology based on fuzzy theory for facility layout problem. Their proposed approach defines the demand of market as fuzzy numbers and minimizes the material handling cost. Castillo and Peters (2002) integrated the unit load and material handling in a layout problem. They proposed a non-linear mixed integer mathematical model for the problem and solved the model using a simulated annealing algorithm.

Irohara et al. (2004) presented a mathematical model for layout and buffer space allocation, based on approximate Markov chain. They called their model as the stochastic facility layout problem (SFLP). In their model, instead of minimizing the material handling cost, the lead-time is minimized. They used a genetic algorithm to solve their model optimally. Kulturel-Konak et al. (2004) considered uncertainty and routing flexibility to formulate an unequal area facility layout model. They used a simulation approach to estimate the expected value and variances of material flow. A tabu search algorithm was applied to solve the model. Enea et al. (2005) took into account the uncertainty of product demand in a capacitated production rate of departments to formulate their facility layout problem. They used fuzzy

theory to handle the uncertainty in their mathematical model. To solve the model, genetic algorithm was applied. Tavakkoli-Moghaddam et al. (2007) proposed a mathematical model for layout problem in cellular manufacturing systems under variable demand. The objective of their model was to minimize inter-cell and intra-cell material handling cost. To verify the model, some test problems were solved by lingo optimization software.

In order to take into account the uncertainty in multiple demand scenarios, Krishnan et al. (2008) proposed a mathematical model for layout problem that minimizes the maximum loss in material handling cost. Their model is applicable in both single and multiple periods. They solved the model using a genetic algorithm. Irohara and Yamashita (2009) developed a new solution methodology which was the expansion of the space partitioning method for their previous proposed SFLP model. The objective function of their model was the minimization of lead-time. Krishnan et al. (2009b) proposed a facility layout model that deals with the uncertainty in the products demand. They developed two approaches for the problem. The first one determined the risk associated with each layout while the second procedure minimized the risk of designed layout. Jithavech and Krishnan (2010) in order to take into account the uncertainty in production demand, proposed a simulation based approach to predict the uncertainty of a designed layout. They used three scenarios to validate their methodology.

## PROBLEM FORMULATION

In this section, a mathematical model for layout problem in a cellular manufacturing system is proposed that considers the demand has a uniform stochastic distribution. The objective of the model is to minimize the total material handling cost (both inter-cell and intra-cell material handling cost). In this study, it is assumed that the cell formation stage has been done beforehand, and the layout of facilities and machine cells are U-shaped. In addition, the demands for products are independent.

### Notations

#### Indices:

P: Index for products  
 R: Index for processes  
 i, k: Indices for locations  
 j, l: Indices for machines  
 m, n: Indices for machine cells

#### Parameters:

$n_m$ : Number of machines and also number of available locations in the system.

**Table 1.** Machine grouping data.

Cell	Number of facilities	Facilities
1	3	1, 2, 3
2	2	4, 5

$n_c$ : Number of machine cells in the manufacturing system  
 $n_p$ : Number of products  
 $R_p$ : Number of processes in the Pth product  
 $U_p$ : Upper bound of demand for the Pth product in the planning horizon  
 $L_p$ : Lower bound of demand for the Pth product in the planning horizon  
 $D_p$ : Demand of the Pth product in the planning horizon  
 $\beta$ : The cumulative probability of occurrence for the demand scenarios  
 $C_{jl}$ : Unit distance cost between machine j and machine l  
 $MF_{jl}$ : Material flow between machine j and machine l  
 $Dist_{ik}$ : Distance between location i and location k  
 $NMC(C_m)$ : The number of machines in the machine cell  $C_m$

**Variables:**

$$F_{PRj} = \begin{cases} 1 & \text{If product P is produced in its} \\ & \text{Rth process on machine j} \\ 0 & \text{Otherwise} \end{cases}$$

$$X_{ij} = \begin{cases} 1 & \text{If machine i is assigned to} \\ & \text{machine location j} \\ 0 & \text{Otherwise} \end{cases}$$

$$Y_{im} = \begin{cases} 1 & \text{If machine i belongs to} \\ & \text{machine cell } C_m \\ 0 & \text{Otherwise} \end{cases}$$

**Mathematical model**

Assume that the demand for each of the products in the planning horizon has a uniform distribution. The amount of demand for each product will be calculated as follows:

$$D_p = L_p(1 - \beta) + U_p\beta \tag{1}$$

Then material flow for the planning horizon is

$$MF_{jl} = \sum_{P=1}^{n_p} \sum_{R=1}^{R_p} D_p * F_{PRj} * F_{P(R+1)l} \tag{2}$$

The mathematical model for facility layout problem in cellular manufacturing systems can be formulated as follows:

$$Min \sum_{i=1}^{n_m} \sum_{j=1}^{n_m} \sum_{k=1}^{n_m} \sum_{l=1}^{n_m} \sum_{m=1}^{n_c} \sum_{n=1}^{n_c} MF_{jl} C_{jl} Dist_{ik} X_{ij} X_{kl} Y_{im} Y_{kn} \tag{3}$$

$$\sum_{i=1}^{n_m} X_{ij} = 1 \quad j = 1, \dots, n_m \tag{4}$$

$$\sum_{j=1}^{n_m} X_{ij} = 1 \quad i = 1, \dots, n_m \tag{5}$$

$$\sum_{m=1}^{n_c} Y_{im} = 1 \quad i = 1, \dots, n_m \tag{6}$$

$$\sum_{i=1}^{n_m} Y_{im} = NMC(C_m) \quad m = 1, \dots, n_c \tag{7}$$

The objective function (3) minimizes the material handling cost on the shop floor. Constraints (4) and (5) ensure that each machine is only assigned to one facility location, and each machine location is assigned to one machine. Then, constraints (6) and (7) make sure that each machine is only assigned to one machine cell, and to each cell assigns the same number of machines belongs to that machine cell.

**ILLUSTRATION**

In this section, the proposed mathematical model is solved for several cases by two solution approaches, Lingo 12.0 optimization software, and an enumeration method on a T2600 2.16 GHz with 2 GB RAM. All the cases have been generated randomly and due to the complexity of the model, only small size cases can be solved by these solution methods in a reasonable time. In the following the data set for each of the cases has been provided.

**Data sets**

In order to examine the issue of stochastic demand on the layout of facilities, the following two cases problems have been generated randomly. In the first case, four products ( $n_p = 4$ ) are produced on five machines ( $n_m = 5$ ). As, can be seen from Table 1, the system includes two machines cells ( $n_c = 2$ ); three machines (1,2,3) for the first machine cell and two machines (4,5) for the second one. The demand and production sequence for each of the four products is given in Table 2.

Let in the second case, the system includes seven machines ( $n_m=7$ ). As it can be seen from Table 3, the machines have been distributed in three machine cells ( $n_c = 3$ ); three machines in the first machine cell (1,2,3), two machines for the second machine cell (4,5) and two machines (6,7) in the third one. The production sequence of these six products ( $n_p = 6$ ) which are produced in the system and the demand for each of them are shown in Table 4.

**RESULTS AND DISCUSSION**

In this section, both cases have been solved by the

**Table 2.** Products' demand and process sequences.

Product	Uniform distribution		Production sequence
	Lower bound	Upper bound	
1	20	240	2- 3- 1- 5
2	100	180	3- 2- 4- 5
3	30	40	3- 2- 1
4	20	40	5- 4

**Table 3.** Machine grouping data.

Cell	Number of facilities	Facilities
1	3	1, 2, 3
2	2	4, 5
3	2	6, 7

**Table 4.** Products' demand and process sequences.

Product	Uniform distribution		Production sequence
	Lower bound	Upper bound	
1	20	240	3- 1- 2- 4
2	40	100	1- 3- 5- 4
3	30	60	1- 2- 3
4	20	160	4- 5- 7
5	40	80	2- 3- 6- 7
6	20	60	3- 2- 1- 7

**Table 5.** Computational results for the cases.

Test problem	$\beta$	Inter-cell layout		Intra-cell layout		Cost (\$)
1: $(n_P = 4), (n_m = 5)$ $(n_c = 2)$	0.2	Cells:	1- 2	Cell 1: Cell 2:	s1- 3- 2 4- 5	948
	0.4	Cells:	1- 2	Cell 1: Cell 2:	3- 2- 1 4- 5	1224
	0.6	Cells	1- 2	Cell 1: Cell 2:	3- 2- 1 5- 4	1484
2: $(n_P = 6), (n_m = 7)$ $(n_c = 3)$	0.4	Cells	1- 2- 3	Cell 1 Cell 2 Cell 3	1- 2- 3 4- 5 7- 6	1684
	0.8	Cells	1- 2- 3	Cell 1	3- 1- 2	2554
				Cell 2 Cell 3	4- 5 7- 6	

solution methods for different demand scenarios. Table 5 includes the obtained results in terms of the inter-cell layout, intra-cell layout and material handling cost.

The results in Table 5 show that any change in the demand scenario by varying in the value of  $\beta$  (cumulative probability of demand), leads to a different arrangement for facilities. For example, in the first case, when  $\beta$  equals to 0.2, the arrangement of facilities in the first machine

cell is as follows: (1-3-2), but when the value of  $\beta$  is replaced by 0.4; the arrangement changes to: (3-2-1). In both demonstrated cases; fluctuation in product demand has caused some changes in material flow between of the facilities, which has resulted in different layout of facilities. The results are evidence to reveal the sensitivity of the layout problems to variation in demand and indicate that uncertainty in the product demand can cause

inefficiency in the layout of facilities.

## Conclusion

In this paper, a stochastic nonlinear mixed integer mathematical model was developed that determines the layout of facilities in cellular manufacturing systems. The model based on the cumulative probability for occurrence of demand scenarios, arranges the machine cells within the shop-floor and the machines within each of the machine cells to minimize the material handling cost. To validate the model, several cases were solved by two solution approaches; Lingo 12.0 optimization software, and an enumeration method. Solving the model for different demand scenarios, which leads to different layouts, shows that layout problems are sensitive to any change in the demand of products.

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